Abstract

A Dynamic Analysis of Mergers

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The dissertation examines the dynamics of firm behavior using computational methods, with the goal of modeling mergers more realistically than previous studies in order to examine policy implications. In this model, mergers, investment, entry and exit are endogenous choice variables rationally chosen by firms in the industry in order to maximize expected future profits. The model differs from previous studies in two ways: first, it directly specifies a process through which firms can merge and predicts which mergers will occur at different states; second, it incorporates dynamic determinants of firm behavior (namely entry, exit and investment). In Chapter 1, I discuss criteria for designing an endogenous merger process and I detail the processes chosen and corresponding results. The results show that by not incorporating the effects of investment and entry on the merger behavior of firms, the static literature is overpredicting the probability of mergers; this type of result has not been shown before since it can only be shown with an endogenous model. In Chapter 2, I examine antitrust policy implications using the dynamic model of endogenous mergers and compare these to the planner and colluder solutions. The results show first that a wide range of intermediate antitrust policies leads to a

higher total welfare than either a complete merger prohibition or no antitrust law. In addition, they show that with production cost differences, merger synergies and barriers to entry, the implications of different antitrust policies are surprising and very different than those predicted by static non-endogenous merger models. Finally, the model uses its rational framework to explain the puzzle, discussed in the empirical literature, of why acquired firms seem to capture all of the gains from mergers. In Chapter 3, I discuss the issues surrounding the design and computation of the model. In particular, I examine when equilibria to the merger processes are likely to exist, the choice of specifications necessary to allow for computation, and the computational details of the processes. This discussion is of interest in order to further develop computational merger models and to design computational models of other oligopolistic games.

A Dynamic Analysis of Mergers

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Introduction:[1]

The purpose of this dissertation is to analyze the effects of mergers between competitors in an industry. The main goal of the analysis is to predict which mergers occur and what the consequences of the mergers that do occur are, in order to derive more realistic results than have been obtained by previous models. Mergers are of interest to economists and policy-makers due to their frequency and their potential effects on the economic performance of industries. Because of their importance, mergers have been studied by many authors with many different goals. Over the past century, different schools have studied the collusive effects of mergers, the empirical efficiency gains from mergers, and the potential of antitrust law to encourage inefficient industry structures. Notable among these schools of study is the Chicago School. One of the main contributions of the Chicago School was to caution that strict antitrust policy might have deleterious dynamic effects;; however, the proponents of the Chicago School did not attempt to model the merger process mathematically. Most recent advances in studying mergers have used game theory, in order to be able to mathematically predict which mergers are profitable. Thus, typical recent studies have attempted to find which mergers yield a static gain to

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producers and which mergers are socially optimal in a variety of settings, including: a Cournot model, a Bertrand model, a Cournot model with cost differences, a Hotelling model, a model where merging firms are perfect complements, and a model where there are synergies between merging firms. While the game theory models have examined different issues, they have mostly used the same paradigm: that there is a merger process, followed by some one-shot production game that is played by the firms that remain after the merger process is completed. Thus, the game theory literature is not able to incorporate the important effects that dynamic determinants, such as investment, entry and exit, will have on the outcome of the merger process. In addition to the game theory literature on mergers, there is a related literature, on endogenous coalition formation, that has attempted to explore which coalitions are likely to result from a given set of agents with defined payoffs. As coalitions that cannot be broken are mathematically identical to mergers, this literature is also relevant to any study of mergers. However, the endogenous merger literature has thus far used very simple conceptions of firms and has not been able to yield predictions for which coalitions will form except in some restrictive cases.

While mergers have been studied a great deal by previous works, none of the papers in the literature has examined mergers in a setting where firms can choose merger decisions according to some well-specified process, which provides the modeler with the ability to examine which mergers firms choose. Alternately put, none of the papers in the literature has endogenized the merger process. Because none of the papers in the literature has examined mergers in a fully endogenous, dynamic setting, none of the papers has been able to analyze the central questions of which mergers will occur, and what the welfare and other effects of the mergers that do occur will be. The primary contribution of this dissertation is that it is the first model which predicts the patterns and occurrences of mergers in industries, and then uses these predictions in order to develop antitrust policy implications for a variety of industry phenomena. The model predicts the patterns and occurrences of mergers using a well-defined model where firms rationally choose merger decisions, as well as entry, exit and investment decisions in order to maximize the expected discounted value of their net future profits, conditional on their information. In addition to this primary contribution, the other contribution of this dissertation is in furthering the theories of coalition formation, auctions with externalities and the general computational modeling of oligopolistic industry structures.

As I discussed above, the model that I present in this dissertation is able to explain mergers in ways that no other model has attempted. The primary reason why the model is able to endogenize the merger process is that, while others have attempted to derive analytical solutions to their merger models, I am using computational methods. The basic methodology that I use is to choose parameter values, compute the equilibrium for the parameter values and then explore the welfare and industry implications that result from the given parameter values. In order to examine the effects of different policies and different types of industries, I change the parameters to reflect different industries and examine how this changes the results of the model. In order to determine the sensitivity of the model to different specifications, I change the specifications and repeat the process. With a computational model such as this one, I am able to examine a more complex vision of an industry than analytical models. Thus, I am able to have an endogenous merger process, incorporate dynamic determinants of firm behavior and evaluate how the industry performance changes for different policies and institutions, all due to the computational nature of my model.

Because I am answering central questions of the merger debate that have not been answered before, the discussion in this dissertation has many interesting facets. The discussion explores what criteria one needs to examine in designing a dynamic, endogenous model of mergers and examines the generality of the model presented by evaluating the results. In addition, the model is able to determine the size and direction of the bias that static models will have by not incorporating dynamics in their models. With the model, I examine the implications of antitrust policies and explore optimal policies and first-best solutions. I also examine how optimal antitrust policies change when such factors as barriers to entry or production cost differences are present in the industry. Finally, I use the model to reexamine the empirical paradox of why firms choose to make acquisitions when the gains from mergers appear to accrue exclusively to the acquired firm. In addition to the results of the model, I discuss the theoretical basis for the endogenous merger mechanisms used and the techniques used in computing the merger and overall dynamic processes for the model.

While the discussion of the model as presented in the dissertation is self-contained, the model developed here is of use in future research. In particular, as the parameters of the model can be changed to examine different industries, the model can serve as a basis for structural empirical work. In the future, economists can estimate relevant structural parameters for industries of interest, and use variants of this model to examine the implications of different antitrust policies for these industries. Although this is a difficult task, it will allow economists to incorporate the effects of dynamics and consistently estimate the effects of government antitrust policies on the future performance of industries. While this task has not been accomplished before, it is important to policy-makers in determining which policies to enact. More generally, one can use the model with stylized versions of industries in order to examine the size and direction of the bias that is introduced into the results by examining the effects of mergers in a static setting and by not endogenizing the merger process. Thus, applications of the merger model to sectors that are the subject of antitrust interest, such as the hospital industry, the cement industry and the timber industry, are likely to be feasible in the near future.

In spite of the wide range of issues that I discuss in this dissertation, there are limitations to the issues that I can examine and the generality of the results. In particular, because of the computational nature of the model, the results of the model depend on the parameters in ways that cannot be determined fully. The best technique that can be used to determine the generality of the results is to vary the parameters and compare the results of the model with different parameters. However, the ability to do this is limited by the computational complexity of the model, which is substantial. In addition, because I am specifying processes by which firms merge, invest, exit and enter, each process is structural and hence the choice of structure is arbitrary. Finally, because of the nature of the model, given a set of arbitrary parameter values, one cannot be assured that the equilibrium that the algorithm converges to is unique, that the algorithm will even converge to an equilibrium, or that there even exists an equilibrium. Thus, the results presented here should not be thought of as the last word on modeling mergers endogenously, but rather the first word. While I am able to obtain many interesting and surprising results with this model, and while this is the first model to predict the occurrence of mergers, there is room to make the endogenous merger process and other processes more general and to make the model less cumbersome.

The remainder of this dissertation is divided as follows. Chapter 1 discusses the choice of model and examines the specification of the model and the necessity of incorporating dynamics in the model. Chapter 2 discusses many different antitrust policy implications of the model and examines the paradox of the splitting of gains from mergers. Finally, Chapter 3 provides a discussion of the computational issues of the merger model, including an examination of the computation of auctions with externalities.

Chapter 1:

A Dynamic Model of Endogenous Horizontal Mergers.

Section 1: Introduction

Economists have long been interested in studying horizontal mergers, because of the important effects that they have, and because they occur in vast numbers. As mergers are of concern to policymakers only in industries with a limited number of firms, recent advances have analyzed mergers using game theory. The first major paper that analyzes mergers with a game theoretic approach is Salant-Switzer-Reynolds (1983 - hereafter S-S-R), which shows that with Cournot interactions between firms that produce a homogeneous product, mergers for oligopoly will generally not be profitable in a static game, unless the original number of firms is small, and a very large proportion of existing firms actually merge.[1] Other papers in the literature have obtained results for a differentiated products Bertrand setting (see Deneckere and Davidson (1985)), with cost differences (see Perry and Porter (1985)), and when there are synergies between firms (see Farrell and Shapiro (1990)). These papers have all used models that are static in nature and accordingly have not incorporated the effects that dynamic determinants of firm behavior, such as entry, exit and investment, have on the results of the merger process. There are several important economic reasons why entry, exit and investment will have large effects on which mergers occur in an industry, and what the welfare consequences of such mergers will be. For example, although one may think that mergers lower consumer welfare by increasing concentration, entry into the industry may increase if mergers are allowed, and counterbalance the increase in concentration. Or, mergers may allow firms to partially internalize the negative externality from their

[1] The proportion must approach one as the number of firms approaches infinity for any mergers to take place.

investment, and thus increase profits. Because these dynamic behaviors are important economic determinants of mergers, some papers, such as Berry and Pakes (1993) and Cheong and Judd (1993), have analyzed mergers with dynamic game theoretic industry models.

While economists have learned a great deal from the static and dynamic game theoretic merger literature, none of the above papers has provided a framework which predicts which mergers are likely to arise in a given industry. Their approach has largely been, instead, to examine the consequences of particular mergers, and to see whether these particular mergers are profitable, relative to no merger. As a result, one cannot fully understand the importance that dynamics have to the consequences of firm merger behavior. For instance, if entry is possible, firms that would have otherwise chosen to merge may not merge, as they know that future entry would make any profits from the increased concentration transitory. Or, a weak firm may choose not to exit an industry, knowing that if it were to remain, it would be acquired by a large competitor sooner or later. Finally, a firm may choose not to acquire another firm in the current period even if such a merger is profitable relative to no merger, knowing that in the future it will be in a better bargaining position and be able to acquire the firm for a lesser price. These examples show that one cannot fully evaluate the effects of dynamics on mergers without endogenizing the merger process and examining how the mergers actually chosen by firms change when dynamics enter into their decisions.

In addition to determining whether or not a dynamic model is necessary to analyze merger behavior, there are many other reasons why economists would like to be able to evaluate which mergers will take place in an industry. In order to investigate the welfare effects of antitrust or other government policies one must use a model that predicts how policies affect future entry, exit, investment and mergers, and through that the future structures and performance of the industry, as otherwise one cannot evaluate optimal long-run policies. Indeed, any analysis of mergers that does not endogenize the merger process will not be able to predict future actions that result from current policies and hence cannot predict the future welfare and industry effects of current policies.

This paper presents a model of endogenous horizontal mergers in a stylized industry. In the model, firms choose their merger decisions via a simple bidding process or via an auction process. Thus, conditional on parameters, the model is able to predict the equilibrium probability that any firm will choose to merge with any other firm given an industry structure. Because of the potential importance of entry, exit and investment in determining the effects of mergers, the model incorporates dynamics: in addition to choosing merger decisions, existing firms and potential entrants choose entry, exit, and investment decisions in order to compete in an infinitely-lived industry. The primary goal of the model is to serve as a tool for analyzing the effects of different policies. It can accomplish this goal in ways that other models have not been able to because it is endogenizing the merger process and using a dynamic framework. There are several ways in which I have used and am planning to use the model to accomplish its goal. First, in this chapter, I use the model in order to find how large a role entry, exit and investment play in determining the effects of mergers, and I examine to what extent the conclusions of the model are dependent on specifications. In Chapter 2, I use the model in order to analyze the effects of antitrust policy in stylized industries and examine how the optimal antitrust policies change when such factors as barriers to entry and increasing returns are present. Finally, as the ultimate goal of this model is to understand actual industries more precisely, this model serves as a tool for future empirical work. Empirical studies can estimate the underlying parameters of the model for specific industries, and then use the model to evaluate the effects of antitrust and other policies for actual industries.

The dynamic framework of firm behavior that I use in this model is based on the framework developed by Ericson and Pakes (1995), and modifies this framework to allow for endogenous mergers. Accordingly, the conception is of an infinitely-lived industry modeled in discrete time, where current firms merge, invest, exit and produce, potential entrants enter, and all agents rationally make their choices in order to maximize their expected discounted value (EDV) of future profits. For the results presented in this chapter, firms invest in order to increase their capacity levels in future periods (following Berry and Pakes) and statically are Cournot competitors in a homogeneous goods market. While this structure was chosen in order for the stylized results to be simple, it is possible to use the endogenous merger model with different static and dynamic interactions between firms and hence to use this model to examine the implications of mergers in different industries.

Because I am endogenizing the merger, investment, entry and exit processes and modeling dynamics explicitly, the model is elaborate and non-linear. Thus, it is not possible to analytically solve for an equilibrium of the model. Instead, the technique that I use is to pick particular parameter values, compute equilibria numerically for these parameter values, and then simulate the industry dynamics, in order to be able to uncover the distribution of the industry particulars (i.e. investment levels, sizes of firms, lifespans, amount of entry, etc.) that occur in the equilibrium distribution of the industry. While the goal of this chapter is to theoretically analyze the effects of incorporating dynamics in a merger model, as the theory that I am interested in is too complex to obtain analytical results, I instead find numerical answers. Thus, the answers to the theoretical questions are largely in the form of results from computational calculations and simulations of the model, rather than from theorems. In Chapter 2, I attempt to answer policy-relevant questions concerning mergers using these same computational methods and this model as a basis.

With this stylized model, I have computed equilibria and simulated the industry using equilibrium policies in order to uncover the characteristics of the industry, in terms of welfare and policy variables. In order to test the sensitivity of the model to parameter changes, I have computed the models for different parameters and specifications, and illustrated how these specifications change the results of the model. The specifications I have examined include modeling mergers with different coalition formation processes, comparing them to an industry where no mergers are allowed, and to an industry where firms' decisions to merge are based only on current period profits, similarly to the static literature on mergers. The results show, primarily, that is possible to develop a model of endogenous mergers that is computable, incorporates dynamic determinants of firm behavior, and yields reasonable results. In particular, the ordering of the firms in the merger process does not have major effects on the industry dynamics, which demonstrates that the model is not overly sensitive to changes in specification, and changes in the size of the synergy distribution have the effects that one would expect. While the results are different depending on whether a take-it-or-leave-it or auction process is used in making merger decisions, the reason for the difference is easily explained by the nature of the different processes. In addition, the results show that incorporating dynamics into a merger model may lead to important improvements in the predictive power of the model. This is apparent largely for two reasons: first, the level of the principal dynamic components of the model (namely entry, exit and investment) changes when mergers are allowed in the industry; second, the occurrences and consequences of mergers are very different when firms choose whether or not to merge by looking at their future stream of profits than when they make this decision myopically by looking only at current period profits, as in the static literature.

The remainder of this paper is divided as follows: Section 2 describes the choice and specifics of the model; Section 3 describes the results of the model; and Section 4 concludes.

Section 2: The Industry and Merger Model

As I discussed in the introduction, my model differs from previous merger models in that it directly specifies an endogenous merger process and in that it incorporates dynamic determinants of firm behavior, such as entry, exit and investment. Accordingly, in developing a model with which to analyze mergers, the two principal aspects on which I focused are the endogenous merger process and the dynamics. All of the earlier static game theoretic models of mergers, such as S-S-R, Deneckere and Davidson, Perry and Porter, Farrell and Shapiro, McAfee-Simons-Williams (1992), Kamien and Zang (1990) and Gaudet and Salant (1992) have used the same general approach to analyze the effects of mergers. Their approach has been to model a merger process, followed by a one-period production process, where the producers are the firms that remain in the industry after the merger process. Production has been modeled by some standard (i.e. Cournot or Bertrand or Hotelling) static industrial organization production game. Using backward induction, one can solve for the reduced-form profits that accrue to each firm at each information set after the merger process has finished, and use these to evaluate the value to firms of different merger choices. The papers have typically then analyzed which mergers might happen, or what the welfare effects of some mergers might be, without fully endogenizing the merger process.

As can be seen from their setup, none of the papers above has dynamics of any sort in their models, and hence none of them can provide a basis on which to develop the dynamics. For this reason, the dynamics are based on an existing industry model, that of Ericson and Pakes (1995) which I extend to examine mergers. (I discuss the Ericson-Pakes framework fully in Section 2.2.) Recall that in the Ericson-Pakes framework, firms may exist in future periods, and entry, exit, and investment also potentially occur in future periods. Accordingly, in my model, the value that a firm receives from any merger is a sum of discounted future payoffs which incorporates the possibility of entry and exit by rivals, good and bad investment draws by rivals or by the firm itself, and future mergers. Thus, the post-merger payoffs in my model are based on a more elaborate construct than the analogous reduced-form Bertrand or Cournot payoffs used by the static literature.

In spite of the differences in post-merger payoffs between my model and static models, it is still possible to compare the merger process in my model to those used in static models, with the appropriate choice of notation. For the static model, call the post-merger reduced-form profits for every post-merger industry structure V^{PM}.[2] In my model, the analog of these values are the expected discounted values (EDV) of net future profits of firms after the merger process for any period has finished. These values include the possibility of merger, entry, exit and investment by firms or their competitors in future periods. I can call these values V^{PM} again. Although V^{PM} is not as directly computable for my dynamic model, it is well defined: it is the EDV of net future cash flows of the firm conditional on the transition probabilities that result from some Markov-Perfect Nash Equilibrium. Because the merger process reoccurs in future periods, V^{PM} will be endogenous to the merger decision. However, one can still discuss firms' merger decisions as a function of V^{PM}. As I will show in Chapter 3, in equilibrium these decisions in turn generate V^{PM} as the true EDV of net future cash flows.

In the discussion that follows, I examine firms' merger decisions solely with regard to their V^{PM} values. Given a set of post-merger values, the idea of my endogenous merger process would apply equally well to a static model: to find which mergers firms will likely choose given some set of reduced-form post-merger values, and what the resulting pre-merger values are. Accordingly, I have applied some of the ideas from the static literature, such as differentiated firms (from Perry and

[2] In Chapter 3, I formally define the V^{PM} values and illustrate what they are in my model.

Porter) and synergies (from Farrell and Shapiro) to my model. To compare my model to static merger models, one may think of the V^{PM} values as the same as the reduced form profits used in the static literature. Thus, although the model is dynamic, the endogenous merger process applies equally well to static games.

In the remainder of this section, I discuss some of the criteria behind my choice of an endogenous merger process, and then discuss the industry model and merger model that I did use.

2.1 Choice of Endogenous Merger Processes

Designing the process by which firms merge is not a straightforward task. It is for this reason that no other paper has attempted to completely endogenize the merger process. As I am interested in applying the merger model to future empirical work, I want a model where the incidences and patterns of mergers are similar to those observed in mergers in actual industries. In addition, there are several theoretical points that I considered in developing the model; I illustrate them below.

