Economics 520, Homework 6

Due Tuesday, October 25 at beginning of class

For the questions requiring use of Matlab/Octave, please print out any functions you are asked to write, and copy any interactive sessions (editing out extraneous material).

1. Let X given $Y = y$ have a normal distribution with mean y and variance one, and let the marginal distribution of Y be normal with mean μ_Y and variance σ_Y^2. What is the distribution of Y given $X = x$?

2. Write a Matlab function to generate r independent draws from a Binomial distribution with parameters n and p.

3. Using the function from the previous question, generate 1000 draws from a Binomial distribution with $n = 7$, $p = 0.3$. (Do not print out all 1000 draws.) Calculate the sample mean and variance using the built-in functions `mean` and `var`, and compare these to the true mean and variance.

4. Explain how to construct a $Unif[-1, 1]$ (that is, uniform on $[-1, 1]$) random draw, based on the output of the `rand` function in Matlab. Generate 1000 draws for a random variable $Y = X^2$, where X is $Unif[-1, 1]$. Calculate the sample mean and variance, and compare to the true mean and variance.

5. Suppose an organization has three job types, and employees move from one job to another according to a Markov transition matrix:

$$P = \begin{bmatrix} .7 & .2 & .1 \\ .2 & .6 & .2 \\ .1 & .4 & .5 \end{bmatrix}.$$

   (a) Use Matlab/Octave to calculate $P^{(2)}$ (the two-step ahead transition matrix), as well as $P^{(5)}$ and $P^{(10)}$. Does $P^{(n)}$ appear to be converging to some fixed matrix?

   (b) Calculate the long-run limiting probabilities of being in each of the three states. (Note: there is an exact answer which can be solved for without using a computer. You may wish to use Matlab to check your answer, however.)

6. Consider the following sequence of random variables $W_1, W_2, \ldots$ with

$$W_n = \begin{cases} \frac{1}{n} & \text{with probability } .5 \\ 0 & \text{with probability } .5 \end{cases}$$

   Using the definition of convergence in probability, show that $W_n \xrightarrow{p} 0$.