Economics 520, Fall 2006
Homework 2

Due Thursday, September 7 at start of class

1. Two dice are thrown and three events are defined as follows: A means "odd face with first die"; B means "odd face with second die"; and C means "odd sum" (one face is odd, one face is even). If each of the 36 sample points has probability 1/36, then

   (a) Show that the events are pairwise independent.

   (b) Show that the events are not jointly independent.

2. Suppose a breathalyzer has 5% false positives and 8% false negatives. That is, only 5% of the time will it indicate that a person is drunk when he is actually sober and 8% of the time will it indicate that a person is sober when the person is in fact drunk. Using this test, the police spot test a population of drivers, 99% of whom are sober. What is the chance that a person, who tests as drunk, is actually sober?

3. Two telegraph signals are sent. Assume that the two signals were sent independently, and that the (marginal) probability of a dot being sent is 3/7, and the (marginal) probability of a dash being sent is 4/7. The probability that a dash sent is read as a dot is 1/3, and the probability that a dot sent is read as a dash is 1/4. What is the probability for all four combinations to have been sent if two dots are received?

4. Let F be the cumulative distribution function of a random variable X. Find the cumulative distribution function of $Y = \alpha X + \beta$, where $\alpha > 0$ and β are constants.

5. Suppose X is a continuous random variable with pdf $f(x)$ and CDF $F(x)$. For a fixed number x_0 such that $F(x_0) < 1$, define

$$g(x) = \begin{cases} f(x)/(1 - /F(x_0)), & x \geq x_0 \\ 0, & x < x_0 \end{cases}$$

   (a) Show that $g(x)$ is a valid pdf.

   (b) Describe in words the random variable with pdf $g(x)$.

6. Suppose that the random variable X has an exponential distribution with pdf $f_X(x) = \exp(-x)$, $x > 0$, and 0 elsewhere.

   (a) Find the pdf for $Y = 1/X$.

   (b) Find the pdf for $Y = \ln(X)$.

   (c) Find the pdf for $Y = 1 - F_X(X)$.

7. Let $f(x) = 1/3$ for $-1 < x < 2$ and zero elswhere be the pdf for a random variable X. Find the pdf and distribution function for the random variable $Y = X^2$.