Economics 520, Homework 7

REVISED DUE DATE: Due Thursday, October 26

1. Suppose that there is a classroom containing 3 white boards. Initially (at time $t = 1$) the boards are blank. In each time period, a professor either: (a) fills an extra board with equations (with probability .25); (b) erases *all* of the boards (with probability .25); or (c) talks and leaves the same number of boards filled with equations. If all of the boards are already filled with equations, then (a) is not an option, and the professor either erases all of the boards (with probability .5) or leaves the boards as they are (with probability .5). If all of the boards are currently empty, then (b) and (c) both result in the boards being empty in the next time period.

   (a) Let X_t denote the number of boards filled with equations at time t. Write down the Markov chain transition matrix. Do you expect the chain to converge to a stationary distribution? If so what is the stationary distribution? (Give as precise a characterization as possible.)

   (b) Write a Matlab/Octave program to simulate the chain X_t (starting at $X_1 = 0$. Simulate the chain for $t = 1, \ldots, 1000$. That is, at each point in time you should draw the next value of the chain randomly, based on the current state and the transition probabilities in part (a), to create a single "run" of the stochastic process. Calculate the average frequencies of the different possible values of X. Compare the frequencies to your results from part (a).

2. Consider the random walk

$$Y_t = Y_{t-1} + \epsilon_t, \qquad t = 2, 3, \ldots$$

   where the ϵ_t are independent $N(0, 1)$. Suppose that the initial value $Y_1 \sim N(1, 4)$.

   (a) What are $E[Y_t]$ and $V[Y_t]$, as functions of time t?

   (b) In Matlab/Octave, simulate the random walk model for $t = 1, \ldots, 100$. Do this 5 times (getting 5 independent chains), and plot the 5 time series on a single graph.