First, it is important that the merger process not generate multiple equilibria. As the computational algorithm only generates one equilibrium, computation of the infinite-game MPNE with multiple equilibria is not feasible or meaningful. In a dynamic multi-agent model such as this one, it is not generally possible to define an industry game that eliminates multiple equilbria; for instance Ericson and Pakes do not rule out the presence of multiple equilibria in their models. However, many of the merger rules that might seem intuitive yield a vast continuum of equilibria, even conditional on the V^{PM} values. Consider, for instance, processes such as those suggested in Kamien and Zang (1990), where every firm indicates a simultaneous bid for every other firm together with an asking price for itself. In this game, the 'unmerged' result is always an equilibrium result, as are any outcomes where the mergers that occur are more profitable than no mergers; thus computation with such a merger process would not be feasible, without further restrictions. Another game one may think of would be one that uses some concept based on matching lists where, for instance, each firm would indicate its preferred firm to merge with, and if there are any matches a merger would take place. With such a game form, any profitable matching would emerge as a Nash Equilibrium, and thus there are likely to be an abundance of these. The problem of multiple equilibria is the principal reason why no paper in the merger literature has endogenized the merger process.

Second, I want the equilibria conditional on V^{PM} to be "reasonable", in the sense that parties do not merge unless they have some apparent gain to merging given equilibrium beliefs, and the merger that occurs results in the largest apparent gain. In addition, the payments that are made between the firms in the merger should be reflective of their outside alternatives. Without these conditions, the dynamic aspects of the model will not work: firms will have perverse incentives when deciding how much to invest and whether to enter that will be based on the fact that some positions will get very different amounts from mergers than other positions. This may result in non-existence of equilibria, or nonsensical equilibria.

Third, as I am explicitly modeling the other decision rules in this industry structurally, I want to rule out any solution to the merger game that does not have a structural interpretation. For instance, a rule that any merger to occur should be picked from the set of Pareto optimal mergers, or a rule that the merger to occur should maximize the expected sum of future profits, or some non-structural endogenous coalition formation model like Hart and Kurz (1981) are all inappropriate. The reason for the need of structure in the model is that the laws that govern how corporations act prohibit certain types of behavior, and thus impose a structure on the industry which should be reflected in my model. For example, as a non-participant to a merger cannot legally compensate potential merging firms in order to encourage a merger, a Pareto optimal merger will not necessarily occur. As the goal of this model is to mimic institutions and behavior in actual industries, it is not useful to evaluate the model without institutional structures or with different ones from those that exist, except for comparison purposes. Because of the desirability of structural interpretations of behavior, recent advances in the coalition formation literature, such as Bloch (1995 and 1995), have used structural models of the formation process. The technique that has generally been used by such structural models has been to allow some sort of bidding process. While the models developed have yielded significant results,they are not yet simple enough to be feasibly used to model differentiated firms.

Fourth, I would like a model where there is enough flexibility in the outcome of the merger process to allow any possible merger to happen. Thus, a merger process which prevents any two firms from merging in a given period would not be appropriate. The reason for this criterion is that I would like to be able to examine issues such as whether mergers for monopoly always result in this model (as is often the case in the static literature), or whether future entry is a sufficient deterrent to this outcome. Thus, I must allow mergers for monopoly to happen. In general, I want a merger process which allows for any possible partition of the set of initial firms into merged firms at the end of the process, given the initial industry structure.

Fifth, I may want to have some randomness in the merger process. There are two reasons for this. The first reason is that I do not want value function payoffs to be discontinuous in the V^{PM} values of firms, or I may find that no equilibrium exists.[3] Without randomness in the merger process, discontinuities are bound to occur. Consider the following example of a stylized industry with three firms, A, B and C, and only one merger possible, that between A and B: suppose firms A and B have V^{PM} values which make a merger just barely unprofitable. Then, C's (pre-merger) value function will incorporate the fact that no merger will occur with probability one. Now suppose that firms A and B have slightly different V^{PM} values which make a merger just barely profitable. Then, as C's value function incorporates the fact that the merger will occur with probability one, it may be very different from the earlier value function. Thus, a small change in A and B's V^{PM} values can cause a discrete jump in the value function of C, as the probability of merger jumps from zero to one. This shows that without some randomness in the merger process, value functions will not be continuous in V^{PM}. The discontinuity occurs because of the externality that the parties that are merging impose on the non-participant in the merger. I want to preserve the externality (as it is reflective of the actual world, as indicated, for instance, by the classic comment made by Stigler (1968) in discussing the steel industry, that non-participants are likely to benefit the most from mergers) but get rid of the discontinuity. This can be done by incorporating

[3] The value function is the firm's EDV of net future cash flows at the beginning of a period (i.e. before the merger process has occurred). The proof of existence of equilibrium, which is given in Chapter 3, depends on the continuity of the value functions when viewed as a function of earlier perceptions and value functions, and hence as a function of V^{PM} values.

some randomness into the model which would smooth out the probability of a merger occurring (as a function of V^{PM} values) instead of having a discrete jump in the probability. This is the same role that is served by mixed strategies, in ensuring the existence of equilibrium for finite games.

The second reason that I may want some randomness in the merger process is that one can observe uncertain outcomes from mergers in the world. It would be hard to view a model without randomness as realistic. Firms can never simply add two plants together without transaction costs. Indeed, real-world randomness is the basis behind a structural justification for noise in data. It is because of noise in data that economists are able to add randomness to econometric models. This randomness is, in turn, necessary to avoid predicting that the model is misspecified: without randomness, the model could claim to predict the exact outcome of mergers, conditional on any state. If the actual outcome is slightly different from what is predicted, this would indicate an error in the model. With randomness, the model would predict the probability a merger has of happening, which would allow for more credible estimation strategies, and avoid misspecification.[4]

2.2 The Industry Model

I have chosen to extend the model of Ericson-Pakes to incorporate mergers, because this model results in entry and exit correlations and size distributions that closely match empirical data. The class of models developed by Ericson and Pakes

[4] Note that allowing randomness in the merger process may not by itself be sufficient for the model to not be econometrically overidentified. Other types of randomness, such as fluctuations in the demand curve, might also be necessary. In addition, randomness in the merger process may not be necessary as randomness can be explained by such things as measurement error.

describes an infinite-horizon discrete-time industry with endogenous entry, exit and investment where firms choose strategies in order to maximize the expected discounted value (EDV) of their net future profits given their information. In this class of models, I represent industry structures by states, where a state indicates the capacities of the different firms active in the industry.[5] Capacities of firms in the industry evolve from period to period through a controlled Markov process, dependent on the investment of firms; a firm's capacity is a stochastically increasing function of its investment level in the previous period. Because of the stochastic nature of the investment process, an industry's state will never converge to some fixed value; rather (as shown in Ericson and Pakes) a fixed ergodic distribution of states will ensue regardless of the initial condition. Using the concept of Markov-Perfect Nash Equilibrium (MPNE), Ericson and Pakes define and prove the existence of an equilibrium, where the strategy space is investment together with the entry and exit decision. MPNE, as defined by Maskin and Tirole (1988), restricts actions to be a function of payoff relevant state variables, and thus eliminates the possibility of supergame payoffs. By the definition of a Nash Equilibrium, at a MPNE, firms are maximizing the EDV of their net future profits given the correct information about their present and potential competitors' strategies.

Because of its non-linearity and complexity, it is not generally possible to solve the Ericson-Pakes model algebraically. Pakes and McGuire (1994) show how to compute the Ericson-Pakes model numerically using stochastic dynamic programming with advanced computational techniques, and compute the model for a

[5] In the Ericson and Pakes model, the firm's dynamic state variable is not capacity but any variable where the static profits are increasing in that variable. Other typical state variables include marginal costs or quality. For ease of exposition, however, I consider capacity as the state variable.

differentiated products market. Recall that I also compute the merger model numerically. As described in Pakes-Gowrisankaran-McGuire (1993 - hereafter P-G-M), the stage game in Pakes and McGuire is a four-level game as follows: first, incumbent firms decide on exit; second, remaining incumbents decide how much to invest; third, potential entrants decide whether to enter; and fourth, active firms decide how much to produce. In my model, I use a five-level game as follows: first, firms decide whether or not to merge, then in the remaining stages, firms follow the process specified in P-G-M. Provided certain conditions are met, I can prove the existence of equilibrium for my model, but just as in Ericson and Pakes, I cannot prove uniqueness - see Chapter 3 for details of existence and uniqueness issues. However, computationally, there has not been a problem of convergence to different equilibria given different starting values. I detail all of the processes below, except for the merger process, which I discuss in Section 2.3.

Every period, incumbent firms can simultaneously choose whether or not to exit the industry. If a firm chooses to exit, it receives a fixed scrap value in the current period and does not ever receive any future returns. After the exit process, the remaining incumbents (i.e. the firms that do not exit) simultaneously choose investment amounts. Investment is chosen in order to increase or maintain a firm's capacity level.[6] In particular, if a firm invests some amount x, then there is a

[6] There are other models which would naturally use the same state additive property of mergers. For instance, consider a state variable of capital, with capital decreasing the marginal costs of production Then, when two firms merge, they would become one firm with the sum of each of their levels of capital. See Perry and Porter (1985) for an example of such a model. Because it is hard to enumerate Nash Equilibria for the capacity constrained model, analytically, most I.O. models with differentiated firms have used such capital-type models. However, there are examples of industries which have been modeled using capacity as the main differentiating variable. See, for instance, Porter and Spence's (1982) study of the corn wet milling industry. Because of the computational nature of my solution, there is no added difficulty in using computational methods to solve the static Nash Equilibrium problem. In Chapter 2, I examine models where costs as well as capacities differ among firms.

probability of $ax/1+ax$ that its capacity will rise by 1 unit next period, for constant a and subject to depreciation. Depreciation is modeled stochastically and is common across firms at any period: each period, there is a probability δ that every firm's capacity will decrease by 1 unit; with the remaining probability, there is no depreciation. After the investment process, the entry process occurs. This process is modeled by assuming that at period t there is a potential entrant, who may enter the industry at period t+1 by paying some entry fee at time t. The entry fee is known to the entrant before it makes its entry decision and is drawn from a uniform distribution with known density. The entrant will enter at period t+1 with some fixed capacity, subject to depreciation, and cannot invest until period t+1. Recall that firms choose their exit, entry and investment decisions in order to maximize the EDV of net future profits.

At every time period, there is some industry-wide static linear demand curve that is invariant across periods. Every firm active in the industry at a time period produces the same homogeneous good in order to satisfy the demand. Each firm also has no fixed costs and the same, constant marginal cost. However, firms are differentiated by their capacity level: a firm can only produce up to this fixed amount, or equivalently, at the capacity level, its marginal costs are infinite. Firms choose their levels of production simultaneously, again in order to maximize the EDV of net future profits. Because quantity is not a payoff-relevant state variable, the MPNE assumption ensures that the quantities that firms choose will be the Cournot-Nash one-shot quantities, given the demand curve and the cost functions.

The methodology used to solve for the equilibrium of the model is a version of dynamic programming, based on the algorithm developed by Pakes and McGuire;

the reader should consult Chapter 3 for details. In general terms, the idea of the algortihm is to solve for the value function backwards through the industry processes given fixed perceptions of the behavior of competitors, and then to update the perceptions. To illustrate the algorithm used, recall that I defined V^{PM} to be the value to firms at the end of the merger process, and define also V^{BM} to be the value to firms at the beginning of the merger process, and hence at the beginning of the period. Then, the algorithm recursively solves the following equations for every state together:

$$
\begin{gathered}
V^{PM} = \pi[\text{Cournot, reduced form}] + \\
\max_{\text{actions}}\left\{-\text{cost of actions} + \beta E\left[V^{BM} \text{ next period} \middle| \text{perceptions, actions}\right]\right\} \\
V^{BM} = \max_{\text{merger actions}}\left\{-\text{cost of actions}\right. \\
\left. + E\left[V^{PM} \text{ at end of merger process} \middle| \text{perceptions, actions}\right]\right\} \\
\text{Perceptions} = \text{function of actions chosen.}
\end{gathered}
\tag{2.1}
$$

Here, the values and optimal choices for firms at any state are expressed in value-function form using dynamic programming. However, because of the multiple-agent nature of this algorithm, the value functions are correct only given perceptions about the actions of other firms. Thus, at every iteration, the algorithm must update the perceptions in order to incorporate the updated actions of every firm; the changing perceptions is the difference between this algorithm and standard dynamic programming. Given dynamic programming theorems (and as I show formally in Chapter 3) a fixed point of the perceptions and the value functions in (2.1) forms an equilibrium to the model. Therefore, the algorithm attempts to iterate on these variables until convergence to a fixed point.

2.3 The Merger Process

I have left the discussion of the merger process for last, as it is the integral part of the model. In this model, if two firms merge, they combine their capacities and become a larger firm, whose capacity is the sum of the capacities of the two firms separately. In addition, when two firms merge, they pay or receive a cost or synergy, which is drawn from a uniform distribution and is known to the firm before the merger occurs.[7] I have designed a merger model that attempts to incorporate the considerations listed in Section 2.1. Throughout the description, I interchange 'merger' and 'acquisition' as the model does not really distinguish between the two.[8]

In my model, the merger process occurs sequentially: the biggest firm (in terms of capacity) can choose to acquire any one other firm by paying it a sufficient amount, first. (I describe different choices for mechanisms by which it buys a smaller firm shortly and examine different orders for the process in Section 3.) If, on one hand, it declines do so, then the second biggest firm can choose to acquire any firm that is smaller than it, and then the third biggest firm can, etc. If, on the other hand, the biggest firm chooses to acquire some other firm, then the merger occurs, the firms combine their capacities, and a new industry structure results. Again, then, the new biggest firm can choose to merge with any firm that is smaller than it, and then if it chooses not to, the next biggest firm can, etc. Given the merger process, note that any possible partition of firms into merged firms is possible. The process continues until there is only one firm left, or the second smallest firm chooses not to

[7] I evaluate the results with the cost/synergy being mean zero. However, if one wished to investigate pure costs or pure synergies, the variable's distribution can be changed.

[8] Note that there are legal differences between the two processes: with a merger, a new firm must be incorporated, while with an acquisition, the charter of one firm simply disappears. However, it is hard to see how to include these differences using game theory, or what I would gain from including them.

buy the smallest firm. Figure 1 presents the merger process graphically for a state where there are three firms active.

Recall that every time two firms merge, they pay/receive some cost/synergy, which is drawn from some random distribution, and that at any given place in the merger game, one firm is the potential 'acquirer' and all the other firms that are smaller than it are the potential 'acquirees'. I assume that at any given time, each pair of firms in the merger process has a different draw from the cost/synergy distribution. Thus, if there are M potential acquirees (and 1 potential acquirer), there are M draws required. I assume that these synergy draws are unknown until the current round of the merger process at which point all M draws are known to the potential acquiring firm, and that all draws are independently drawn from the same distribution.[9] The justification for this independence is that firms do not know how well they will match with their competitors until they actively start bargaining; admittedly there are problems with the iid assumption, especially for possible estimation work.

With this structure that I have imposed, I have transformed the bargaining problem: without structure, there would be N firms who could form arbitrary mergers among themselves, with externalities to non-participants. Recall that there are now many stages to the merger process. At each stage, I can now formulate the

[9]Note that the actual value of the draws are known to the potential buying firm before the firms have to decide whether or not to merge. One might think that there may be some additional uncertainty in coordination between the firms that may not be known until after they have merged. Although this is likely to be the case, the fact that firms are risk neutral in my model means that I do not have to consider post-merger synergy draws, as I can integrate out over the values of these. For computation purposes, it is necessary that there be some random draw that is realized before the mergers have occurred. Otherwise, the merger decision would always be the same at every state, and the synergies would not smooth out the reaction functions and ensure existence of equilibrium.

problem as having M firms[10] which have something to sell (themselves), and one firm which can buy from only one of the others or from none of them. As firms care whether or not a competitor is acquired, externalities to non-participants still exist in this model. Thus, the problem is an M-person bargaining model with externalities and some hidden information, namely the cost/synergy draws of competitors. Since there is no widely accepted solution concept to this bargaining model, I demonstrate the results using two different structural solution concepts: a simple take-it-or-leave-it (tioli) offer scheme and an auction model. I discuss the tioli and auction models separately.

A simple model for the stage merger process is to have the potential acquiring firm choose any one potential acquired firm and give it a take-it-or-leave-it acquisition offer. In this framework, the acquiring firm would offer to buy another firm for a set price, and if the potential acquired firm turned it down, the acquiring firm would not be able to buy any other firm in the current round.

The main advantage to the tioli model is its simplicity. Because there are no simultaneous actions in the game, there exists a unique equilibrium solution to the game, conditional on the V^{PM} values; see Chapter 3 for details. Recall that firms make their merger (and other) decisions in order to maximize the EDV of net future profits. With this objective function, the solution satisfies that the net gain from acquiring any firm (defined as the combined value of the firms that merge minus their values as separate firms) plus the realized synergy draw must be greater than the net gain for any other acquisition and greater than zero, for the potential

[10] This M varies and is smaller than the previous N, as not all firms can bid for other firms at any given time.

acquiring firm to make an offer. The offer, which will always be for exactly the firm's reservation price, will be accepted. As shown in Figure 2, one can think of the result as being formed from overlapping uniformly distributed bars from which draws are taken. The acquired firm, if any, is the one which has the highest draw and is greater than zero. Thus, in order to compute the pre-merger values of firms, one must integrate out over the joint distribution of net gains plus synergies, in order to find the probability of occurrence of each merger and the resulting value to each firm.

Because the bars are overlapping, at any state, the equilibrium does not enumerate, with certainty, what industry structure will result after the merger process has occurred. Rather, it provides an equilibrium probability distribution over the possible mergers, that acts much like a mixed strategy.[11] In this way, having the cost/synergy smoothes out a firm's perceptions of the probability of occurrence for mergers in which it is not involved and eliminates the problem of discontinuity.

Although the tioli model is easy to solve analytically and computationally, its simplicity may lead to some limitations. In particular, the tioli model cannot fully capture the dynamics between the different selling firms. For instance, if firm A makes an offer to firm C, then firm C knows that if it rejects A's offer, then A cannot make another offer, and the token passes on to B. However, in the merger model, the final coalition structure of the industry results in externalities to non-participants in the merger process. This means that A not merging with anyone might have very

[11] In fact, an alternate interpretation of the cost/synergy is as a structural form of a mixed strategy. A standard result shows that as the synergy distribution converges to a degenerate zero, the equilibrium payoffs will converge to mixed strategy equilibrium payoffs. As noted earlier, this is why the cost/synergy helps ensure the existence of equilibrium.

different implications for C then A merging with B. Applying Proposition 3.2, the reservation price (using the tioli model) for firm C in this example is the value that C faces if A does not merge with any firm. Thus, the tioli model does not directly capture the externality to non-participants of the coalition formation process, which in this case is reflected by the fact that if C refuses A's offer then A might still be able to merge with B.[12] Because of the potential importance of the externalities, one may want to use a richer framework that is able to incorporate the externalities. Additionally, because of the nature of a tioli model, the gains to mergers from the tioli model accrue exclusively to the acquiring firms, which will skew the results, and is at odds with the perceived empirical wisdom that it is acquired firms that exclusively gain from mergers.

A natural framework for the MM coalition formation process is an auction model. In an auction model, firms would bid for the right to buy another firm or to be bought. They would choose their bids in order to form a Nash equilibrium of some well-defined auction mechanism. The modeler can then solve for the Nash equilibrium of the auction mechanism, in order to find their bids. The version of the auction mechanism that I use is that each potential acquired firm simultaneously submits a bid, which specifies a price for which it would be willing to be acquired. The acquiring firm, knowing the synergy draws, chooses one or none from among the potential acquired firms.

Unlike the tioli model, the auction model directly captures the externalities and results in the gains from mergers being shared between the two merging parties. However, because of the simultaneous decision making and the

[12] Note that because of the dynamic nature of my merger process, even the tioli model will capture some of the externality effects because of future plays of the merger game.

externalities, one cannot prove the same properties about the auction model that hold for the tioli model. In particular, it is not always true that, conditional on V^{PM} values, an equilibrium to the auction game exists, and even if one exists, it is not necessarily unique. (Counterexamples are provided in Chapter 3.) One can prove, though, that provided the size of the cost/synergy distribution is sufficiently large, an equilibrium exists.

