

## Economics 520, Homework 10

Due Tuesday, November 22 at beginning of class

1. Let  $Y_1, Y_2, \dots, Y_N$  be independent random variables with common cdf

$$F_Y(y) = 1 - \exp(-\lambda e^y),$$

for  $-\infty < y < \infty$ , for some parameter  $\lambda > 0$ .

- (a) Find a one-dimensional sufficient statistic.
  - (b) Find the maximum likelihood estimator
2. Let  $Y_1, Y_2, \dots, Y_N$  be independent random variables with identical pdf  $f_Y(y, \theta)$ . For each of the following densities find a minimal sufficient statistic for  $\theta$ . (Note: remember that a one-dimensional sufficient statistic does not always exist).
- (a)  $f_Y(y, \theta) = 1/(1 - \theta)$ ,  $\theta < y < 1$ , 0 elsewhere
  - (b)  $f_Y(y, \theta) = \exp(-y + \theta)$ ,  $\theta < y < \infty$ , 0 elsewhere.
3. Check whether the following distributions are members of the exponential family, and find sufficient statistics for random samples of size  $N$  if they are.
- (a) A Gamma distribution with parameters  $\alpha = \theta$  and  $\beta = 1$ .
  - (b) A Gamma distribution with parameters  $\alpha = 1$  and  $\beta = \theta$ .
  - (c) A Chi-square distribution with  $\theta$  degrees of freedom.
  - (d) A binomial distribution with parameters  $N = 5$  and  $p = \theta$ .
  - (e) A uniform distribution on the interval 0 to  $\theta$ .
4. Let  $Y_1, Y_2, \dots, Y_N$  be a random sample from a distribution with probability density function
- $$f_Y(y, \theta) = 2y/\theta^2 \quad 0 < y < \theta, \quad 0 \text{ elsewhere}$$
- (a) Show that  $W = 3\bar{Y}/2$  is an unbiased estimator for  $\theta$ .
  - (b) Find the Cramér–Rao bound.
  - (c) Show that the variance of  $W$  is less than the Cramér–Rao bound.
5. CB 7.40.
6. CB 7.46.