Economics 520, Fall 2008
Homework 10
Due Tuesday, December 2 at beginning of class

1. Let $Y_1, Y_2, \ldots, Y_N$ be independent random variables with identical pdf $f_Y(y,\theta)$. For each of the following densities find a minimal sufficient statistic for θ. (Note: remember that a one–dimensional sufficient statistic does not always exist).

   (a) $f_Y(y,\theta) = 1/(1-\theta)$, $\theta < y < 1$, 0 elsewhere

   (b) $f_Y(y,\theta) = \exp(-y+\theta)$, $\theta < y < \infty$, 0 elsewhere.

2. Check whether the following distributions are members of the exponential family, and find sufficient statistics for random samples of size N if they are.

   (a) A Gamma distribution with parameters $\alpha = \theta$ and $\beta = 1$.

   (b) A Gamma distribution with parameters $\alpha = 1$ and $\beta = \theta$.

   (c) A Chi–square distribution with θ degrees of freedom.

3. Let X and Y have the following joint probability density:

$$f_{X,Y}(x,y) = \frac{1}{2\mu^2} \exp\left(-\frac{x}{\mu} - \frac{y}{2\mu}\right).$$

   (a) Are X and Y independent? What are their marginal densities?

   (b) Find the Cramer-Rao bound for unbiased estimators of μ.

   (c) What is the MLE? Is it minimum variance unbiased?

4. Let $X_1, X_2, \ldots, X_N$ be independent random variables, all with a binomial $\mathcal{B}(2,p)$ distribution.

   (a) Find the maximum likelihood estimator for p.

   (b) Is the maximum likelihood estimator the minimum variance unbiased estimator?

   (c) Let $N = 100$, $\sum_{i=1}^{N} x_i = 40$, and $\sum_{i=1}^{N} x_i^2 = 48$. Calculate the MLE.