Although the computation of the auction model is less straightforward than for the tioli model, it is quite possible. The solution method is to find a vector of bids for the acquired firms that forms an equilibrium, noting that the acquired firms choose their bids in order to maximize the EDV of net future profits, conditional on information. Given a vector of bids, the acquiring firm will choose the firm which has the highest normalized bid (defined as the combined value of the firms that merge minus the acquiring firm's value alone minus the bid) plus the realized synergy draw, provided that is greater than zero. Thus, in equilibrium, there will again be a probability distribution over mergers, with the selected merger chosen as in Figure 2, and the pre-merger value functions can again be computed by integrating out over the joint distribution of normalized bids plus synergy draws.

In the results presented in Section 3, both the auction and tioli processes are used. While it is possible to get results from both types of processes, I prefer the auction process, because of the richer interactions between firms. Although comparing the merger processes to the ways in which merger possibilities are evaluated in actual industries would lend credence to the merger framework, it is not possible to do this, because of the large variety and idiosyncrasy of processes across firms. Thus, although the merger processes, as presented, are natural ones, they are by no means the only possible ones. In particular, one can define other processes where the ordering is more flexible, or processes with different auction mechanisms. However, I made the choice of particular mechanisms because it is possible to computationally solve for the equilibria of the model with these processes, and simulate the industry in order to characterize the industry equilibria. In addition, while the derivation of the computational techniques and properties of the model, as presented in Chapter 3, are quite cumbersome, the statement of the model, as presented above, is straightforward. In order to understand the results as presented in Section 3, it is not necessary to have read Chapter 3.

Section 3: Results

In order to evaluate whether the merger model can provide realistic answers as to which mergers are likely to happen and whether dynamics are important determinants of these answers, I have computed results from the model described in the previous section. The results that I present in this section are based on parameter values that specify static demand, investment, production, entry and exit costs, the discount and depreciation rates, and the synergy distribution. The exact parameters are detailed in Table 1. There is no modeling of antitrust law of any kind in the results that I present; however, in Chapter 2, I discuss antitrust law in depth.

The parameters used are relatively arbitrary as they are not chosen with any specific industry in mind. There are, however, two motivating factors behind the choice of parameters. First, the algorithm is too large to be computable for more than 4 firms. Thus, I chose parameters that result in industries where the concentration is most often 2 or 3 firms, in order to make sure that the results are not much affected by the bound on the number of firms. Second, besides mergers, there are two ways in which firms can increase their capital: incumbents can invest, while potential entrants can enter. Mergers allow for the gains from these methods of investment to be transferred across firms. Accordingly, if the cost of entry is lower than the cost of investment, then allowing for mergers in an industry allows for huge welfare gains, as incumbents can stop investing, and instead buy up new entrants. In order to prevent this possibility, I set the entry costs so that the cost of capital for entrants is roughly the same as for incumbents.[13] I can approximate the cost of capital for incumbents by giving the incumbent the same dollar value to spend on capital over two or three periods and examining what is the maximum expected increment in capacity that the incumbent can attain with the additional dollar amount of capital.

For this section, the methodology that I use is to compute the equilibrium of the industry and then to simulate the industry given the equilibrium policies, in order to find the distribution of these policies in equilibrium, as well as the distribution of welfare and other specifics that result from the equilibrium. There are three general types of statistics that I have presented: first, there are industry specific discrete variables, like the number of active firms at each period. For these variables, I have presented the percentage of time that the variable takes on each value. Second, there are industry specific continuous variables, like excess capacity or total investment at each period. I have listed the mean and standard deviation of

[13] In Berry and Pakes, which uses a similar model of an industry, the authors chose entry costs that result in the cost of capital being a couple of orders of magnitudes lower for entrants than for incumbents. The motivation for choosing different parameters here than used by Berry and Pakes was in order to avoid the implications of this difference in capital costs, which would not be appropriate in the context of an endogenous merger model.

these variables. Third, there are firm specific variables, like firm production levels and capacity levels. For these variables, I have counted each firm existing at each period as one observation, and presented the mean and standard deviation. Finally, I note that I have not listed standard deviations for any of the distributions (although as discussed, the continuous distributions are themselves described by means and standard deviations). The reason is that I have simulated the industry for a large number (50,000) of time periods, and thus most of the statistics presented are very accurate. The one exception to this is that for the discrete variables, events that happen very infrequently (such as having three mergers in one period) will have large standard errors relative to their values.

In Section 3.1, I evaluate different merger models in order to find to what extent and for what reason a change in specification changes the results. In Section 3.2, I use the merger model to examine to what extent the static literature is obtaining inconsistent results by not incorporating dynamic firm behavior.

3.1 Comparison of different specifications for the merger model

Table 2 presents the results for the two merger models, the auction model and the take-it-or-leave-it (tioli) model. (Table 2 also presents the results for the 'myopic' and no merger models, which I discuss in Section 3.2.) Both the auction model and the tioli model show that mergers often happen in the industry. What are the reasons behind firms' decisions to merge? The reader will note that there are no differences in the production costs between firms. As firms are not merging in order to reduce costs, mergers serve five main purposes. First, they decrease the competition in the industry and increase profits that way, as after a merger, the Cournot production game is played with fewer firms. Second, by decreasing competition, mergers allow firms to partially internalize the negative externality that their investment has on other firms in the industry. Third, they are a mechanism for increasing capacity, which substitutes for investment. As the investment process as specified allows only for slow growth, mergers are a way in which firms can grow more rapidly. Fourth, they are a way of transferring the capacity of weak firms that would otherwise have exited the industry and received only the scrap value for their capacity. Fifth, they are a means of capturing positive synergy draws, as synergy draws can only be obtained by merging. It is important to note that in the models I have presented, as in actual industries, there are occasions when these positive effects of mergers result in firms merging, and other occasions when they do not result in firms merging. The frequency of mergers and other industry characterizations depends on the type of process (i.e. auction or tioli) that is used.

The most immediately noticeable difference between the auction and tioli models is that the auction model results in a much less concentrated industry than the tioli model: in the auction model, a duopoly is often observed, while in the tioli model, there is almost always a monopolistic structure. In addition, the auction model has much more entry, exit, investment and even mergers than the tioli model. This results in the auction model having a lower producer surplus than the tioli model, however, the lower industry concentration results in a higher consumer surplus than the tioli model.

The reasons for these differences between the two models stem from the specifications of the merger models: in the tioli model, a merger will occur whenever one is mutually beneficial to both the buying and selling firm. In the auction model, this is not necessarily the case: selling firms will set their bidding prices high enough that optimal mergers will not always occur, in order to capture more of the rents from the mergers that actually occur. Thus, the lower concentration and lower producer surplus in the auction model are the result of the auction bargaining game, which is less efficient for the firms than a take-it-or-leave-it bargaining game. With a tioli model, new firms in the industry do not capture any of the gains from a merger; this is why entry is much lower in the tioli model. The smaller number of mergers in the tioli model is simply a result of the much smaller number of entrants; conditional on entering, a potential entrant is more likely to be acquired in the tioli model than in the auction model.

Thus, in the tioli model, the industry structure tends to be one large monopolist, who has little excess capacity, invests comparatively little and buys up any firm that enters very quickly. It is because the tioli firms are able to largely internalize the externality of investment that the tioli model results in a higher producer surplus than the auction model. In the auction model, there tends to be one or possibly two smaller firms, who together have more excess capacity. If an entrant enters, it will typically stay in the industry for a few periods, bidding sufficiently high so that only a high synergy draw will cause another firm to buy it. In both models, the chance of exit (except by being acquired) is quite remote: about 1 in 5 entrants later exit the industry in the auction model, while fewer than 1 in 25 entrants exit in the tioli model. (Again, the smaller ratio is explained by the fact that some efficient mergers which would have prevented exit do not happen in the auction model.)

While the results presented show that the auction and tioli industries behave as one would expect, this is not sufficient indication as to whether the general results of this endogenous merger process are significant, or so arbitrary as to be useless. In order to test how significant are the results that I presented are, one must determine how changing the specification of the models would affect the conclusions. This can also inform us at to whether there are general properties that can be determined about the results of the model, or whether the results are subject to wild fluctuations depending on the parameters. If I could evaluate the solution to this model analytically, I could perhaps also characterize the solution as satisfying certain properties, in order to be able to generalize the answers. This model, though, cannot be solved analytically, and it is not possible to characterize properties about the equilibrium analytically either, because of its complexity.

The difficulty in obtaining analytical results as to the sensitivity of the model does not mean that the task of examining the sensitivity of the model is hopeless. In fact, there is a natural way to proceed: this is to change the specification of the model, recompute the equilibrium for the new specification, and examine how the implications of the model change with different specifications. I have recomputed the results of the model for some different specifications, and reported these results in Tables 3 and 4.

One of the possible concerns with this model is that the results are driven by the cost/synergy distribution, which is influencing which mergers happen to a large extent. In order to determine to what extent the synergy distribution affects the results, I examine the auction and take-it-or-leave-it models, when I increase the size of the synergy distribution from $[-0.2,0.2]$ to $[-0.25,0.25]$ and $[-0.3,0.3]$. I did not evaluate the results for smaller synergy distributions, because of the difficulty in getting the program to converge for smaller distributions. The results are presented in Table 3. The results show that larger synergy distributions have a negligible effect in the tioli model, and a slightly larger effect in the auction model. In both models, the effects of a larger synergy distribution is to have a lesser concentration level in the industry. From the table, one can see that the mean realized synergy draw rises significantly when the size of the distribution is increased. Additionally, one can see that not only does the mean realized synergy draw rise when the size of the distribution is increased, but the quantile of the mean realized synergy draw rises when the size of the distribution is increased, which is because firms that are likely merger partners wait longer in order to receive a sufficiently large synergy draw before merging, given a larger synergy distribution.

The result of firms waiting before merging is that in general, consumers gain from having more competition in the industry. Accordingly, consumer surplus rises with larger synergies, while producer surplus falls slightly or remains constant, depending on the model. In the auction model in particular, it appears that increasing the size of the randomness makes the process less efficient for the firms but more efficient for consumers and overall; however, it is not known whether this result holds in general. Additionally, in both models, because firms are not merging so often, the firms that are there are smaller. Changing the synergy distribution appears to have negligible effects on the amounts of entry, exit and investment in the model.

From the table, one can see that for all of the models, the mean realized synergy draw is always positive. In the auction model, in particular, the standard errors show that firms rarely, if ever, merge when there are negative synergies. These results indicate that part of the realized gain from mergers in this model is the synergies. However, from changing the distribution of the synergies, one can see that the size of the distribution has little effect on any variable except for the realized synergy draw, for the range of values tried. This suggests that the reason that firms only merge when there are high synergies, in the auction model, is that because of the first-price auction mechanism, they must bid high in order to capture some of the realized synergies, and not because mergers would not be profitable if the synergies were not positive. Thus, the synergies do not appear to be the primary determinants of merger decisions between firms. Rather, it would appear that the use of acquisitions as a substitute for investment and in order to reduce competition in the industry are the major determinants of the merger decision.

A second possible concern with the model is that the ordering of the merger process is influencing the results a great deal, by giving an advantage to one or the other firm. In order to examine to what extent the ordering of potential buying firms in the merger process affects results, I have evaluated the auction model with five different ordering schemes, including the regular ordering. The results from these experiments are presented in Table 4. The ordering schemes presented are as follows: the regular ordering, a reverse ordering, an inside first ordering, an outside first ordering and a shifted ordering. The table lists the order that each firm would be the buying firm under each ordering, assuming that there are five firms and the capacities of them satisfy $A > B > C > D > E$.[14]

From the results, one can see that most of the industry characteristics change only a small amount, between the different models. In particular, entry, exit, investment, firm sizes, and welfare variables are all very close together. As a measure of closeness, I note that the range of these variables over the five different ordering schemes does not overlap with any of the other models presented, such as the tioli model, the model with no mergers, the myopic model or the models with larger synergy distributions. The variables which exhibited the largest change are the concentration measures. Here also, the amount of change is quite small (and the range does not overlap): the percent of periods with one firm active ranges from 58.1 percent to 69.7 percent, while the percent of periods with two firms active ranges from 29.2 percent to 39.8 percent. In general, the reverse orderings tended to lead to a less concentrated industry, because the pairings of firms that can merge in this case is not optimal for the firms, which causes them to wait until subsequent periods before merging.

The reasons for the magnitudes of the changes are quite straightforward. Because of the dynamic structure of the model, the merger process repeats itself every period with the firms that are active in that period. This means that to a certain extent, even if a particular merger can only occur at a certain point in the

[14] Another ordering scheme that can potentially be used is a random ordering, where each period, firms are randomly drawn out of a hat and go through the merger process in the order in which they are drawn. Although the conclusions of the random ordering may be interesting, it is much more computationally intensive than a fixed ordering, which is why I have not presented results for the random ordering.

merger process, this will not affect the results: Since the process keeps repeating itself, there is an endless cycle over all the merger possibilities that repeats itself from period to period. Different orderings then do not have very different implications in the overall structure of the game. In addition, in the auction model, unlike in the tioli model, the gains from the mergers will be split pretty evenly between the buyer and the seller, because of the decisions underlying the choice of optimal bid in the auction process. Thus, different orderings do not distribute the gains from mergers, conditional on them happening, very differently either. Because of these reasons, which are verified by the equilibrium results, the ordering of the firms will not be an important factor in determining the implications of the model.

3.2 The importance of dynamics in modeling mergers

Now that I have established results as to the sensitivity of the endogenous merger process, I would like to evaluate whether dynamic aspects of firm behavior are important in determining which mergers happen. In order to compare my model to the static literature on mergers, I have presented results from two different comparable models in Table 2, along with the base results. The first model that I discuss is a model of a similar industry, but where mergers never happen, and where firms know that mergers never happen. This comparable industry is then just a particular version of the Ericson-Pakes model.

The most striking difference between the industry with mergers and the industry without mergers is that when moving from allowing for mergers to not allowing for mergers, the amount of investment, entry and exit in the industry changes dramatically. In both the tioli and auction merger models, the effect of allowing mergers is to increase the amount of entry in the industry, relative to having no mergers: with mergers, a new entrant typically enters to eat away at the monopoly rents in the industry, knowing that this may cause the dominant firm in the industry to buy it, which it typically does. Without mergers, new entrants do not disappear from the industry so soon, because they cannot be bought by other firms, and because there are no monopoly rents to earn. Similarly, the ability of incumbent firms to buy struggling firms decreases the amount of exit as a percentage of the amount of entry. Finally, investment by incumbents drops relative to the model with no mergers, as mergers (or external investment) substitute for internal investment and firms are able to partially internalize the negative externality from their investment. (When adding to investment by incumbents the amount spent on capacity by new entrants, the tioli model has less investment than the model without mergers, while the auction model has more.)

In addition to these differences in entry, exit and investment, I observe that, under both merger models, the industry is much more concentrated than without mergers, and that the average firm is substantially larger. The reason for the increased sizes of firms is that with mergers, firms tend to buy small firms in order to monopolize the industry, which increases their capacities more than in the no merger case. Not surprisingly, when mergers are allowed, the increased concentration results in lower consumer surplus than when mergers are not allowed, and higher producer surplus. The total surplus is also lower with mergers, largely because of the simple static reason that there is a deadweight loss from increased concentration.

These major differences between the model with mergers and the model without mergers listed above underscore the importance of endogenizing entry, exit and investment. These variables all change a great deal depending on whether or not mergers are allowed in an industry, and their change has a significant effect on which mergers happen and what their consequences are. In particular, entry ensures that firms do not simply merge for monopoly and earn monopoly rents forever. In addition, this points out the need for an endogenous merger process, in order to properly evaluate the effects of dynamics. Because previous static and dynamic merger models have not endogenized investment and entry, they have not fully examined the differences in entry and investment behavior that occur when mergers are allowed, and hence they have not examined how different entry and investment levels will affect the welfare calculations and other variables of interest. In sum, because the decisions to merge in the static literature do not take account of their dynamic effects on future investment and entry, the conclusions of the static literature as to the effects of mergers are likely to be flawed. And, it is only with a dynamic model of endogenous mergers that one can fully understand the scope of the importance in dynamics.

The model without mergers shows that dynamic variables change greatly when mergers are allowed. In order to further show that the static literature may lead to misleading conclusions about mergers, I have examined what the conclusions of the merger model would be if firms did not base their decisions on dynamics, but instead made their merger decisions statically, as in the traditional merger literature. Accordingly, I have presented the results from a 'myopic' version of the auction model, also in Table 2. Here, the firms face the same auction process as in the regular auction model, however, they make their decisions to merge assuming that at the end of the merger process, they will receive the one-shot Cournot profits from whatever industry structure is present at the end of the merger process. Thus, this corresponds to changing the V^{PM} values used in the merger decisions from the true EDV of future profits to reduced-form Cournot profits. While investment, entry and exit decisions continue to be made rationally, merger decisions are now made in order to maximize static profits, and therefore ignore the dynamics behind their decisions.

The results show that the conclusions of the 'myopic' model differ greatly from the conclusions of the regular auction process. In particular, in the 'myopic' model, there is almost no entry, and the industry is almost always a monopoly with the resulting low consumer surplus, high producer surplus and low total surplus, similarly to much of the static literature. These extreme results are not surprising: firms do not want to enter, knowing that if they enter, they will be bought up cheaply, because their future dynamic profits do not factor into their merger reservation values. Similarly, incumbents are willing to buy them, foolishly thinking that they will earn monopoly profits, and not considering the effects of future entry on these profits.

To further examine how static and dynamic decision-making for the merger process yields different results, I have taken a particular industry structure, one where there are four symmetric firms active, each with state 4 (and hence capacity 0.4), and compared the predictions of my model directly to that of a static model. Thus, I am directly applying my endogenous merger process to a static game where the payoffs are the reduced-form Cournot profits, and then I am comparing the static endogenous merger process to the dynamic one. As even endogenous static models do not predict the long-run equilibrium distribution of industry structures, the way to compare a dynamic model to a static model is to run the model for one period and evaluate the results for the static model and for the dynamic model. Because of the random cost/synergy draws of my model, I do this by repeatedly running the model for one period and evaluating the distribution of mergers that would occur, both for the rational model (which is valid if firms incorporate dynamics into their behavior), and for the myopic model (which is valid if firms do not incorporate dynamics into their behavior).

The results, presented in Table 5, show that if firms make their merger decisions myopically, the outcome will be greatly different. In particular, with non-myopic dynamic decision-making, firms were unlikely to merge with this structure, because of the threat of entry, and thus a merger occurs less than 3 percent of the time. Furthermore, given that a merger does occur with non-myopic decision-making, the most likely outcome is an industry structure of $[8,4,4,0]$. With myopically static decision-making, firms choose to merge far more often: in about 75 percent of the periods. In addition, they most often choose to merge into two firms, with a resulting industry structure of $[12,4,0]$, and quite frequently into monopoly.

Both examples given with the myopic model as well as the model without mergers show that the assumption that firms make their merger decisions without thinking about the dynamic consequences of their decisions leads to very different results from the assumption that firms make their merger decisions knowing the dynamic effects of these decisions. In particular, one can see that a static model of mergers is likely to predict a much more concentrated industry structure than a dynamic model. Given that firms in the real-world do account for dynamics in their decision process, this shows that the results from the static literature are inconsistent, and that dynamics are necessary in order to define a process that models mergers realistically.

