

Economics 520, Homework 5

Due Thursday, September 28

1. The random variable X has a probability density function

$$f(x; \lambda) = \frac{\lambda^x \exp(-\lambda)}{x!}$$

for $x = 0, 1, 2, \dots$ (i.e. X has a Poisson distribution with parameter λ). In a lengthy manuscript, it is discovered that only 13.5 percent of the pages contain no typing errors. If we assume that the number of errors per page is a random variable with a Poisson distribution, find the percentage of pages with exactly one error.

2. Suppose in a shop on average twenty customers come in per hour.
 - (a) How long do you expect to have to wait for the next customer to come in?
 - (b) What is the probability when you enter that you would have to wait more than ten minutes for the next customer to come in?
3. Let X be a continuous random variable with pdf $f(x) = \lambda \exp(-\lambda x)$ for $x > 0$ and zero elsewhere. Calculate $E[X^3]$.
4. Show that each of the following distributions is an exponential family:
 - (a) Gamma distribution.
 - (b) Beta distribution.
 - (c) Chi-squared distribution.
5. Let X have a normal distribution with mean μ and variance σ^2 . Find the probability density function of $Y = \exp(X)$. This is known as a lognormal distribution.

Matlab Questions

The following questions should be done using either Matlab or Octave. Please print out any functions you are asked to write, and copy any interactive sessions (editing out extraneous material).

1. Write a function to generate r independent draws from a Chi-squared distribution with parameter k . [Hint: Let X_1, X_2, \dots, X_k be independent $N(0, 1)$ random variables. Then $Y = \sum_{i=1}^k X_i^2$ has a Chi-squared distribution with degrees of freedom equal to k .]
2. Using the function from the previous question, generate 10 draws from a Chi-squared distribution with $k = 2$. Calculate the sample mean and compare it to the true mean.
3. Repeat the previous question but taking 1000 draws. Is the sample mean closer to the true mean than with only 10 draws?