Economics 520, Fall 2005
Midterm, October 13, 2005

Please write your answer to each question on a separate sheet of paper, and put your name on every sheet of paper. The midterm is worth a total of 100 points; each question is worth 20 points. Note: you must show all your work!

1. Suppose X is a continuous random variable that describes the amount of time (in days) that a person is unemployed. Suppose X has a PDF

$$f_X(x) = \frac{1}{50} e^{-x/50}.$$

   (a) Find the probability that the person is unemployed for less than 30 days.

   (b) Let $Y = 24 \cdot X$. What is the interpretation of Y? Find the PDF of Y.

2. Prove Markov's inequality: if X is a nonnegative random variable, then for any $a > 0$,

$$Pr(X \geq a) \leq \frac{E[X]}{a}.$$

3. Suppose that the joint PDF of X and Y is

$$f(x, y) = \frac{3x + y}{7} \qquad \text{for } 0 < x < 2, 0 < y < 1.$$

   (a) Find the marginal density of X.

   (b) Find the conditional density of Y given X.

4. Suppose Y and X are independent binary random variables, with $P(X = 1) = p$ and $P(Y = 1) = q$. Let $Z = X + Y$.

   (a) What is the PMF of Z? What is $E(Z)$?

   (b) Find the conditional PMF of Y given $Z = z$.

5. Suppose that X is distributed Uniform on $[0, 1]$ and that Y is a random variable with

$$E[Y|X = x] = \alpha + \beta x^2.$$

   (a) Calculate $E[Y]$.

   (b) Let $U = Y - \alpha - \beta X^2$. Calculate the covariance between U and X.