Section 4: Conclusions and Further Research

In this paper, I have proposed a model of endogenous horizontal mergers for firms in a dynamic setting, evaluated the model numerically, and discussed how the conclusions of the model change depending on the specifications. Finally, I have used the model to examine the size and direction of the bias introduced by examining mergers in a static setting, without incorporating entry, exit and investment into the decision-making process of firms. The results show that the industry performance will vary depending on the type of merger process that one uses: an auction type process will give less concentrated industry structures than a take-it-or-leave-it type process. However, varying the ordering in which firms merge within these structures and varying the size of the synergy distribution when they merge have much more minor effects on the resulting industry structure. In addition, the results show that the static literature is likely to obtain inconsistent results that overstate the amount of mergers that will occur in an industry.

Recall that the goal of this paper is to develop a coherent model of endogenous mergers and apply this model in order to find out the importance of dynamics in determining firms' merger behavior. While this is not the first paper to look at endogenous coalition formation and it is not the first paper to examine the dynamic implications of mergers with game theory, it is the first paper that combines all of these elements in order to be able to predict when mergers occur in an industry and how the mergers that occur change with the specification. Thus, the research presented above should not be thought of as the last word on the modeling of endogenous mergers with dynamics, but rather as the first word.

As this is the first model of its kind, there are many ways in which the model could be made more realistic. In particular, I designed the merger process in order to try to have the technology of merged firms be the combination of technologies of the unmerged firms. Although this is the case for the static production possibility set, it is not the case for the dynamic investment possibility set: one of the potential differences between a merged firm and its two unmerged counterparts is that when two firms merge, they go from each having an uncertain investment or R&D process, to having only one uncertain investment process. Thus, the stochastic investment realization possibilities of a merged firm are different than the sum of the investment realization possibilities for the two unmerged firms. This difference between unmerged and merged firms may create incentives to merge or to not merge that probably do not reflect reality. The solution to this problem is to allow for multiplant firms. Then, when two firms merge, they would simply keep all of their plants, or shut some of them down, as they wished. Allowing for multiplant firms will also allow for the examination of mergers in differentiated products markets. In future research, I will be examining mergers in this multiplant setting. In addition to this problem, one of the limitations of this model is that I am only able to compute it for four or at most five firms. The solution to this problem is faster computers and better algorithms for computing industry equilibria, both of which are being developed at a rate that suggests that computation of the model for many more firms may soon be feasible.

In spite of the many limitations of the model, a dynamic model of endogenous mergers such as this one is useful in many ways. In particular, a model such as this one can help answer policy-relevant questions concerning the impact of antitrust law in different types of industries. Because of the computational nature of this model, it is easy to add government antitrust policy to the model and to examine the welfare and other implications of mergers for different policies and different industries. This is the focus of Chapter 2, where I use the model developed here to examine the implications of mergers and antitrust law given such phenomena as barriers to entry and increasing returns.

More generally, I view this model as being a potential basis for future empirical work with mergers. In the long run, one could conceivably estimate the dynamic equilibrium in an industry structurally, allowing for firms in the industry to merge according to the processes specified. This would then serve as a useful tool for governments to determine what the effects of different antitrust policies would be. Although potentially attractive, realistic dynamic structural estimation remains largely infeasible for most industries. In addition, governments will likely be extremely hesitant before understanding or accepting the conclusions of a non-linear structural dynamic estimation process. Accordingly, I expect that the bulk of estimation work on the effects of mergers and antitrust law will continue to be static. However, a dynamic model of endogenous mergers can help even if fully dynamic estimation of mergers cannot be done: by allowing economists to examine the implications of policies in this dynamic setting for different models, we can examine different industries to see how different the conclusions of the dynamic setting are from those of the static setting. Thus, a dynamic computational model of mergers can help determine the size of the bias and imprecisions, for different industries, caused by ignoring the dynamics underlying firms' decisions. This is evidenced by the results that compare the dynamic model to a static version, which show how large the bias will be for a static merger model. This type of analysis can be extended to examine the directions of the bias for many different static specifications, which will allow economists to further understand the impact of mergers in different industries.

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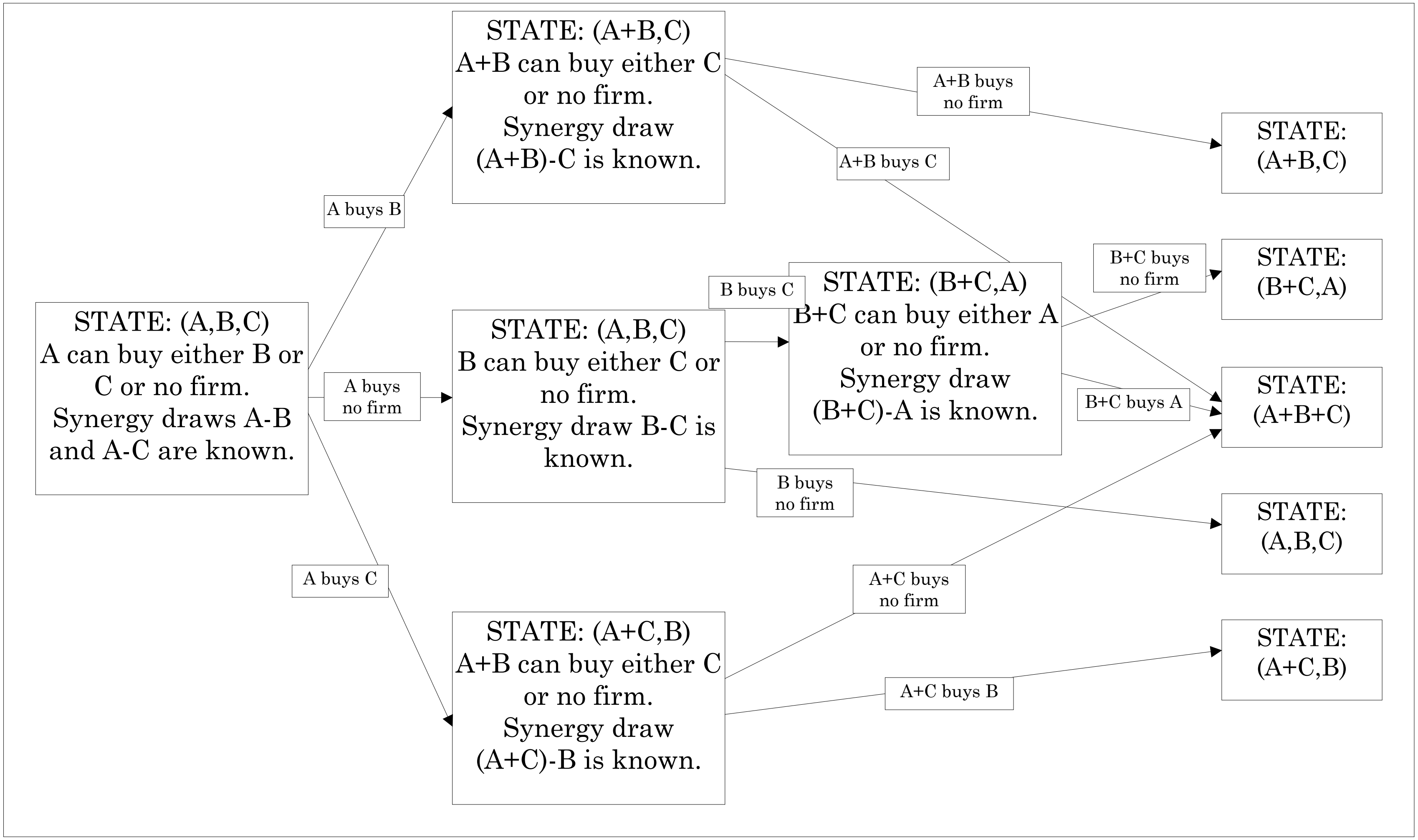

Figure 1

Merger Process for a state with three firms A, B, and C, where A > B > C, and B + C > A.

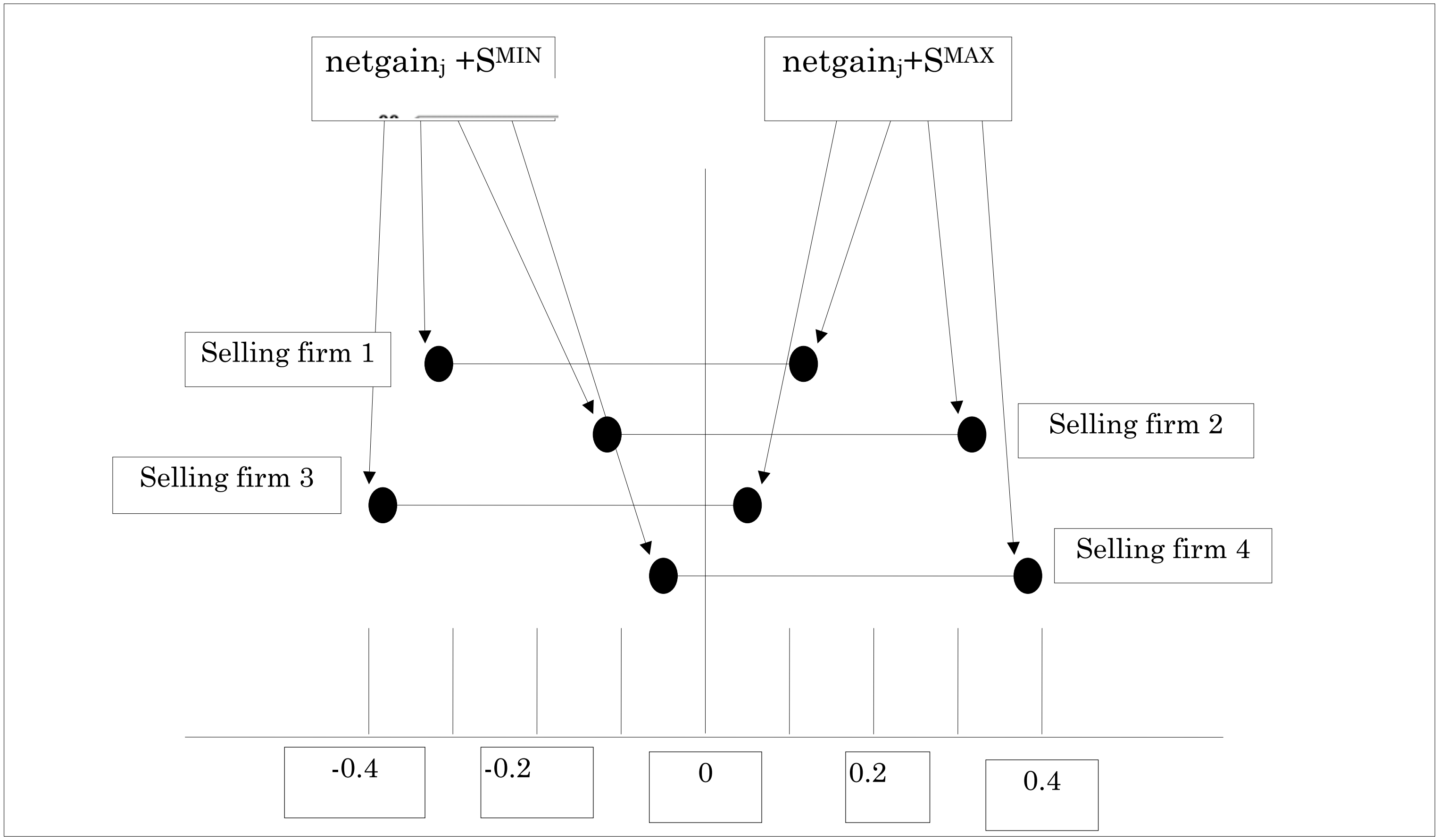

Figure 2

Outcome of MM^{tioli} merger game -
Each selling firm gets a uniform draw from its bar;
the buying firm makes an offer to the firm with highest draw, if greater than zero;
this firm will then be bought.

Parameter	Explanation	Value
α	Ratio of capacity to state	0.1
a	Investment efficiency parameter	35
δ	Industry capacity depreciation rate	0.7
β	Discount rate	0.925
D	Demand for good, i.e. $Q^D(P)=D-P$	4
mc	Marginal cost	2
N	Maximum number of firms	4
k_E	Entrant's initial capacity	2
Φ	Exitor's scrap value	0.1
S^{MIN}	Minimum cost/synergy from merger	-0.2
S^{MAX}	Maximum cost/synergy from merger	0.2
x_E^{MIN}	Minimum entry cost	1.1
x_E^{MAX}	Maximum entry cost	1.5

Table 1

Parameters used in model.

Value in Industry Equilibrium of:	Merger model with auction process	Merger model with take-it-or-leave-it process	Merger model with auction and myopic behavior	Model with no merger process
% of periods with 1 firm active	58.1%	98.4%	99.94%	0.29%
% with 2 firms active	39.8%	1.62%	0.053%	83.9%
% with 3 firms active	2.02%	0.003%	0.008%	15.7%
% with 4 firms active	0.005%	0%	0%	0.05%
% of periods with entry	13.1%	3.9%	0.03%	0.36%
% of periods with exit	2.2%	0.12%	0.01%	0.35%
Mean total investment (standard deviation)	0.231 (0.377)	0.102 (0.224)	0.077 (0.045)	0.166 (0.105)
Mean inv. by incumbents only (standard deviation)	0.083 (0.064)	0.058 (0.027)	0.077 (0.036)	0.162 (0.071)
Mean excess capacity (standard deviation)	0.482 (0.274)	0.213 (0.173)	0.033 (0.076)	0.133 (0.195)
Mean firm production (standard deviation)	0.727 (0.350)	0.972 (0.116)	0.842 (0.170)	0.552 (0.150)
Mean firm capacity (standard deviation)	1.061 (0.609)	1.182 (0.235)	0.875 (0.212)	0.614 (0.209)
% of periods with no mergers	89.5%	96.3%	99.98%	100%
% of periods with 1 merger	10.1%	3.71%	0.015%	0%
% of periods with 2 mergers	0.37%	0.04%	0.008%	0%
% of periods with 3 mergers	0.003%	0%	0%	0%
Mean consumer surplus (standard deviation)	0.549 (0.084)	0.489 (0.041)	0.369 (0.129)	0.732 (0.233)
Mean producer surplus (standard deviation)	0.780 (0.381)	0.898 (0.228)	0.870 (0.126)	0.751 (0.123)
Mean total surplus (standard deviation)	1.33 (0.392)	1.39 (0.245)	1.239 (0.247)	1.48 (0.286)

Table 2
Comparison of equilibrium results for merger models.

Value in Industry Equilibrium of:	Merger model with auction process			Merger model with tioli process		
Uniform distribution of cost/synergy with range:	-0.2,0.2	-1/4,1/4	-0.3,0.3	-0.2,0.2	-1/4,1/4	-0.3,0.3
% of periods with 1 firm active	58.1%	50.9%	45.4%	98.4%	97.9%	97.6%
% with 2 firms active	39.8%	46.3%	51.1%	1.62%	2.13%	2.36%
% with 3 firms active	2.02%	2.84%	3.42%	0.003%	0.003%	0.005%
% with 4 firms active	0.005%	0.013%	0.05%	0%	0%	0%
% of periods with entry	13.1%	12.1%	12.1%	3.9%	4.4%	4.1%
% of periods with exit	2.2%	1.9%	1.89%	0.12%	0.15%	0.16%
Mean total investment (standard deviation)	0.231 (0.377)	0.229 (0.366)	0.234 (0.367)	0.102 (0.224)	0.109 (0.237)	0.105 (0.231)
Mean inv. by incumbents only (standard deviation)	0.083 (0.064)	0.091 (0.065)	0.096 (0.066)	0.058 (0.027)	0.060 (0.029)	0.059 (0.030)
Mean excess capacity (standard deviation)	0.482 (0.274)	0.453 (0.268)	0.450 (0.272)	0.213 (0.173)	0.208 (0.176)	0.215 (0.177)
Mean firm production (standard deviation)	0.727 (0.350)	0.700 (0.337)	0.682 (0.114)	0.972 (0.116)	0.966 (1.131)	0.966 (0.135)
Mean firm capacity (standard deviation)	1.061 (0.609)	0.998 (0.585)	0.966 (0.578)	1.182 (0.235)	1.169 (0.249)	1.177 (0.252)
% of periods with no mergers	89.5%	90.0%	90.2%	96.3%	95.9%	96.1%
% of periods with 1 merger	10.1%	9.69%	9.47%	3.71%	4.08%	3.83%
% of periods with 2 mergers	0.37%	0.28%	0.35%	0.04%	0.07%	0.005%
% of periods with 3 mergers	0.003%	0%	0.003%	0%	0%	0%
Mean realized synergy draw (standard deviation)	0.161 (0.024)	0.207 (0.029)	0.251 (0.034)	0.056 (0.084)	0.079 (0.097)	0.115 (0.109)
Mean consumer surplus (standard deviation)	0.549 (0.084)	0.570 (0.103)	0.586 (0.114)	0.489 (0.041)	0.488 (0.045)	0.490 (0.040)
Mean producer surplus (standard deviation)	0.780 (0.381)	0.781 (0.370)	0.777 (0.373)	0.898 (0.228)	0.892 (0.242)	0.898 (0.236)
Mean total surplus (standard deviation)	1.33 (0.392)	1.35 (0.389)	1.36 (0.395)	1.39 (0.245)	1.38 (0.261)	1.39 (0.251)

Table 3

Changes in equilibrium results from different cost/synergy sizes.

Value in Industry Equilibrium (for merger model with auction process) of:	Model with regular ordering	Model with reverse ordering	Model with inside first ordering	Model with outside first ordering	Model with shifted ordering
Ordering of buying firm	A-B-C-D-E	E-D-C-B-A	C-D-B-E-A	A-E-B-D-C	D-E-A-B-C
% of periods with 1 firm active	58.1%	68.6%	61.0%	53.0%	69.7%
% with 2 firms active	39.8%	30.3%	37.1%	44.6%	29.2%
% with 3 firms active	2.02%	1.2%	1.9%	2.4%	1.1%
% with 4 firms active	0.005%	0.004%	0.01%	0.02%	0.002%
% of periods with entry	13.1%	12.2%	13.1%	12.0%	12.1%
% of periods with exit	2.2%	1.8%	2.2%	1.9%	1.7%
Mean total investment (standard deviation)	0.231 (0.377)	0.211 (0.369)	0.229 (0.379)	0.227 (0.365)	0.214 (0.374)
Mean inv. by incumbents only (standard deviation)	0.083 (0.064)	0.073 (0.061)	0.080 (0.064)	0.090 (0.065)	0.071 (0.060)
Mean excess capacity (standard deviation)	0.482 (0.274)	0.414 (0.269)	0.471 (0.285)	0.439 (0.272)	0.439 (0.270)
Mean firm production (standard deviation)	0.727 (0.350)	0.774 (0.342)	0.736 (0.354)	0.708 (0.339)	0.781 (0.340)
Mean firm capacity (standard deviation)	1.061 (0.609)	1.085 (0.567)	1.07 (0.612)	1.00 (0.582)	1.12 (0.575)
% of periods with no mergers	89.5%	89.9%	89.5%	90.1%	89.4%
% of periods with 1 merger	10.1%	9.8%	10.1%	9.6%	10.2%
% of periods with 2 mergers	0.37%	0.31%	0.39%	0.30%	0.31%
% of periods with 3 mergers	0.003%	0.004%	0.004%	0.002%	0.004%
Mean consumer surplus (standard deviation)	0.549 (0.084)	0.529 (0.067)	0.540 (0.076)	0.563 (0.098)	0.528 (0.064)
Mean producer surplus (standard deviation)	0.780 (0.381)	0.802 (0.373)	0.784 (0.383)	0.780 (0.369)	0.800 (0.378)
Mean total surplus (standard deviation)	1.33 (0.392)	1.33 (0.381)	1.32 (0.392)	1.34 (0.387)	1.33 (0.386)

Table 4

Comparison of equilibrium results from different ordering of firms.

