Economics 520, Fall 2007
Homework 7
Due Tuesday, October 30 at beginning of class

1. Consider the Example on P.5 of Lecture Note 9, of a sequence of random variables that does not converge almost surely. Construct a new sequence of random variables that have the same *marginal* distributions as the example, but which converges almost surely, and show explicitly what the almost sure limit is. (The marginal distribution of X_1 should be the same in both cases, the marginal distribution of X_2 should be the same in both cases, etc.)

2. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables with finite mean and variance, and let $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Let $Z_n = 1/(\bar{X}_n)$. Find the limiting distribution of Z_n.

3. Suppose $Y_1, \ldots, Y_n$ are IID discrete random variables with

$$\begin{aligned} P(Y_i = 0) &= \theta_0, \\ P(Y_i = 1) &= \theta_1, \\ P(Y_i = 2) &= \theta_2, \end{aligned}$$

   where the parameter vector $\theta = (\theta_0, \theta_1, \theta_2)$ satisfies: $\theta_j \geq 0$ and $\sum_{j=0}^{2} \theta_j = 1$.

   (a) Calculate $E[Y_i]$ and $E[Y_i^2]$, and use the results to derive a method of moments estimator for the parameters (θ_1, θ_2).

   (b) Show that the maximum likelihood estimator for $\theta = (\theta_0, \theta_1, \theta_2)$ is

$$\begin{aligned} \hat{\theta}_0 &= \frac{1}{n}\sum_i 1(Y_i = 0), \\ \hat{\theta}_1 &= \frac{1}{n}\sum_i 1(Y_i = 1), \\ \hat{\theta}_2 &= \frac{1}{n}\sum_i 1(Y_i = 2). \end{aligned}$$

   (Hint: be sure to impose the constraint that $\theta_0 + \theta_1 + \theta_2 = 1$.)