Economics 520, Fall 2007
Homework 6
Due Tuesday, October 23 at beginning of class

1. Suppose that U, V and W are independent random variables (not necessarily identically distributed) with strictly positive variance. Let $X = U + V$, $Y = W + V$, and $Z = W - U$. Show that X and Y are positively correlated, Y and Z are positively correlated, but X and Z are negatively correlated.

2. Consider the following sequence of random variables $W_1, W_2, \ldots$ with

$$W_n = \begin{cases} \frac{1}{n} & \text{with probability } .5 \\ 0 & \text{with probability } .5 \end{cases}$$

   Using the definition of convergence in probability, show that $W_n \xrightarrow{p} 0$.

3. Suppose that ϵ_t (for $t = 0, 1, \ldots$) is a white noise process (not necessarily Gaussian). For $t = 1, 2, \ldots$, let

$$Y_t = \alpha + \epsilon_t + \frac{1}{2}\epsilon_{t-1}.$$

   Let $\bar{Y}_T = \frac{1}{T}\sum_{t=1}^{T} Y_t$. Show that $\bar{Y}_T$ converges to α in probability as $T \to \infty$.

4. Continue the setup of the previous question, but now assume that ϵ_t is IID Gaussian white noise. Derive the (marginal) distribution of

$$\sqrt{T}(\bar{Y}_T - \alpha).$$

   Does $\sqrt{T}(\bar{Y}_T - \alpha)$ converge in distribution?

5. Show that if the sequence $X_1, X_2, \ldots$ converges to a constant θ in probability, then it converges to θ in distribution (that is, it converges in distribution to a degenerate random variable equal to θ with probability 1).

6. R Exercise: Suppose that X_i are IID Uniform on the unit interval, for $i = 1, \ldots, 1000$. Simulate the X_i and the running average:

$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i,$$

   for $n = 1, \ldots, 1000$. (Hint: try using the `cumsum` function in R.) Plot X_n against n, using `plot(x, type="l")` to get a line plot.