Result of Merger Process given a Starting Structure of [4,4,4,4]:	Probability of Occurrence of Structure	
Final Industry Structure:	Regular (Dynamic) Model	Myopic (Static) Model
[4,4,4,4]	97.1%	25.6%
[8,4,4,0]	2.51%	19.2%
[8,8,0,0]	0%	0%
[12,4,0,0]	0.35%	38.7%
[16,0,0,0]	0.08%	16.6%

Table 5

Final industry structure resulting from one merger process in regular and myopic industry equilibrium, given that the industry is started with a structure of [4,4,4,4].

Chapter 2:

Policy Implications of a Dynamic Horizontal Merger Model.

Section 1: Introduction

While antitrust laws restricting horizontal mergers have existed in the United States for over a century, there has been and continues to be much debate regarding their role and effectiveness. Economists agree that there are several reasons why firms might merge horizontally. These include managerial advantages, strategic improvements, shared technology and cost and efficiency gains; the reader may consult Posner (1976) or Scherer and Ross (1990) for a complete discussion. In spite of the near consensus on the reasons for mergers, economists remain divided on the desirability of different antitrust policies. Over the years, the arguments proposed in this discussion have belonged largely to two schools: on one hand, there is the Chicago School, which has tended to be more laissez-faire, and on the other hand, there are the more interventionist economists.

The standard interventionist justification that mergers may be undesirable is quite straightforward. As Hay and Werden (1993) write, "The main social cost that has been associated with horizontal mergers arises from their potential to raise price and restrict output either by altering the outcome of an unchanging oligopoly interaction, or by causing there to be a different sort of interaction."[1] This potential price-raising effect resulting from a merger, or more generally from increased concentration, has generally been referred to as collusion or market-power. In addition, as Scherer (1980) points out, if policy-makers are in doubt as to whether a merger should be prevented for economic reasons, it may be better to err on the side of preventing too many mergers, because increased concentration may be

[1] p. 1.

intrinsically bad for non-economic reasons such as the ability of big firms to influence government policy.[2]

Although these arguments against mergers are very direct, the Chicago School of antitrust has refuted them, by discussing reasons why they might not be valid. One of the principal arguments that the Chicago School uses is that potential future entry may lessen the likelihood of collusion. For instance, when discussing the Von's Grocery case of the 1950s, Posner argues that the proposed takeover of Von's Grocery by Shopping Bag would not have had large deleterious effects, because of the ease of entry in the retail supermarket industry and the absence of significant economies of scale. In general terms, the premise of the Chicago School has been that, with few exceptions, the market operates efficiently without any antimerger law except for prohibitions of mergers for monopoly. Thus, by attempting to change the interactions of a market that is already performing efficiently, antitrust law rarely helps as intended, and may often have unforeseen harmful consequences. As an example of such consequences, consider the Brown Shoe case (also from the 1950s) where the courts argued that Kinney should not be allowed to acquire Brown Shoe, in order to preserve small firms, such as Brown Shoe, in the shoe retailing business. Posner argues here that the main result of the prevention of this acquisition was to decrease the value of owning a small business (such as Brown Shoe), as the option value from owning one would then be smaller as it would no longer incorporate the possibility of being acquired. As another example, Posner argues that if there are large economies of scale in an industry, a concentrated structure is the most viable outcome, and thus the industry is likely to become

[2] p. 546.

concentrated in the long-run whether or not mergers are allowed. In this case, antitrust prohibitions on mergers will not lessen the long-run concentration but may simply act to slow down the efficient allocation of resources, as companies will have to grow by slower internal expansion. In both of these examples, the long-run effects of strict merger prohibition would be to hurt the parties that the law is trying to help and to lower the overall efficiency of the economy.

From the preceding paragraph, the reader can see that the Chicago School antitrust arguments are dynamic in nature: their results account for the fact that firms make their decisions to merge, exit, enter and invest knowing that these decisions have long-run consequences and that changes in policy will affect the decisions chosen and through them the performance of the industry. Unlike the standard interventionist view of antitrust policy, the Chicago School's view incorporates the fact that although the immediate effect of a prohibition of a merger is to keep the industry less concentrated, the long-run effects may be quite different. However, while the Chicago School does take account of the dynamic determinants of firm behavior, they have not modeled these dynamics explicitly. The approach used by Posner and others has instead been to state these dynamic reasons in order to refute the standard interventionist criticisms of mergers. Thus, it has not been possible to find out how important these dynamic effects are relative to the simple static effect of an increase in concentration. Because of this, it has not been possible to evaluate for which situations the Chicago School arguments are more correct and for which situations the interventionist arguments are more correct, and accordingly what optimal antitrust policies should look like. It is because it has not been possible to evaluate the scope of these dynamic effects that the antitrust debate has not been settled and that economists continue to disagree as to the optimal scope of antitrust policy.

With the advent of game theory to industrial organization, it became possible to analyze the effects of mergers quantitatively. This led to the development of a game theory literature on mergers and antitrust law, characterized by papers such as those by Salant, Switzer and Reynolds (1983), Deneckere and Davidson (1985) and Perry and Porter (1985). Unlike the earlier antitrust literature, these game theory models were able to explicitly evaluate the welfare result of any particular merger. However, the game theory literature has, thus far, been limited in the range of questions that it can answer, because it has not fully endogenized the merger process and has largely not incorporated dynamic determinants of mergers in its models. Thus, game theoretic models have been able to discuss which set of mergers would yield a (static) surplus gain over no merger, and which set of mergers firms would prefer to no merger. However, the game theory literature has thus far not been able to answer questions posed by the Chicago School such as when the effects of entry (as cited by Posner) are large enough to mitigate the negative effects of mergers and render a merger socially optimal.

In general terms, the game theory literature has not been able to answer the principal questions of the antitrust debate, which are to evaluate what effects different antitrust policies would have on the structure and performance of different industries and accordingly what optimal antitrust policies look like. The reason that the game theoretic models have not been able to answer these questions is exactly because, in order to examine the long-run effects of antitrust policy, one must have a model that endogenizes the merger process and incorporates the dynamic determinants of firm behavior. Because the literature has lacked a model which is able to predict equilibrium merger behavior as a result of policy changes, it has not been able to shed new light on these principal questions.

The goal of this chapter is to quantitatively analyze the effects of antitrust policy on long-run industry structure and performance using a framework that incorporates the dynamic determinants of firm behavior. With the model that I developed, I am able to directly analyze issues central to the debate on antitrust law, such as what the optimal antitrust policies should be in the presence of barriers to entry or increasing returns. Thus, I am able to directly examine when dynamic effects are important and the Chicago School arguments are correct and when the dynamic effects are not sufficient to outweigh the standard interventionist justifications for antitrust policy. The reason that I am able to accomplish this goal, which has not been done before, is that I am using a model that endogenizes mergers and that incorporates the dynamic determinants of firm behavior. No previous paper in the merger literature has used a model which is able to directly specify the equilibrium probabilities that different mergers will happen, given some starting industry structure.

The model I use, which is developed in Chapter 1, specifies a conception of an infinitely-lived industry, where current and potential firms rationally make entry, exit, investment and acquisition decisions in order to maximize the expected discounted value (EDV) of net future profits. Randomness is present in the model, in terms of uncertain returns to the R&D investment process, random sunk costs of entry, and a random cost or synergy for merging firms. As I discuss in Chapter 1, because of the complexity of a dynamic model that endogenizes entry, exit, investment and merger decisions, it is not possible to analytically solve the equilibrium of this model. The approach that I use instead, is to numerically solve the equilibrium of the model given specific parameters and evaluate the equilibrium structures of the industry using simulation.

In the following sections, I first present the results of the model for different antitrust policies, and compare these to first-best and perfectly collusive solutions. In order to characterize the equilibrium, for each antitrust policy I have listed equilibrium values of industry variables, such as concentration, number of mergers, size of firms, entry, exit, investment and welfare indicators. These values are presented in table format, and the total welfare indicators are also presented graphically. As discussed above, the purpose of these results is to answer the central questions of what roles antitrust serves, how close can antitrust policy bring us to the first-best solution and how bad is pure collusion. In addition to these basic questions, much of the debate on antitrust has focused on analyzing the role of government policy in the presence of different factors that might make mergers more or less troublesome. I examine some of these factors, by investigating the effects of mergers in the presence of production cost differences, merger transaction costs, synergies and barriers to entry. Finally, I examine the empirical literature that uses stock market values to analyze how the gains from mergers are split. As this literature has obtained results that have defied explanations with static non-endogenous merger models, I have compared the model's predictions of how gains from mergers are split to those of the empirical literature to see if this model can explain the empirical results better. The results presented in the following sections show many surprising patterns, including the fact that both the Chicago School and the interventionist school of antitrust may be correct and that in certain cases, stricter antitrust policies may actually lead to increased industry concentration.

The remainder of this Chapter is divided as follows: Section 2 examines the basic model and antitrust results; Section 3 examines the effect of production cost differences on antitrust policy; Section 4 examines the effects of merger transaction costs and synergies and sunk costs of entry; Section 5 examines the splitting of gains from mergers as evidenced by the change in stock market values of firms involved in mergers; and Section 6 concludes.

Section 2: The Model and Antitrust Results

The results that I discuss in this Chapter are all based on the model developed in Chapter 1. Recall that the model that I use incorporates endogenous processes for mergers, investment, entry and exit. The model takes a version of the Ericson and Pakes (1995) industry model, which specifies a framework for an infinitely-lived industry where firms choose entry, exit and investment decisions in order to maximize the expected discounted value (EDV) of net future profits conditional on their information, and adds a merger process to the beginning of each period. In this model, firms are differentiated by their capacity level; a firm's capacity is the maximum amount that it can produce. When two firms merge, they combine their capacities and become one firm with a capacity given by the sum of the two original capacities. I solve the model in order to find a Markov-Perfect Nash Equilibrium (MPNE), as in Maskin and Tirole (1988), where the state space is the capacity level of a firm and of all of its competitors. As the equilibrium is too complex to solve algebraically, I use computational methods in order to solve for an equilibrium. The process used is to specify parameter values, compute equilibrium policies and value functions for these parameter values, and then simulate the industry in order to find the distribution of various indicators in equilibrium. The purpose of this specification of the model is to examine the implications of different antitrust policies with a simple dynamic model that incorporates the major dynamic determinants of firm behavior, namely entry, exit and investment, and allows for these variables as well as the merger decision to be endogenous choice variables of the firms in the industry.

Recall that I have imposed structure on the merger process, by separating the merger process into stages, where at each stage one firm can acquire any one other firm, where the order is such that the biggest firm can move first and acquire any smaller firm, and then the next biggest firm can acquire any firm that is smaller than it, etc. Recall also that firms receive draws from a cost/synergy distribution when they merge. Just as in Chapter 1, within each stage game the results that I discuss incorporate two different merger processes, a take-it-or-leave-it (tioli) process and an auction process. For this section, I illustrate results from both processes in order to examine how the results differ depending on the process. Because of its ability to incorporate externalities more directly and share the bargaining power between the firms (which I detailed in Chapter 1) I generally prefer the auction process. In addition, the auction model has proved to be easier to compute solutions for, with convergence being less of a problem than for the tioli model. For these reasons, in subsequent sections of this paper, I primarily use the auction model when examining how changes in industry parameters affect merger outcomes.

2.1 Extensions to the Model

In addition to these two solution mechanisms, I examine two different single-agent solutions, for comparison purposes. First, I examine the solution that would be chosen by a benevolent social planner, or equivalently, the first-best solution. Second, I examine the solution that would be chosen by a perfectly colluding cartel or equivalently by a multiplant monopolist, which could choose to have entry, exit or investment of new plants as it saw fit and all of which it controlled. The general methodology used to solve for the single-agent problems (without mergers) as developed by Pakes and McGuire (1994), is quite straightforward. In Pakes-Gowrisankaran-McGuire (1993 - hereafter P-G-M) and Pakes and McGuire, the exact technique used to solve for the single-agent analog of the dynamic equilibrium model without mergers is discussed. In order to solve either problem, one must define the return function for the (unique) agent in that problem. In the case of a multiplant monopolist, the return is the total producer surplus (including investment, entry and exit expenses), while for the social planner, the return is the sum of the producer and consumer surpluses.

In order to evaluate single-agent results for the merger model, I must first extend the single-agent decision process to the merger game in such a way that the technology is the same for both models. Thus, for the single-agent game, I use the same stage game and ordering scheme as for the MPNE solution and the same cost/synergy draws when firms merge. In order to make the cost/synergy framework consistent with the MPNE model, I assume that the buying firm of the single agent knows all of its current synergy draws before it decides which merger to enact. Within each stage game, the single-agent decision problem is much simpler than the MPNE decision problem. First, it is no longer necessary to separate the solutions by the auction and tioli mechanisms, as both game forms yield the same optimal solution, modulo details about the payment amounts, which are not defined. Using either mechanism, a merger will take place if the net gain (defined as the combined value to all of the plants in the industry from the merger minus the combined value to all of the plants in the industry from no merger) plus the realized synergy draw is greater than the net gain plus synergy from any other merger and greater than zero. The reason for this result is that there is one agent controlling both sides of the bargaining process, which ensures that the optimal solution will occur, as the agent is concerned only with maximizing its total return and not with distribution effects. Just as in the MPNE results, different mergers may occur here depending on the realizations of the cost/synergy draws. In order to solve for the pre-merger values, the technique that I use is to integrate out over the joint net gain plus synergy distribution to find the probability of each merger occurring and the resulting value given that merger. As the reader can see, the technique used to solve for the single-agent model is very similar to that used to solve for the tioli model, as described in Chapter 1.

In addition to the single agent problem, the other new variable that I introduce in this chapter is antitrust policy. In the United States, prohibitions against horizontal mergers today stem largely from three acts:[3] Section 2 of the Sherman Act prohibits actual or attempted monopolization of an industry; Section 7 of the Clayton Act prohibits acquisitions of the stocks of companies, where this acquisition would substantially lessen competition; and the Celler-Kefauver Act eliminates a loophole in the Clayton Act by prohibiting the acquisition of the assets of companies, where this would substantially lessen competition. Since 1968, the United States Department of Justice and the Federal Trade Commission have jointly published *Horizontal Merger Guidelines*, in order to aid firms in determining which mergers are likely to be challenged in court under the antitrust laws. The *Guidelines* specify whether the government is likely to challenge a merger, as a function of the change in market concentration resulting from the merger and the criterion used to define a market. While market definition issues are outside the scope of this model as presented (because the model assumes a perfectly homogeneous good) policies of challenging mergers depending on the concentration change can be analyzed by this model.

In order to model government antitrust policy, I simplify the idea of the *Guidelines* and assume that the government always challenges mergers which result in concentration levels above its specified threshold, that the government always wins in such a case and that it is able to assess arbitrary punishments on firms that violate antitrust laws. In particular, I assume that the government specifies criteria (which are common-knowledge) under which it views specific mergers as illegal. If an acquisition of a firm is illegal, the firm will receive an infinitely bad payoff from the merger, as punishment from the government. Thus, illegal mergers do not

[3] See Breit and Elzinga (1982) for details.

happen. The government antitrust criterion that I have chosen is as follows: the government sets a maximum concentration ratio allowed. This concentration ratio specifies the combined maximum allowed production ratio of any two firms that want to merge. Thus, defining the production levels to be q_j, the concentration ratio for a merger between i and j is $C_{ij} = \left(q_i + q_j\right) / \sum_k q_k$. The merger policy is implemented so that if C_{ij} is at least as high as the maximum allowed concentration ratio, then a merger will be prohibited; if it is smaller than the maximum allowed, then there is no prohibition of the merger. This simple rule is similar to the initial merger guidelines, introduced by the Department of Justice in 1968, which prevented mergers based on the concentration ratio. Later versions of the guidelines used the Hirschmann-Herfindahl index, but it is not clear what effect this change had.[4]

In the Department of Justice guidelines, pre-merger quantity levels are used in determining the market shares and concentration levels.[5] In order to have the same criteria, I use the market shares from the Cournot game for the existing structure, in determining the concentration levels. Note that in this way, I am basing antitrust laws in my dynamic model on static criteria in order to emulate actual antitrust laws, which are also based on static criteria. With this setup, it is very easy to implement any level of antitrust policy. The algorithm does this by redefining a firm's combined value from a merger to be the value from the merger, if the current merger is not prohibited by law, and to be a very negative number otherwise. This programming forces firms to choose mergers from the set that is not

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[4] See Rogowsky (1984) for a complete description and examination of the different merger guidelines.
[5] See, for instance, *Horizonal Merger Guidelines* (1990).

prohibited by law. In addition, note that although mergers that are not prohibited will use the same elements to determine their value as they otherwise would have in the absence of antitrust law, their actual values will be different because of the dynamic effects of prohibiting certain mergers. Thus, in equilibrium, the value of every element of the value function for firms will be different when antitrust law is activated.

2.2 Results of the Model

Tables 1, 2 and 3 and Figure 1 list the results for this section. Table 1 indicates the results for the auction model; Table 2 describes the tioli model and Table 3 the single-agent model. Figure 1 graphically displays the total welfare resulting from every antitrust policy for all the models in Tables 1 through 3. The parameters that I used for these results are the same as those used in Chapter 1, except for the addition of antitrust law. See Chapter 1, Section 3 for a description of the methodology and of the different parameters, and Chapter 1, Table 1 for a list of the parameter values. In particular, the results presented indicate the ergodic (or long-run) distribution of several indicators that characterize the industry, such as concentration and production levels, amount of mergers and welfare indicators, given a set of equilibrium policies for every agent. In order to ensure accuracy of the results, all results are presented using a 150,000 period simulation. Tables 1 and 2 show the results for 9 different antitrust law parameters.

The results for both the auction and tioli models (in Tables 1 and 2) show that either a total prohibition on mergers (i.e. a maximum allowed concentration of

0) or no antitrust law (i.e. a maximum allowed concentration of 1) are not optimal policies, as intermediate antitrust policies give a higher total welfare than either extreme. For the parameter values that I tried, antitrust policy parameter values of 0.6 to 0.7 seem to be the optimal choices, from a total welfare standpoint. For both models, producer surplus peaks when there is no antitrust law whatsoever. For the auction model, consumer surplus peaks at intermediate values, while for the tioli model, consumer surplus is at its highest when there are no mergers allowed.

For both the models, there are several trends that one can observe across the antitrust levels. First, as antitrust law becomes more strict, the amount of mergers in the industry decreases. The result is exactly what one would expect: with stricter antitrust policies, there are less legal opportunities to merge and thus less times when a legal opportunity to merge coincides with firms wanting to merge. In spite of the fact that less mergers occur with more strict antitrust policies, other consequences that one might expect, such as having concentration decrease with stricter policies, do not hold because of the dynamic effects. Second, as antitrust law becomes more strict, the mean realized synergy draw decreases. The reason for this is because firms must then take advantage of a legal merger situation without waiting for a better synergy draw. Third, as antitrust law becomes more strict, the amount of entry in the industry decreases. The reason for this is because the option value of an entrant is smaller for more strict antitrust policies: with stricter antitrust policies, the entrant has less of a chance of being acquired by another firm, which is potentially valuable. Fourth, as antitrust policy becomes more strict, the sizes of firms becomes smaller, as firms must increasingly use the slower internal investment process instead of the faster acquisition process to expand when there are more strict antimerger laws. Fifth, apart from the cases where merger for monopoly is allowed, the amount of total investment is very similar across different antitrust policies and across the auction and tioli models. As entry increases with more lenient antitrust policies, I conclude that, for more lenient antitrust policies, investment decreases. The reason for the decrease in investment by incumbents is that, with an increased possibility of present or future mergers, firms are able to internalize the negative externality of their investment to a greater extent. However, as firms cannot internalize the negative externality of entry, this decrease in investment is almost exactly counterbalanced by the increase in entry. Sixth, as antitrust law becomes less strict, the amount of exit as a proportion of entry decreases. The reason for this is because, in this model, exit will be inefficient except in the rare case where the value to another firm of the exitor's capacity is less than the (small) scrap value of the capacity.[6] Thus, as antitrust policy becomes more lenient, likely exiting firms are increasingly able to find a legal opportunity to be acquired that is more valuable to both parties than exit.

