

Economics 520, Fall 2008
Homework 11

Due Tuesday, December 9 at beginning of class

1. Continue with the setup of Question 4 in Homework 10. What is the large sample distribution of the MLE? (Give as precise a characterization of the distribution as possible.)
2. Let X_1, X_2, \dots, X_N represent a random sample from a Poisson distribution with parameter λ .
 - (a) Find the maximum likelihood estimator for λ and its asymptotic distribution.
 - (b) Suppose we are interested in the probability of a count of zero, $\theta = Pr(X = 0) = \exp(-\lambda)$. Find the maximum likelihood estimator for θ and its asymptotic distribution.
3. Let X_1, \dots, X_{10} be a random sample of size 10 from a normal distribution with mean zero (known), and variance σ^2 . We want to test the null hypothesis $H_0 : \sigma^2 = 1$ against the alternative hypothesis $H_a : \sigma^2 > 1$. Consider a test that rejects the null hypothesis if $\sum_i x_i^2 > C$, where C is a constant. Find the constant C so that the test has significance level .05. (Hint: use a table of the chi-square distribution.)
4. Suppose we want to test if a coin is fair (that is, the probability of heads is $p = .5$). We toss the coin 1000 times, and record the number of heads. Let T denote the number of heads divided by 1000. Consider a test that rejects the null hypothesis that $p = .5$ if $T > c$.
 - (a) Write down a formula for $P(T > c | p = .5)$. Provide an outline of how to write an R program that would calculate this number for any value of c . You do not need to give exact code, just explain the steps in enough detail so that an experienced R programmer could implement it.
 - (b) Give a large-sample approximate formula for $P(T > c | p = .5)$ based on the central limit theorem.
 - (c) Using the large-sample approximation, calculate the appropriate value c for a test at the 0.05 percent level. You can use the fact that if $Z \sim N(0, 1)$, then $P(Z > 1.645) = 0.05$. Would your test reject the null hypothesis if we observed 560 heads and 440 tails?