

Economics 520, Fall 2008
Homework 2

Due Tuesday, September 16 at start of class

1. Two dice are thrown and three events are defined as follows: A means “odd face with first die”; B means “odd face with second die”; and C means “odd sum” (one face is odd, one face is even). If each of the 36 sample points has probability $1/36$, then

- (a) Show that the events are pairwise independent.
- (b) Show that the events are not jointly independent.

2. Suppose a breathalyzer has 5% false positives and 8% false negatives. That is, only 5% of the time will it indicate that a person is drunk when he is actually sober and 8% of the time will it indicate that a person is sober when the person is in fact drunk. Using this test, the police spot test a population of drivers, 99% of whom are sober. What is the chance that a person, who tests as drunk, is actually sober?

3. Suppose X is a continuous random variable with pdf $f(x)$ and CDF $F(x)$. For a fixed number x_0 such that $F(x_0) < 1$, define

$$g(x) = \begin{cases} f(x)/(1 - F(x_0)), & x \geq x_0 \\ 0, & x < x_0 \end{cases}$$

- (a) Show that $g(x)$ is a valid pdf.
- (b) Describe in words the random variable with pdf $g(x)$.

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5. Suppose that the random variable X has an exponential distribution with pdf $f_X(x) = \exp(-x)$, $x > 0$, and 0 elsewhere.

- (a) Find the pdf for $Y = 1/X$.
- (b) Find the pdf for $Y = \ln(X)$.
- (c) Find the pdf for $Y = 1 - F_X(X)$.

6. Let $f(x) = 1/3$ for $-1 < x < 2$ and zero elsewhere be the pdf for a random variable X . Find the pdf and distribution function for the random variable $Y = X^2$.

7. R Exercise: write R function definitions for the following functions. Print out the function definitions and check them by calculating some specific values. Also, use the `curve` function in R to plot these functions over the unit interval $[0, 1]$.

- (a) $f(x) = 1 + 2x - x^2$.
- (b) $f(x) = \log(1 + x)$.
- (c) $f(x) = e^{-x}$.