In spite of the similarities of trends between the tioli and auction models, there are some differences between them. The differences stem from the fact that the tioli model is a more efficient bidding process for firms, which occurs because all of the power is given to the potential acquiring firm. Thus, unlike the auction model, with the tioli model, mergers that will be good for firms will generally happen. One can observe this fact by noting that the mean realized synergy draws is much higher for the auction model than for the tioli model. This is because in the auction model, potential buying firms set their bid high enough that optimal mergers do not occur,

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[6] This is also the rationale behind the failing firm defense in the *Horizontal Merger Guidelines*.

in order to capture more of the synergy draw for mergers that do occur. The result of the inefficiency is that allowing mergers for monopoly results in a much more concentrated industry in the tioli model than in the auction model. In addition, one notices that for intermediate antitrust policies from 0.6 to 0.9, while the auction model sometimes has an industry structure with 3 firms, the tioli model almost always has two firms. The reason for this is that obtaining a monopolistic industry structure through merger would be illegal, and having a structure with three firms would be less efficient. While the firms in the tioli model can effectively merge to the maximum legal level, the auction model's firms cannot. Because of its greater industry concentration, the tioli model results in much fewer mergers than the auction model across policies. And, because acquired firms do not have as much to gain from a merger, its has less entry, which is particularly pronounced in the case of no antitrust policy, where the tioli model has much less entry and through that much less total investment. In addition, the tioli model's greater efficiency leads to less exit as a percentage of entry than the auction model. In terms of welfare, the result of the differences in the model is that the tioli model yields a much lower total and consumer welfare than the auction model for the case with no antitrust policy, while for intermediate antitrust policies, the tioli model yields slightly lower total and consumer welfare. In terms of producer surplus, the greater efficiency results in the tioli model having a higher welfare for lenient antitrust policies.

From the fact that different merger policies yield different surpluses, one can see that mergers can be both harmful and productive. Mergers may be good for firms and for consumers because they allow capacity to be transferred to where it is most needed, which allows firms to spread the riskiness from the uncertain investment process, to grow more quickly, and to productively use the capacity of dying firms that would otherwise exit the industry. In addition, mergers allow firms to partially internalize the negative externality from investment, and provide a cost/synergy payment to firms that merge, which is beneficial if positive. However, mergers also have negative effects. The most apparent negative effect of mergers is that they increase the concentration of the industry and hence increase prices above the competitive level, leading to a deadweight loss. In addition, mergers can have other deleterious effects, such as increasing the amount of nuisance entry, which occurs because of the strategic interaction between firms in the industry. Of all these reasons given, the cost/synergy draw is probably the one which has the least relevance to reality, as any synergies between firms in actual industries would not necessarily be of the same form. However, one can see that even in the case with the most mergers (which occurs when the auction model is used with an antitrust parameter of 1) the mean surplus coming from the synergy draw is less than 0.02. This amount is an order of magnitude smaller than the differences between the surpluses resulting from different antitrust policies, as these are in the range of 0.2. This shows that the synergies are not the main reason why there are differences in welfare between different antitrust levels.

One of the features of the results presented is the fact that, for both models, intermediate antitrust policies are optimal. The reason for this result is that the negative effects of mergers occur more for large mergers, while the positive effects of mergers occur even for small mergers. The result is interesting, because it shows that the United States government's policy, which is to disallow big mergers but allow small mergers, is correct. In addition, this model achieves this result in a rational game theoretic setting. In spite of the intuitive nature of this result, it has not been shown by previous models. The reason that it has not been shown is exactly because previous models have not endogenized mergers and have not looked at a dynamic conception of an industry and therefore have not been able to predict the welfare and policy implications of different antitrust policies.

In addition to the fact that intermediate antitrust policies may dominate either extreme, the other noticeable results is that there are a wide range of antitrust policies that yield very similar welfare results. In particular, the biggest jump in the surpluses occurs when mergers for monopoly are prevented. Apart from this jump, antitrust policy has relatively little effect on the producer or consumer surplus. As antitrust policy becomes more strict, firms are able to substitute internal investment for entry and mergers. It is only when there is no antitrust policy or a ban on most mergers that antitrust policy starts to have noticeable deleterious effects. Within the intermediate range, changes in the antitrust policy do not even change the industry concentration, as the drop in entry compensates for the decrease in allowed mergers. The reason for this result is that even without any mergers, this is an industry where there are likely to be only two firms active. Thus, by preventing mergers for monopoly, the government is preventing the main change that mergers can enact on the industry, which is to have a monopolistic structure.

In the past, Chicago School adherents like Posner (1976) have argued that only mergers for monopoly should be challenged. Others, like Adams and Brock (1991) or Scherer (1980) have argued that many more mergers are undesirable. This model shows that both sides of this debate may be correct: for a wide range of intermediate policies, the total welfares, consumer surpluses, producer surpluses and even the industry concentrations are very similar. This result also shows that even though antimerger policies vary a great deal across countries, most of the policies in place will lead to welfare levels near the maximum achievable through antitrust policies. Note that this result differs vastly from the results shown in the literature. The reason that this model is able to obtain this result is because it is able to evaluate the extent to which firms substitute equilibrium behaviors given different antitrust policies. It is only by examining mergers in an endogenous equilibrium framework that one can evaluate the extent of the substitutions involved. This underscores the need to examine mergers with an endogenous model in order to properly understand antitrust policy implications.

There is one more interesting feature of the debate between strict and lax antitrust policies to which the model can add. Many economists have argued that large firms may be inherently bad, for political economy reasons, and hence that one of the goals of antitrust policy should be to encourage small business. As Adams and Brock (1991) write, "disproportionate private economic size and power" might be feared "because of its unique political capacity to manipulate government"[7] and because concentration of power is undemocratic. While the model does not incorporate any of the political economy considerations of concentration, it points out the validity of a counterargument. In response to this criticism of lax antitrust law, Posner (1976) writes that: "I am not prepared to argue that it has no merit whatever. I am, however, confident that antitrust enforcement is an inappropriate method of trying to promote the interests of small business as a whole."[8] As can be

[7] p. 119.
[8] p. 19.

seen from Tables 1 and 2, the model supports Posner (1976)'s conclusions, because, so long as mergers for monopoly are prohibited, antitrust law does not have a large effect on concentration in the industry. Furthermore, as the reader will see in the following sections, in certain cases, the effect of more strict antitrust policy is to actually increase long-run industry concentration.

2.3 Single-Agent Results

Table 3 details the single-agent equilibrium policies and values for both the multiplant monopolist and the social planner. For both cases, the agent never chooses to have any merger other than merger for monopoly, regardless of the antitrust policy. Indeed, both agents almost always keep no more than two active firms in the industry, which renders mergers for oligopoly impossible. As there is only one agent in this model and no mergers happen when the antitrust maximum concentration allowed is less than 1, it follows that the equilibrium will be the same for any antitrust policy less than 1.[9]

From the social planner's solution, one can see that the first-best solution yields a welfare of 1.845, which is significantly higher than the highest welfare (across antitrust policies) achieved by a Markov-Perfect Nash Equilibrium (MPNE) model, which is 1.518. The social planner achieves this level of welfare by maintaining a very concentrated industry, typically with one very large firm producing a great deal. The first-best solution involves mergers in 0.67 percent of

[9] Note that with multiple agents, this is not true. For this reason, it is not redundant to print the equilibrium values for different strict antitrust policies where no mergers happen, as the results are not the same across these policies.

the periods. While far less than the amount of mergers in the MPNE model, this is still significant. When mergers are allowed, the social planner is able to obtain a higher surplus as compared to the industry when mergers are not allowed. Just as in the MPNE model, there are several (socially optimal) reasons why the social planner might choose a merger between two firms, including risk-sharing, faster growth, capturing the capacity of exiting firms and capturing synergy draws. Again, I note that the gain from synergy draws accounts for approximately 0.0003, two order of magnitude less than the gain of 0.025 from allowing for mergers, and thus synergies are not a large part of the explanation. The principal reason why mergers benefit the planner is that mergers allow the planner to control for bad draws in investment by bringing in new firms and then merging these with existing firms. When mergers are not allowed, the only way that the planner can use new firms to control for bad draws in investment is by keeping both firms in business or allowing one firm to exit. Because of this, when mergers are allowed, the amount of entry increases dramatically, as entrants can be acquired by big firms and are not only useful in the case of exit.

From the perfectly colluding cartel solution, one notices that the industry concentration is very high, just as for the social planner solution. The main difference between the industry structures for the cartel and the planner is that with the cartel, firms are much smaller, and produce much less. Not surprisingly, this results in the mean social welfare for the perfectly colluding cartel being less than even the lowest welfare of the Markov solution. Just as for the first-best solution, mergers occur with some small frequency, 0.96 percent, when permitted. Thus, although authors like Posner (1976) have argued that mergers function as a second-best substitute for collusion when collusion is not allowed, mergers are still useful here, even when total collusion is permitted. Just as in the first-best model, the main gain from mergers is not in terms of the synergy payments but rather in controlling for bad draws to the investment process, by allowing for two R&D processes to be run simultaneously and for their products to be combined as necessary. Additionally, mergers in the collusion model are beneficial both to consumers and producers. The reason for this is that, in the collusive model, mergers are not necessary in order to increase market power, and thus mergers do not affect the colluder's pricing behavior. As their only use is to increase the efficiency of the investment process, both consumers and producers benefit from the increased efficiency, according to their elasticities.

From the single-agent results, one can see that both the perfect colluder and social planner results yield industry structures that are more concentrated than the MPNE results. For this reason, they also have much less mergers, entry and exit. However, one can see from the MPNE results that making antitrust policies strict enough to decrease the amount of mergers to the amount chosen by the social planner is not the optimal antitrust policy. The implication of this follows the theory of the second best: given that the government cannot enforce first-best marginal cost pricing, the best antitrust policy is to allow more mergers than would occur given optimal merger choices with marginal cost pricing. In this way, it is not meaningful to look at the first-best optimal amount of mergers.

Section 3: Differences in Production Costs

The results discussed so far do not allow for production cost differences between different firms or alternatively put, economies of scale. There are important reasons why one might want to consider increasing returns. Survey evidence shows that firms often cite economies in administration and management and in the integration of facilities as the primary reasons why they choose to merge.[10] When the production costs of merged firms are lower than the costs of the two firms involved in the merger, mergers have the possibility to vastly increase the efficiency resulting from an industry. Because of this potential for efficiency gains, there is an added possible societal benefit to allowing a lenient antitrust policy, when there are increasing returns. Indeed, the central justification for why governments have allowed any mergers is that there is a potential for cost savings.

In spite of the potential gain from mergers when there are economies of scale, there is also a larger potential loss in this case. If merged firms have lower costs, then they may be able to more successfully deter entry and exercise market power over other firms with higher costs. Thus, the ultimate effect of the cost advantages may be to lower consumer and total welfare. Because of these opposing effects of economies of scale, the court's attitude towards them has been very mixed. In the Proctor & Gamble case of the 1960s, the court's opinion was that "Possible economies cannot be used as a defense to illegality,"[11] while at other times, the court has accepted efficiency gains as a reason for merger. The court's ambiguity points

[10] See Scherer and Ross (1990), p. 166.
[11] Excerpted from Breit and Elzinga (1982).

out the need for economic analysis of mergers in industries with increasing returns, in order to sort out the magnitudes of the different effects.[12]

Accordingly, many economists have modeled such cost differences in the past. One major work is that by Williamson (1968), which attempted to characterize the levels of cost savings that would be sufficient for a merger to yield a higher total welfare, given the increased market power. While Williamson's work was able to bound the amount of cost savings necessary for a merger to decrease static surplus, it did not explicitly model the costs of a merged firm as a function of the costs of its component firms. In order to analyze which type of mergers the government should prohibit, it is necessary to explicitly model the production costs resulting from mergers. For this reason, more recent models of cost differences have explicitly modeled costs, with a game theoretic setting.

One principal paper that has analyzed mergers with explicit cost differences is Perry and Porter (1985). The framework used by Perry and Porter is to assume that each firm has some level of capital k_i and that capital serves to decrease the production costs of the firm. With this framework, there is a natural concept of what happens when two firms merge: they combine their capital stocks and have the cost function for their new capital stock. The major problem in implementing the Perry and Porter setting is that there are infinitely many ways in which one could define the cost curve as a function of the capital stock. Perry and Porter choose a natural specification where capital affects costs inversely and quantity affects costs

[12] Note that there are two possible economies of scale resulting from mergers: cost savings for higher levels of production, and cost savings for larger companies regardless of the level of production. In this paper, I am referring primarily to the second type.

quadratically. For their model, the variable costs of a firm producing quantity q_j is then:

$$C(q_j, k_j) = a\frac{q_j^2}{k_j},$$

for some constant a. Thus, with this functional form, capital lowers the slope of the cost function. As when two firms merge, they have more capital than either one separately, they have a cost function that is lower for any quantity than either firm's cost function separately.

For this section, I have adopted the Perry and Porter framework of having capital stock affect cost, but I have modified it in order to match it with the model as previously defined.[13] The functional form I use is that higher capital leads to a lower but still constant marginal cost, and a higher capacity level. In particular, the specification is:

$$C(q_j, k_j) = \begin{cases} c_1 + c_2 \cdot e^{-k_j \cdot c_3}, \text{if } q_j \le k_j \\ \text{infinite, otherwise} \end{cases},$$

where (c_1, c_2, c_3) are parameters. Note that with this capital/cost structure, as I raise c_2, larger firms in the industry receive larger cost advantages. However, the cost to a sufficiently large firm remains the same for different values of c_2. For this reason, the socially optimal solution will be roughly the same, regardless of c_2. With this framework, the model will approach the base case model, for low values of c_2. The rationale behind this framework is that, even with cost differences, the primary goal of a firm's investment is to increase its capacity. However, with more capacity, firms learn how to produce more efficiently, and thus, increasing one's capacity has the side effect of lowering one's costs.

[13] If a reader wishes to compare the results to those of Perry and Porter, it is a straightforward task to use Perry and Porter's exact specification with this model.

For the results I present, the non-cost related parameters are chosen to be the same as those used earlier, except for the size of the cost/synergy distribution, which was increased from 0.2 to 0.3 in order for the program to converge.[14] For the cost related parameters, I have chosen values of $c_1 = 1.8$ and $c_3 = 6$. The choice of c_2 depends on the experiment. The results for the models with different cost differences are presented in Tables 4a through 4c and Figure 2. Table 4a presents the model with no production cost difference, 4b with small differences and 4c with large differences, while Figure 2 displays the total welfare from each model graphically. The corresponding values of the parameter c_2 for the three models are 0, 1 and 3 respectively.

The results show that regardless of antitrust policy, the industry will become more concentrated as the cost differences between firms rise. The reason for this is that with larger cost differences, it is harder to sustain multiple firms that have enough capital to have low costs. Because of this trend towards greater concentration, the welfare generated by the MPNE solution moves further and further away from the first-best welfare level and closer to the welfare generated by the perfect colluder, as cost differences grow larger. In fact, because of the colluder's ability to build large amounts of capital without fear of competition, for the model with large cost differences, the perfect colluder with no antitrust policy provides a social welfare that is just as high as the MPNE model with no antitrust policy and higher than the MPNE model with complete merger prohibitions. This result is exactly what one would expect: with increasing returns, a competitive market will

[14] Recall from Chapter 1 that larger cost/synergy distributions may be necessary for an equilibrium to exist and hence for the algorithm to converge.

perform increasingly poorly with higher cost differences. In addition, one can observe more mergers in the industry in this case. However, as welfare falls with higher cost differences regardless of the antitrust policy, restricting mergers is not a solution to this problem.

In addition to this welfare effect of increasing costs, there are several trends that one can observe in the industry, regardless of cost differences. For instance, regardless of the level of cost differences, as antitrust law becomes more strict, one can observe declining entry (as the option value of entrants becomes less), increasing exit as a percentage of entry, and a slightly declining capital stock. In addition, the results show that regardless of the level of cost differences, an intermediate antitrust policy leads to higher total welfare than either extreme policy and that there is a wide range of acceptable intermediate policies. This is interesting, because, in a static non-endogenous setting, one would expect that the consequences of moving away from the optimal antitrust level would become more sharply negative. However, by endogenizing mergers, one can see that in equilibrium, even in the presence of sharp cost differences, firms are able to substitute internal investment for external investment and obtain similar welfare levels, for a wide range of antitrust policies.

While these general patterns above hold, many of the conclusions are different depending on the size of the production cost differences. A comparison of the results with small cost differences (Table 4b) to the results with no cost differences (Table 4a) shows that the change in the results tends to follow the patterns that one would expect. In particular, with no antitrust prohibition and cost differences, firms obtain a very large capital stock, with a mean size of 1.089, compared to a size of less than 0.8 for every other antitrust level. In spite of the size and efficiency increase, the welfare effects from having no antitrust prohibitions are much worse when there are cost differences. The reason for the lower welfare is that it is harder for entrants to earn a profit in the industry with cost differences, and so firms that merge are able to sustain market power for longer. With small cost differences, it is therefore more crucial to protect the industry from monopoly.

In spite of the increased necessity of a prohibition on monopoly, there is no increased necessity of allowing other mergers, in the presence of small cost differences. In particular, although a static analysis indicates that a complete prohibition on mergers would yield a much lower surplus in the model with cost differences (as such a prohibition would prevent firms from achieving the necessary efficient scale) this is not the case, as firms are able to achieve close to the same level of capital stock, even in the absence of mergers. Although the government's intermediate antitrust policy has been based on the fact that allowed mergers have the potential to lower costs and through that increase welfare, this model suggests that this is not an appropriate reason for this policy. In equilibrium, firms are able to grow to an efficient size regardless of merger policy and cost differences. The gain from allowing mergers occurs because of the reasons discussed in Section 2, i.e. risk sharing, using the capacity of likely exitors, etc. Thus, while allowing an intermediate antitrust policy is beneficial in the presence of cost differences, it is also equally beneficial in the absence of cost differences.

Turning to the model with large cost differences, it is apparent that some of the results that one might expect from antitrust law in this case no longer hold for this specification. In particular, there do not appear to be large differences between the welfare levels for different levels of antitrust enforcement, including allowing mergers for monopoly. The reason for this result is that with this level of cost differences, the equilibrium industry structure will be very concentrated, regardless of the antitrust policy. Thus, the negative effects of having no antitrust law, which is that the industry may become more concentrated, no longer hold. Surprisingly, the largest effect of allowing more lax antitrust policy is that there tends to be more consumer surplus and less producer surplus in this case. The reason is that increased entry causes the industry to have larger firms, which helps consumers, but that this entry hurts producers who must buy up the entrants in order to keep their market power. In fact, when mergers for monopoly are allowed, the industry tends to be slightly less concentrated than when no mergers at all are allowed. This result, which occurs because allowing for mergers increases the value to entrants in the industry (because of the option value of being acquired), cannot be explained using static intuition. While an intermediate antitrust policy continues to be optimal even in the presence of large cost differences, the results show that for sufficiently large cost differences, vigilant antitrust policy may no longer matter, because preventing concentration may be hopeless.

