Economics 520, Homework 8

DUE DATE: Due Thursday, November 2

1. Consider the Example on P.5 of Lecture Note 9, of a sequence of random variables that does not converge almost surely. Construct a new sequence of random variables that have the same *marginal* distributions as the example, but which converges almost surely, and show explicitly what the almost sure limit is. (The marginal distribution of X_1 should be the same in both cases, the marginal distribution of X_2 should be the same in both cases, etc.)

2. Consider the following stochastic process: $Y_1 = 0$ with probability 1, and for $t = 2, 3, \ldots,$

$$Y_t = \beta Y_{t-1} + \epsilon_t,$$

where the ϵ_t are $N(0,1)$, independent across t. Assume that $0 < \beta < 1$.

   (a) Show formally that the sequence Y_t converges in distribution. What is the limiting distribution? In answering this question, it may help to know two facts about the normal distribution: sums of independent normal random variables are normal, and the CDF of the normal distribution is everywhere continuous.

   (b) If $\beta = 1$, would your result still hold?

   (c) Write a Matlab program to simulate independent realizations of Y_{100} with $\beta = 0.6$. Generate 1000 draws for Y_{100}, and plot the histrogram of the draws. Compare this to the distribution you derived in part (a).

3. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables with finite mean and variance, and let $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Let $Z_n = 1/(\bar{X}_n)$. Find the limiting distribution of Z_n.