Economics 520, Fall 2007
Homework 10
Due Tuesday, November 20 at beginning of class

1. X_1 and X_2 are independent normally distributed random variables with mean μ and variances σ_1^2 and σ_2^2 respectively. The variances are known and we are interested in estimating the means. Consider estimators of the form $W_{\lambda,\delta} = \lambda X_1 + \delta X_2$. Find the minimum variance unbiased estimator in this class of estimators.

2. X has a binomial distribution with parameters $N = 1$ and $p = 1/2$. Y, which is independent of X, has a normal distribution with mean μ and variance 1. Consider the estimator for μ of the form $W_1 = Y + 2X - 1$.

   (a) Is W_1 unbiased?

   (b) What is the variance of W_1?

   (c) Consider the estimator $W_2 = E[W_1|Y]$. Is W_2 unbiased? How does its variance compare to that of W_1?

3. Let $Y_1, Y_2, \ldots, Y_N$ be independent random variables with identical pdf $f_Y(y, \theta)$. For each of the following densities find a minimal sufficient statistic for θ. (Note: remember that a one–dimensional sufficient statistic does not always exist).

   (a) $f_Y(y, \theta) = 1/(1 - \theta)$, $\theta < y < 1$, 0 elsewhere

   (b) $f_Y(y, \theta) = \exp(-y + \theta)$, $\theta < y < \infty$, 0 elsewhere.

4. Check whether the following distributions are members of the exponential family, and find sufficient statistics for random samples of size N if they are.

   (a) A Gamma distribution with parameters $\alpha = \theta$ and $\beta = 1$.

   (b) A Gamma distribution with parameters $\alpha = 1$ and $\beta = \theta$.

   (c) A Chi–square distribution with θ degrees of freedom.

5. R exercise: Suppose that $X_1, \ldots, X_5$ are IID draws from a normal distribution with mean μ and variance σ^2. Let $\hat{\sigma}^2$ be the maximum likelihood estimator of σ^2. Also, consider the alternative estimator

$$\tilde{\sigma}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X}_n)^2,$$

   where $\bar{X}_n$ is the sample average of the X_i. Suppose that the true value of μ is 1, and the true value of σ^2 is 2. Conduct a simulation exercise to evaluate the distribution of $\hat{\sigma}^2$ and $\tilde{\sigma}^2$: randomly generate 5 draws from the $N(1, 2)$ distribution, and calculate both $\hat{\sigma}^2$ and $\tilde{\sigma}^2$. Repeat this 1000 times, saving each estimate $\hat{\sigma}^2$ and $\tilde{\sigma}^2$. Calculate the approximate mean and variance of each estimator, and the approximate mean squared error. Which estimator appears to be "better"?