From the results presented, one can see that much of the model's conclusions in the case of cost differences are due to the dynamic effects of firm behavior. In order to see how the results are different from previous static models, one can compare the results to previous models. Williamson (1968) calculates that (in his simplest setting) mergers with cost advantages are rarely bad, as the market power effects would have to be very significant to dominate the efficiency gains. Because of the endogenous merger process and the dynamics, Williamson's results no longer hold true, for two opposing reasons. First, market power in a dynamic setting is very different than in the static setting, because of potential entry. Thus an industry where there are two moderate-sized firms active is much less concentrated than it would appear, statically. Second, even if mergers are not allowed, firms will tend to grow until they have low enough costs. Thus, the cost savings from a merger would occur in the long run even if the merger was not allowed. These two opposing effects result in a wide range of antitrust policies that yield surpluses near the maximum, which is very different from the conclusions of a static model. Perry and Porter (1985) show that, in their model, mergers will often occur in instances where they would not be profitable, if there were no cost differences. Just as in Perry and Porter's model, the results presented here show that the frequency of mergers increases when there are cost differences: without cost differences, mergers occur in approximately 3 percent of the periods; with small cost differences, the figure is 12 percent, and with large cost differences, it is 14 percent. Because Perry and Porter's model is static and does not fully endogenize the merger process, it does not attempt to evaluate which mergers actually occur. Thus, it is not possible to compare the other predictions of this model to those of Perry and Porter.

In spite of the differences in conclusions between this model and static models, I note that many of the dynamic conclusions of this model have been discussed, though not modeled directly, in previous works. In particular, the results here follow the arguments proposed by Posner (1976) when discussing the Brown Shoe case. In this case, Posner argues that "If an integrated firm like Brown has lower costs than nonintegrated retailers, it will expand its share of the retail shoe market whether or not it acquires Kinney. The principal effect of a very strict rule against horizontal mergers is not to retard economic progress; it is to reduce the sale value of small firms by making it difficult for their large competitors … to buy them."[15] In my model the same is true: if there are large cost advantages, the industry will become very concentrated; and the largest effect of not allowing mergers is to decrease the option value of new firms, which is evidenced by the tiny level of entry in this case. Finally, I note that Posner argues that monopolists may have lower costs than competitive firms, because they are able to capture all the gains from their R&D investment.[16] In this model, while it is generally true that the equilibria with no antitrust enforcement and the perfect colluder equilibria have lower costs than other cases, this does not necessarily lead to higher social welfare levels.

Section 4: Other Cost Differences

In this section, I examine two other cost differences that may affect the consequences of mergers and antitrust law. These cost differences are the occurrence of synergies or of transaction costs between merging firms and the presence of barriers to entry.

4.1 Synergies or Transaction Costs for Mergers

For many reasons, one might expect that different merging firms have synergies or transaction costs when they merge. Reasons for synergies include

[15] p. 105.
[16] p. 15.

managerial efficiencies, or sharing of patents or other proprietary technology, while the reasons for costs might include corporate culture clashes or legal fees. Furthermore, when firms discuss the cost savings from mergers, they often are not referring to the well-specified cost savings that I discussed in Section 3, but rather to cost savings for reasons such as those described above. This is the basis behind Farrell and Shapiro's (1990) model, which examines the gains from mergers in the presence of synergies. These synergies or transaction costs differ from changes in regular production costs (as discussed in Section 3) in two ways. First, they are generally not observable to or contractible on by the government (and often not to the firm either) as they are not merely a function of the sizes of the companies. Thus, antitrust law cannot be made conditional on these synergies. Second, these synergies or transaction costs would not happen in the absence of mergers. One would expect these differences to lead to changes in the optimal antitrust policies.

I have modified the model in order to examine the effects of mergers when there are possible transaction costs or synergies. Recall that the model already has a cost/synergy draw that arises when firms merge, which is mean zero. Because this variable is built in to the model, examining merger synergies or transaction costs is quite straightforward: in order to examine pure synergies, I have made the synergy random variable always positive, and in order to examine pure transaction costs, I have made it always negative. The major limitation of this structure is that in many models with synergies, the synergies are implicitly assumed to be persistent and thus affect the profit function in subsequent periods. Here, for simplicity, the synergies are only a one-time payment. In this section, the parameters are the same as in the base case, except for the synergy distribution parameters, which are listed below.

The results for the models with merger transaction costs or synergies are presented in Table 5a-c and Figure 3. Table 5a contains the results for pure synergies, where the synergies are distributed $U[0,0.4]$; 5b for merger transaction costs, where the synergies are distributed $U[-0.4,0]$; and 5c the base-case results of mean zero costs/synergies with distribution $U[-0.2,0.2]$, while Figure 3 displays the total welfare for all models graphically. The results show that there are not huge differences in the welfare and other variables between the three different models. In particular, even in the presence of positive synergies, an intermediate antitrust policy yields a higher surplus than allowing mergers for monopoly or allowing very few mergers. The reason for this is that even if the synergy distribution were higher, the gain from the synergies still does not outweigh the loss from the increased market power.

The most noticeable difference between the three models is that, for most levels of antitrust policy, the model with synergies yields the highest welfare, while the model with transaction costs yields the lowest welfare. The primary reason for this result is not that the higher synergies are being captured by firms, as these synergies are only a small fraction of the increase in welfare, especially since firms compensate for the better synergy distribution by accepting an average draw from a lower quantile of the distribution. Rather, the reason for the higher surplus in the MPNE model is that with a higher synergy distribution, less firms exit the industry, and firms wait longer before merging, because the industry is more profitable. Thus, the model with synergies is able to yield a higher surplus primarily by decreasing the concentration in the industry and by decreasing the amount of exit. The increase in surplus is largely not because the industry is intrinsically more profitable: the social planner solution has a welfare increase of only 0.014 with synergies and a welfare decrease of only 0.002 with merger transaction costs, while the MPNE solution without any antitrust law has a welfare increase of 0.088 with synergies and a decrease of 0.004 with transaction costs. Rather, the increase is due to the merger process, which indirectly causes firms to wait in the industry longer, and accordingly deters some of the negative effects of mergers.

The results suggest two policy implications for the government. First, for moderate sized synergies, i.e. where the largest size of the synergy is approximately one fifth of the industry demand with perfect competition, the government should not pay too much attention to claims of synergies in determining whether or not to block a merger, especially when such synergies may be unknown, as the optimal levels of antitrust policy do not change a great deal with synergies. Second, a policy of increasing the paperwork and legal expenses required to complete a merger in order to deter mergers does not seem to work, for in this model, the effects of this type of policy are to decrease consumer surplus and increase concentration in the industry. This result may seem counterintuitive because, in the base model, intermediate antitrust policies do yield higher surpluses than no antitrust policy. The reason for the result is that antitrust policy works by disallowing mergers which result in a large concentration, not by randomly deterring arbitrary mergers.

4.2 Barriers to Entry

Barriers to entry have concerned industrial organization economists for many years. In general terms, barriers to entry are defined as non-recurring costs to an entrant in the industry. Thus, if an entrant must pay a sunk cost that is not borne by incumbents in order to enter the industry or if an entrant consistently faces higher costs than an incumbent, these are considered barriers to entry. Note that cost advantages for large firms as used in Section 3 are not barriers to entry, because they only reflect the necessary scale of production, which apply both to incumbents and to new entrants.

There are, in general, many reasons why barriers to entry might exist in an industry, and many examples of industries where such barriers have been thought to exist. An example of such an industry is the U.S. airline industry in the late 1980s, where following many mergers, including those by TWA & Ozark, and Northwest & Republic, concentration was high enough that entry might have been viable, but firms that wanted to enter had to lease gates from existing airlines. Another example is the camera industry which Kodak dominated for many years, in spite of the absence of any legal barriers to entry. Although economists have observed barriers to entry in different industries, there are also theories such as contestable markets which explain why barriers to entry might not exist.[17] Some economists believe that barriers to entry are quite rare. As Posner (1976) writes: "there is grave doubt as to whether there are important nonrecurring costs of entry - barriers to entry in the true sense."[18]

[17] See Adams and Brock (1991) for details regarding this entire discussion.
[18] p. 92.

It is not the goal of this section to examine whether or not barriers to entry exist in different industries. However, the goal is to determine what effect such barriers to entry might have on the consequences of mergers and on optimal antitrust policies. With a contestable markets type of model, potential entrants serve almost completely as competitors in the industry. Thus, in this model, any market-power gains from mergers would be short-lived, and mergers may not have much negative effects. When there are substantial barriers to entry, one might expect that a merger would result in decreased competition over a long period of time, as new firms could not easily enter the industry without paying large sums of money. Because of this, conventional wisdom suggests that in the presence of substantial barriers to entry, the government would want to adopt a vigilant antitrust policy.

In this model, it is very easy to examine the effects of barriers to entry. Recall that any entrant must pay some scrap value in order to enter the industry and then receive some fixed capacity. In the base case results, this sunk cost of entry is chosen so that its mean value is approximately the same as the mean cost that an incumbent would incur in raising its capacity by the level that the entrant gets. Thus, the base case results correspond to free entry. In order to examine the effects of entry barriers, I have raised the sunk cost of entry to levels above the free entry threshold. The results are presented in Tables 6a-c and Figure 4. Table 6a presents the 'free entry' results, which involve an entry cost that is distributed uniformly over the interval $[1.1,1.5]$; 6b presents results for small barriers to entry, where the entry cost is distributed uniformly over $[1.8,2.2]$ and 6c presents results for large barriers to entry, where the entry cost is distributed uniformly over $[2.8, 3.2]$, while Figure 4 displays the total welfare for all models graphically.

The results show that regardless of the antitrust policy, when barriers to entry are added to the model, the industry becomes more concentrated and has lower social welfare. Because of this, as barriers to entry are increased, the MPNE solution of the industry moves increasingly far away from its potential maximum (i.e. the social planner's solution, which changes very little with entry barriers) and very close to the colluder's solution. The reasons for this behavior are quite clear: with increased barriers to entry, there is less entry and less threat of entry. Thus, firms that are acquired by other firms or that exit the industry are not as easily replaced, which increases the market power of existing firms. Furthermore, because of the entry barriers, the incentive to invest for strategic reasons (such as entry deterrence) is less, and because of the higher concentration there is no longer a negative externality for investment. For these reasons, investment drops as entry barriers increase. Finally, note that as entry barriers increase, the amount of mergers in the industry decreases. The reason for this is that a higher cost for current and future entry translates into a higher purchase price for acquired firms. Thus, internal investment is now a cheaper vehicle for increasing capacity than is buying other firms, as these firms must pay larger entry costs in order to enter.

While the results show that with increased barriers to entry the performance of the industry worsens regardless of antitrust law, the size of the change is not the same for every level of antitrust law. In particular, moving from no barriers to entry to small barriers to entry, note that the change in welfare of the industry when there is no antitrust prohibition changes by only 0.009, while the effect on industries with stricter antitrust prohibition is quite large: when mergers are prohibited, the effect of small barriers to entry is to decrease welfare by 0.157. With both small and large barriers to entry, allowing for every merger actually yields a higher total surplus than allowing no mergers, while with large barriers to entry, allowing for any merger yields the highest surplus overall.

The reason for these results is that allowing for mergers increases the value of entering, because the entrant has the option value of being acquired by the dominant firm in the industry. The fact that lenient antitrust policies encourage entry is evidenced by the vastly increased entry levels when mergers are allowed. With substantial barriers to entry, there will be a high industry concentration, even if mergers are prohibited. Thus, with barriers to entry, the negative effect of increased concentration occurs whether or not mergers are prohibited. But, when mergers for monopoly are allowed, this negative effect is mitigated somewhat because of the entrant's ability to share the 'rents' from being an incumbent in the industry that occur because of the barriers to entry. Note that these rents are actually higher than in the free entry case, because entrants do not come along very often. Thus, when mergers for monopoly are allowed, the increased barriers to entry are partially compensated by the additional rents that entrants will earn when they are acquired, which in effect decreases the real amount of the barriers to entry. When mergers for monopoly are not allowed, potential entrants are less able to capture rents through acquisition, and thus there is a higher effective entry barrier, which leads to a higher industry concentration. This inability to capture rents when mergers for monopoly are not allowed is particularly pronounced in the case of large barriers to entry, because entrants can never be acquired since the industry structure is always a monopoly.

The results presented here show that the model supports the conclusion reached by the traditional literature that barriers to entry are worrisome, as they lead to lower total welfare. In this model, with large barriers to entry, the total welfare from the MPNE solution is comparable to that of a perfectly colluding cartel. However, the model shows that the traditional literature is wrong in its estimation of the effects of mergers on these barriers to entry. The model shows that strict antitrust policy is not the solution to the problem of barriers to entry. Strict antitrust enforcement does not lead to an unconcentrated market, as firms that disappear are unlikely to be replaced by new firms in the absence of the possibility of mergers. In fact, by decreasing the option value of entrants, strict antitrust enforcement actually leads to a more concentrated market. Thus, if there are high barriers to entry, the best antitrust policy would appear to be a lenient one. This counterintuitive result is due to the fact that the dynamic effects of mergers in the presence of barriers to entry outweigh the simple static effects. Still, lenient antitrust policy is not a solution to the problem, as the effects of barriers to entry are still negative. This suggests that a better solution to barriers to entry is to deal with such barriers directly and lower them.

Note that the reason that the model obtains the surprising results presented here is that it is endogenizing the merger process, and thus is able to examine the long-run consequences that barriers to entry will have on industry structures. With barriers to entry, static intuition dictates that mergers will have deleterious effects, as the increase in competition will not easily be abated by future entry. However, the model shows that the effect that strict antitrust policy has in decreasing the option value of entry is more important than this simple static effect of the rise in concentration. The results show that, in order to examine the effects of antitrust policy with barriers to entry, endogenizing the merger process is essential.

Section 5: Stock Market Values

One aspect of mergers that has puzzled industrial organization and financial economists for a long time has been the pattern of the change in stock market valuations resulting from mergers. The empirical studies on this subject have attempted to quantify the effect of mergers on the stock market values of the acquiring and acquired firms, both before and after a merger occurs. For the acquired firm, one can measure gains from a merger by looking at the ratio of the price that the acquired firm is bought for to the value that its stock was trading at several weeks or months before the merger announcement, while for the acquiring firm, one can measure the gains by looking at the ratio of its stock market value several months after the merger to at the time of the merger, or before the merger. There is a consensus among economists that acquired firms gain significantly from a merger, as the ratio of the value they are paid to the value their stock was trading at is significantly higher than one.[19] However, the evidence for the acquiring firms is much more ambiguous. Most studies agree that acquiring firms typically do not gain

[19] See Magenheim and Mueller (1988) for estimation of the size of the gains as a function of the time-frame examined.

any value following a merger, while some studies have shown that in the years following a merger, the acquiring firm's value actually declines on average.[20]

Given an efficient markets hypothesis, stock market valuations represent the EDV of net future profits conditional on the market equilibrium. Thus, the empirical findings appear to show that the acquiring firms should not merge, as mergers do not increase their expected profits and often may decrease them. Economists have long attempted to explain this paradox of why acquiring firms choose to merge anyhow. Explanations that have been proposed include: that people are myopic and therefore the stock market is not efficient; that tax advantages or other non-market reasons explain this phenomenon; and that managerial incentives may result in unprofitable acquisitions (i.e. that management of the acquiring firm may gain from the increased prestige of managing a larger company). While these explanations may have some validity, none of them have proved fully satisfactory. In particular, all of these explanations have as an assumption that there is some firm-level behavior that is not rational, as they assume that acquiring firms could have earned more profits from not completing the acquisition. In addition, these explanations have generally been refuted by empirical testing, which suggests that none of them is correct. [21]

It is interesting to see if my model can shed some light on why the acquired firms seem to capture all of the gains from mergers. Recall that my model uses rational expectations; thus there is no possibility that firms are acquiring other

[20] See Scherer and Ross (1990) p. 168-170 for a survey of the literature. Different papers in this literature differ in the length of the pre- and post-merger time span that they consider, in terms of how they derive the expected returns for each firm and in terms of the data that they use.

[21] See Jarrell, Brickley and Netter (1988) for details.

firms because of some sort of myopic behavior. If my model shows the same patterns in its simulated data that appear in the stock-market data (and have been analyzed in the empirical literature), then this indicates that there is a perfectly rational, profit-maximizing explanation for why firms' stock-market valuations respond to mergers in the way that they do.

Recall from Chapter 1 that in order to solve for the equilibrium of my model, I compute value functions, which are the EDV of future equilibrium profits for firms at every state. As stated earlier, stock market values will represent EDV of future profits, given an efficient markets hypothesis. In order to define the stock market values of firms by their value functions, I assume that firms do not borrow or lend money with the exception of the acquisition of another firm, when they borrow the purchase price, net of their synergy draw realization, and pay the market interest rate, which is equal to the discount rate. The assumption that firms borrow when they buy another firm is necessary in order to ensure that the stock market value of the firm does not jump by the price of the new firm, which would be unrealistic as it is only possible if shareholders gave money to the company to cover the expenses of buying a new firm. This setup also requires that firms which earn excess profits over their investment levels in any period must distribute them to shareholders as dividends.

With these assumptions, I can represent the stock market values of firms by examining their value functions. For the acquired firms, the model generates a price paid whenever a firm is bought. Similarly to the literature, I examine the ratio of this price to its stock market value. In order to compare these results to the literature, I evaluate the stock market value at two points: the beginning of the period when the firm was bought, and the beginning of the previous period.[22] For the acquiring firm, I examine the ratio of its future value to its value at the beginning of the period when it acquires the other firm. Again, in order to compare the results, I examine two different future values: the value at the end of the merger process of the period with the acquisition, and the value at the beginning of the next period. Recall that for the post-merger value, I subtract the present value of the price paid for the firm minus the amount received from the synergy draw. In the results presented, if a firm makes two acquisitions in any period, it is counted twice in the sample. In addition, if an acquiring firm does not exist next period or at the end of the current merger process (because it has been bought up by another firm) or if an acquired firm did not exist in the previous period, then it is not included in the sample. As it is rare that the acquiring firm exits the industry before the start of the next period, this does not affect the results much. However, there are often new entrants which are bought before they start production; thus the values for the acquired firm based on last period exclude a large number of observations, which may bias their results. Note that the results which are based only on current period values are mostly unaffected by this sample selection issue and also are not dependent on any assumption about when firms distribute dividends to shareholders, and thus are more reliable than the next and previous period results.

The results for this section are presented in Table 7. The first row of results shows the values to the participants in mergers when the merger process that is used is the standard auction process. With the auction process, I observe that

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[22] Recall that for the results presented, the discount rate, β, is set at 0.925, which ties down a period to be equal to one year.

acquired firms are paid, on average, a 15.6 percent premium over their stock market value at the beginning of last period, while in the period following a merger, acquiring firms have on average a stock market value 3.7 percent lower (or 96.3 percent) than their value at the start of the period in which they acquire another firm. As discussed earlier, because of the sample selection issue, and in order to avoid having to deal with dividends, I also present the results for the firm values using only the current period. I find that at the end of the merger process, acquiring firms have an average market value almost exactly equal to their market value at the beginning of the period, with a gain of 0.4 percent, while on average acquired firms are bought for 5.0 percent more than their value at the beginning of the period. Both sets of results support the data that has been reported by empirical papers in the literature. In particular, they show that acquired firms gain significantly from acquisitions, whether one compares the price that they are paid to their value at the beginning of the period or to their value last period. In addition, the results show that acquiring firms do not gain relative to no merger in the short-run, and lose money from the merger in the long-run.

Recall from Chapter 1 that I presented results where the ordering of firms in the merger process was different than the largest-to-smallest ordering that I use regularly. In order to determine the cause of the above results, I have reversed the order of the merger process and evaluated the results for the new ordering. The results for the smallest-to-largest ordering, presented in the second row of Table 7, show the same general trends as for the regular ordering. In particular, the acquired firm captures most of the gain and the acquiring firm loses value one period after the merger. Here, the acquiring firm has an average value next period of only 39.8

percent of its original value. This low ratio occurs because the monopoly rents it earns in the period of the merger are quite large relative to its original valuation.

