Economics 520, Homework 11

Due Tuesday, December 6 at beginning of class

1. Let X_1, X_2, ..., X_N represent a random sample from a Poisson distribution with arrival rate λ.
   (a) Find the maximum likelihood estimator for λ and its asymptotic distribution.
   (b) Suppose we are interested in the probability of a count of zero, $\theta = Pr(X = 0) = \exp(-\lambda)$. Find the maximum likelihood estimator for θ and its asymptotic distribution.

2. Let the random variable X have probability density function $f_X(x;\theta) = (1/\theta)\exp(-x/\theta)$ for $x > 0$. Consider the simple hypothesis $H_0: \ \theta = 2$ and the alternative hypothesis $H_a: \ \theta = 4$. Let X_1, X_2 denote a random sample of size two from this distribution. Show that the best test of H_0 against H_a may be carried out by use of the statistic $X_1 + X_2$. Find the optimal critical region for $\alpha = 0.1$.

3. Let $X_1, \ldots, X_{10}$ be a random sample of size 10 from a normal distribution with mean zero and variance σ^2. Find a best critical region for testing $H_0: \ \sigma^2 = 1$ against $H_a: \sigma^2 = 2$ of size $\alpha = 0.05$. Is this also a best test of the same null hypothesis against the alternative hypothesis $H_a: \ \sigma^2 = 3$? And against the alternative hypothesis $H_a: \ \sigma^2 > 1$?

4. Suppose we want to test is a coin is fair (that is, the probability of heads is $p = .5$). We toss the coin 1000 times, and record the number of heads. Let T denote the number of heads divided by 1000. Consider a test that rejects the null hypothesis that $p = .5$ if $T > c$.
   (a) Write down a formula for $P(T > c|p = .5)$. Provide an outline of how to write a Matlab program that would calculate this number. You do not need to give exact code, just explain the steps in enough detail so that an experienced Matlab programmer could implement it.
   (b) Give a large-sample approximate formula for $P(T > c|p = .5)$ based on the central limit theorem.
   (c) Using the large-sample approximation, calculate the appropriate value c for a test at the 0.05 percent level. You can use the fact that if $Z \sim N(0, 1)$, then $P(Z > 1.645) = 0.05$. Would your test reject the null hypothesis if we observed 560 heads and 440 tails?