Economics 520, Fall 2007
Homework 11
Due Tuesday, December 4 at beginning of class

1. CB 7.40

2. Let $X_1, X_2, \ldots, X_N$ be independent random variables, all with a binomial $\mathcal{B}(2, p)$ distribution.

   (a) Find the maximum likelihood estimator for p.

   (b) Is the maximum likelihood estimator the minimum variance unbiased estimator?

   (c) Let $N = 100$, $\sum_{i=1}^{N} x_i = 40$, and $\sum_{i=1}^{N} x_i^2 = 48$. Find the MLE and its large sample variance.

3. Let $Y_1, Y_2, \ldots, Y_N$ be a random sample from a distribution with probability density function

$$f_Y(y, \theta) = 2y/\theta^2 \quad 0 < y < \theta, \quad 0 \text{ elsewhere}$$

   (a) Show that $W = 3\bar{Y}/2$ is an unbiased estimator for θ.

   (b) Find the Cramér–Rao bound.

   (c) Show that the variance of W is less than the Cramér–Rao bound.

4. Let the random variable X have probability density function $f_X(x; \theta) = (1/\theta)\exp(-x/\theta)$ for $x > 0$. Consider the simple hypothesis $H_0 : \ \theta = 2$ and the alternative hypothesis $H_a : \ \theta = 4$. Let X_1, X_2 denote a random sample of size two from this distribution. Show that the best test of H_0 against H_a may be carried out by use of the statistic $X_1 + X_2$. Find the optimal critical region for $\alpha = 0.1$.