As another comparison, I present the results for the take-it-or-leave-it (tioli) process (as discussed in Section 2) in the third row of Table 7. Recall that the tioli model gives all of the actual surplus from mergers to the acquiring firm, because of the nature of a tioli process. In spite of this, for the next period values, the tioli model again shows the same general pattern, which is that the acquired firm gains, while the acquiring firm loses slightly. For the current period values, the values are both very close to 100 percent. This can be explained as follows: Recall from Table 1 that for the tioli model, there are almost always only one or two firms active and that the mean realized synergy draw is very close to 0, which suggests that most of the mergers happen at states where mergers will almost always happen. Because of these two facts, neither acquiring nor acquired firms can earn a substantial premium or loss from mergers in the current period.

While the current period results for the tioli model are different from the auction results, I note that the tioli offer scheme is very similar to a tender offer, where a potential acquiring firm makes a take-it-or-leave-it offer to buy the shares of a target firm. As Jensen and Ruback (1983) find, with tender offers, the gains are split much more evenly between the acquired and acquiring firm than with mergers. Thus, the take-it-or-leave-it nature of the offer scheme partly explains why the gains are split more evenly for this model as well as in real-world tender offers.

The results show that the model explains the real-world data, using only rational behavior on the part of firms. Note also that while the model (and real-world data) show that the gains from mergers over the pre-merger values accrue exclusively to the acquired firm, this occurs in the model in spite of the fact that the true gains will be split evenly because of the nature of the auction process. The conclusion is that the empirical literature may be incorrect in assuming that the true gains from mergers accrue exclusively to the acquired firm, as there is an explanation consistent with the data where this is not the case. Because the same trends appear for the reverse ordered process, it follows that the reason for the results is not because larger firms gain more than smaller firms from merging. Rather, the reason is that for the potential acquirer, the value from not acquiring any firm is substantially less than its overall value. Thus, even though firms appear to lose from buying other firms, if one were to examine a firm's value if it were prevented from choosing an acquisition that it wanted to, one would find that its value would be lower.

In general terms, the results show that even though previous empirical studies have been puzzled by the split of gains from mergers, there is a fully rational explanation for this puzzle of why the gains to mergers appear to be one-sided. The reason that previous studies have found the gains to be one-sided is that they have wrongly interpreted the pre-merger value functions as the values from not merging. Thus, they have not allowed for the fact that pre-merger values might incorporate the possibility of mergers, and therefore, for the acquiring firm, the value from not merging may be less than the pre-merger value. This assumption, that pre-merger values do not incorporate the possibility of any merger, is made because none of these models has endogenized the merger process as an expected part of future events and they have all instead implicitly made the simplifying assumption that all future mergers are unexpected. Because stock-market values are expected discounted values of discounted profits, they are fully based on dynamic determinants of behavior which include the possibility of mergers and the changes that these cause. By incorrectly interpreting stock market values as not reflecting the possibility of these future mergers, the literature has not been able to incorporate the effects that future mergers will have on the future values, and accordingly has not correctly evaluated the values from not merging. This underscores the importance of using a dynamic model that endogenizes the merger process in order to understand empirical stock-market merger data.

Section 6: Conclusions

In this paper, I have examined the implications of different antitrust policies for stylized versions of industries and examined the effects of production cost differences, merger synergies and transaction costs, barriers to entry and the stock market values of firms. The results of this model, as presented in the preceding sections, reveal many new insights that can help economists understand the consequences of mergers in different contexts. By endogenizing the merger process, one can observe many surprising conclusions. For instance, the results show that for both types of merger processes that I tried, intermediate antitrust policies yield higher total surpluses than either extreme of allowing all mergers or no mergers. In addition, the results show that because firms are largely able to substitute policies depending on the severity of the antitrust law, a wide range of intermediate policies leads to a total welfare close to the maximum achievable by a Markov-Perfect Nash Equilibrium. With production cost differences, the necessity of preventing mergers for monopoly may be increased, but there is a still an equally wide range of acceptable policies. In the presence of synergies, the model predicts that the total welfare of the industry will be higher, for any level of antitrust policy. However, the presence of synergies does not change the optimal antitrust policies, and thus synergies are not a reason for a more lenient policy. In addition, with larger barriers to entry, the industry will yield much lower welfare levels, regardless of antitrust policy. However, the decrease in welfare is most apparent for lenient antitrust policies because these policies decrease the option value from entering. This suggests that antitrust policy is not a solution to the problem of barriers to entry. Finally, the model shows that the paradoxical empirical result that acquired firms seem to capture all of the gains from mergers may not be true: the model predicts the same pattern of stock market changes as the empirical literature and accordingly shows that the reason that empirical studies have obtained this result is that they are improperly assuming that pre-merger stock market values do not incorporate the possibility of future mergers.

In spite of the wide range of questions that this model has answered, there are many limitations to the range of issues that the model can incorporate and questions that it can answer. In particular, much of the antitrust literature has used the paradigm that mergers increase concentration and that increased concentration increases the ability of firms to collude. There is a rich literature describing when collusion is likely to result from more concentration and what its effects might be.[23] As evidence that collusion has been central to the debate of antitrust policy, the courts in the United States have made a distinction between mergers for market-

[23] See Hay and Werden (1993) for a survey of this literature.

power or collusive reasons and for efficiency increasing reasons: Breit and Elzinga (1982) note that the courts have allowed mergers within reason because "the reduction in competition … is ancillary to the main purpose which is to bring about a greater degree of efficiency."[24] In my model, I am not modeling collusion between firms directly. Instead, there are two ways in which collusion appears. First, with static decisions (i.e. pricing and quantity), as I am not modeling collusion over the one-shot Cournot level, the effect of increased concentration is that there are less firms playing a one-shot Cournot game. Thus, the amount of pricing collusion in this framework is a decreasing function of the number of firms in the industry at any time. Second, with dynamic decisions (i.e. entry, exit, investment and mergers) as firms make decisions rationally in my model, they are made knowing that they will potentially have some effects on the efficiency and also some effects on the rents that they will earn, because of the change in concentration. Thus, in my model, dynamic decisions are always made by firms who are aware of the possible collusive consequences of their decisions, and so the market power gained by firms that merge is never 'ancillary'. It is therefore not possible to separate the different effects of collusion and efficiency gains in the way that the courts have discussed in the past.

In addition to collusion, there are many other reasons for mergers that have been studied, such as predation, multimarket contact and deep pockets. All of these reasons are important determinants of the effects of mergers in many different cases, but they are outside the scope of this model. In addition, the version of antitrust policy that I have modeled is much simpler than any antitrust policy that the government uses. In the real-world, issues of market definition render market

[24] p. 65-66.

shares a much trickier concept than in my model and governments may incur substantial enforcement costs in finding and prosecuting antitrust offenders.[25]

Despite the limitations of the model, the results show that, by using an endogenous dynamic model of mergers, I have obtained results that provide new insight into which mergers will happen in an industry. In many instances, the dynamics of the model show that the insights that one might expect will hold. For instance, the model shows that intermediate antitrust policies nearly always dominate either extreme antitrust policy. However, in other cases, the conclusions of the model show that by incorporating dynamic effects of firm behavior and an endogenous framework, the conclusions that result are surprising and very different from what one might expect. For instance, the results show that a wide range of antitrust policies yields surplus near the maximum and that with large barriers to entry, strict antitrust policies might actually increase the industry concentration and decrease welfare in this way. In general, the conclusions show that it is necessary to examine mergers in a dynamic, endogenous framework in order to determine when static intuition about mergers will be correct and when unexpected dynamic effects will yield different results. At present, the model is sufficiently realistic that may use the model with different industry characterizations (i.e. barriers to entry, increasing returns to scale, fluctuations in demand, etc.) in order to determine the signs and magnitudes of antitrust policies in different industries. The ultimate goal of this research is to estimate the parameters of the model in order to ascertain the occurrences and consequences of mergers for different industries without the inconsistencies of previous models.

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[25] See Breit and Elzinga (1976) for a discussion of optimal antitrust policies with enforcement costs.

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Antitrust active with max. concentration allowed:	0	0.2	0.4	0.5	0.6	0.7	0.8	0.9	1
% of periods with 1 firm active	0.21%	0.27%	0.08%	0.29%	0.28%	0.54%	3.33%	9.26%	58.1%
% with 2 firms active	84.8%	83.6%	84.2%	92.3%	97.4%	97.0%	93.2%	88.6%	39.8%
% with 3 firms active	14.9%	15.5%	15.6%	7.4%	2.28%	2.40%	3.39%	2.11%	0.02%
% with 4 firms active	0.07%	0.60%	0.14%	0.05%	0.08%	0.06%	0.06%	0.02%	0.01%
% of periods with entry	0.42%	0.48%	0.77%	1.3%	1.77%	1.83%	2.40%	2.80%	12.67%
% of periods with exit	0.41%	0.46%	0.31%	0.21%	0.24%	0.24%	0.38%	0.60%	2.07%
Mean total investment (standard deviation)	0.168 (0.109)	0.170 (0.115)	0.171 (0.132)	0.168 (0.161)	0.165 (0.177)	0.167 (0.180)	0.171 (0.198)	0.169 (0.208)	0.227 (0.373)
Mean firm production (standard deviation)	0.551 (0.149)	0.548 (0.152)	0.552 (0.145)	0.580 (0.132)	0.602 (0.132)	0.601 (0.134)	0.603 (0.157)	0.618 (0.186)	0.726 (0.349)
% of periods with no mergers	100%	99.9%	99.6%	98.9%	98.5%	98.5%	98.0%	97.8%	89.7%
% of periods with 1 merger	0%	0.002%	0.44%	1.08%	1.42%	1.44%	1.91%	2.13%	9.97%
% of periods with 2 mergers	0%	0%	0.003%	0.02%	0.05%	0.07%	0.05%	0.03%	0.31%
% of periods with 3 mergers	0%	0%	0%	0%	0%	0%	0%	0%	0.001%
Mean realized synergy draw (standard deviation)	n/a	0.050 (0.038)	0.080 (0.076)	0.095 (0.070)	0.143 (0.038)	0.145 (0.037)	0.148 (0.037)	0.142 (0.043)	0.162 (0.024)
Mean consumer surplus (standard deviation)	0.724 (0.235)	0.728 (0.236)	0.733 (0.230)	0.740 (0.201)	0.753 (0.169)	0.750 (0.171)	0.741 (0.170)	0.722 (0.169)	0.550 (0.085)
Mean producer surplus (standard deviation)	0.751 (0.128)	0.747 (0.137)	0.747 (0.146)	0.759 (0.162)	0.766 (0.170)	0.765 (0.173)	0.765 (0.191)	0.775 (0.201)	0.784 (0.376)
Mean total surplus (standard deviation)	1.475 (0.292)	1.475 (0.298)	1.480 (0.300)	1.499 (0.295)	1.518 (0.284)	1.515 (0.287)	1.507 (0.296)	1.497 (0.291)	1.334 (0.388)

Table 1

Base-case auction results with different antitrust policies.

Antitrust active with max. concentration allowed:	0	0.2	0.4	0.5	0.6	0.7	0.8	0.9	1
% of periods with 1 firm active	0.21%	0.25%	0.11%	0.08%	0.20%	0.09%	0.06%	0.11%	98.4%
% with 2 firms active	84.8%	83.4%	90.4%	97.3%	99.6%	99.9%	99.9%	99.9%	1.58%
% with 3 firms active	14.9%	16.2%	9.42%	2.57%	0.23%	0.06%	0.04%	0.02%	0.003%
% with 4 firms active	0.07%	0.11%	0.04%	0%	0%	0%	0%	0%	0%
% of periods with entry	0.42%	0.45%	0.60%	0.80%	0.90%	0.86%	0.85%	0.96%	4.25%
% of periods with exit	0.41%	0.44%	0.19%	0.09%	0.09%	0.07%	0.07%	0.11%	0.13%
Mean total investment (standard deviation)	0.168 (0.109)	0.171 (0.112)	0.165 (0.122)	0.163 (0.134)	0.160 (0.138)	0.159 (0.136)	0.160 (0.134)	0.162 (0.141)	0.107 (0.233)
Mean firm production (standard deviation)	0.551 (0.149)	0.546 (0.151)	0.563 (0.144)	0.578 (0.136)	0.586 (0.133)	0.587 (0.131)	0.587 (0.132)	0.585 (0.136)	0.972 (0.115)
% of periods with no mergers	100%	100%	99.6%	99.3%	99.2%	99.2%	99.2%	99.2%	95.9%
% of periods with 1 merger	0%	0%	0.40%	0.68%	0.79%	0.78%	0.76%	0.85%	4.02%
% of periods with 2 mergers	0%	0%	0%	0.02%	0.006%	0.002%	0.003%	0.002%	0.05%
% of periods with 3 mergers	0%	0%	0%	0%	0%	0%	0%	0%	0%
Mean realized synergy draw (standard deviation)	n/a	n/a	0.004 (0.113)	0.016 (0.110)	0.030 (0.101)	0.017 (0.109)	0.026 (0.104)	0.032 (0.110)	0.053 (0.084)
Mean consumer surplus (standard deviation)	0.724 (0.235)	0.723 (0.239)	0.718 (0.223)	0.705 (0.211)	0.705 (0.205)	0.707 (0.204)	0.707 (0.202)	0.701 (0.204)	0.489 (0.041)
Mean producer surplus (standard deviation)	0.751 (0.128)	0.747 (0.130)	0.759 (0.135)	0.768 (0.142)	0.773 (0.145)	0.774 (0.141)	0.774 (0.139)	0.772 (0.145)	0.893 (0.237)
Mean total surplus (standard deviation)	1.475 (0.292)	1.470 (0.297)	1.476 (0.294)	1.473 (0.296)	1.478 (0.295)	1.480 (0.292)	1.481 (0.288)	1.474 (0.295)	1.382 (0.254)

Table 2

Base-case tioli results with different antitrust policies.

Type of agent:	Social planner		Perfectly Colluding Firms	
Antitrust max. concentration::	0 - 0.9	1	0 - 0.9	1
% of periods with 1 firm active	98.7%	99.8%	99.0%	99.9%
% of periods with 2 firms active	1.25%	0.14%	0.96%	0.12%
% of periods with 3 firms active	0.07%	0%	0%	0%
% of period with 4 firms active	0%	0%	0%	0%
% of periods with entry	0.01%	0.68%	0.05%	0.96%
% of periods with exit	0.01%	0.002%	0.05%	0%
Mean total investment (standard deviation)	0.076 (0.034)	0.076 (0.101)	0.079 (0.045)	0.078 (0.120)
Mean firm production (standard deviation)	1.663 (0.284)	1.727 (0.198)	0.825 (0.189)	0.871 (0.135)
% of periods with no mergers	100%	99.3%	100%	99.0%
% of periods with 1 merger	0%	0.67%	0%	0.96%
% of periods with 2 mergers	0%	0.003%	0%	0.002%
% of periods with 3 mergers	0%	0%	0%	0%
Mean realized synergy draw (standard deviation)	n/a	0.034 (0.097)	n/a	0.027 (0.103)
Mean consumer surplus (standard deviation)	1.450 (0.387)	1.514 (0.321)	0.362 (0.132)	0.389 (0.109)
Mean producer surplus (standard deviation)	0.395 (0.271)	0.356 (0.251)	0.861 (0.138)	0.888 (0.154)
Mean total surplus (standard deviation)	1.845 (0.134)	1.870 (0.139)	1.224 (0.263)	1.277 (0.237)

Table 3

Base-case social planner and perfect colluder results with different antitrust policies.

Ownership structure:	Multiple agents (Markov-Perfect Nash Equilibrium)						Perfect colluder		Social planner
Antitrust max. concentration:	0.0	0.2	0.4	0.6	0.8	1.0	0.0	1.0	1.0
% of periods with 1 firm active	0.02%	0.02%	0.003%	0%	0.005%	0.88%	98.9%	99.6%	99.6%
% with 2 firms active	52.5%	51.5%	63.0%	93.7%	94.9%	93.8%	1.03%	0.42%	0.36%
% with 3 firms active	45.6%	46.8%	36.3%	6.03%	4.93%	5.09%	0.04%	0%	0%
% with 4 firms active	1.88%	1.6%	0.71%	0.28%	0.20%	0.23%	0%	0%	0%
% of periods with entry	0.49%	0.50%	1.07%	3.10%	3.23%	3.65%	0.04%	1.03%	0.81%
% of periods with exit	0.48%	0.48%	0.33%	0.33%	0.34%	0.42%	0.04%	0.002%	0.003%
Mean total investment (standard deviation)	0.198 (0.128)	0.197 (0.127)	0.190 (0.157)	0.180 (0.228)	0.181 (0.232)	0.184 (0.240)	0.079 (0.044)	0.078 (0.124)	0.077 (0.108)
Mean firm capital stock (standard deviation)	0.618 (0.218)	0.620 (0.217)	0.648 (0.210)	0.768 (0.199)	0.769 (0.200)	0.775 (0.213)	1.054 (0.231)	1.106 (0.186)	2.040 (0.229)
% of periods with no mergers	100%	99.9%	99.3%	97.4%	97.2%	96.9%	100%	99.0%	99.2%
% of periods with 1 merger	0%	0.01%	0.71%	2.48%	2.65%	2.91%	0%	1.00%	0.78%
% of periods with 2 mergers	0%	0%	0.01%	0.14%	0.12%	0.15%	0%	0.01%	0.01%
% of periods with 3 mergers	0%	0%	0%	0%	0%	0.003%	0%	0%	0%
Mean realized synergy draw (standard deviation)	n/a	0.145 (0.089)	0.161 (0.097)	0.211 (0.058)	0.216 (0.056)	0.222 (0.055)	n/a	0.089 (0.125)	0.104 (0.117)
Mean consumer surplus (standard deviation)	0.977 (0.303)	0.984 (0.301)	0.974 (0.274)	0.984 (0.156)	0.979 (0.152)	0.975 (0.156)	0.450 (0.150)	0.485 (0.121)	1.895 (0.354)
Mean producer surplus (standard deviation)	0.878 (0.152)	0.877 (0.151)	0.897 (0.168)	0.933 (0.210)	0.934 (0.213)	0.934 (0.220)	1.072 (0.137)	1.100 (0.156)	0.398 (0.284)
Mean total surplus (standard deviation)	1.855 (0.325)	1.861 (0.322)	1.871 (0.316)	1.917 (0.306)	1.913 (0.309)	1.909 (0.316)	1.522 (0.279)	1.582 (0.248)	2.293 (0.144)

Table 4a

Production cost differences: results for model with no cost differences.

Ownership structure:	Multiple agents (Markov-Perfect Nash Equilibrium)						Perfect colluder		Social planner
Antitrust max. concentration:	0.0	0.2	0.4	0.6	0.8	1.0	0.0	1.0	1.0
% of periods with 1 firm active	1.36%	0.80%	0.80%	1.30%	0.32%	47.5%	99.9%	99.8%	99.8%
% with 2 firms active	92.2%	94.2%	87.6%	96.5%	94.9%	49.9%	0.12%	0.19%	0.18%
% with 3 firms active	6.44%	4.93%	11.6%	2.11%	1.87%	2.57%	0%	0%	0%
% with 4 firms active	0.02%	0.04%	0.05%	0.07%	0.02%	0.02%	0%	0%	0%
% of periods with entry	0.14%	0.13%	0.40%	1.91%	2.02%	14.1%	0%	1.04%	0.69%
% of periods with exit	0.13%	0.13%	0.09%	0.18%	0.15%	1.62%	0%	0.002%	0%
Mean total investment (standard deviation)	0.161 (0.082)	0.158 (0.081)	0.163 (0.105)	0.168 (0.185)	0.166 (0.188)	0.250 (0.393)	0.078 (0.036)	0.079 (0.125)	0.076 (0.102)
Mean firm capital stock (standard deviation)	0.719 (0.217)	0.725 (0.214)	0.721 (0.207)	0.778 (0.198)	0.784 (0.205)	1.089 (0.589)	1.079 (0.210)	1.117 (0.180)	2.041 (0.218)
% of periods with no mergers	100%	100%	99.7%	98.4%	98.2%	88.1%	100%	99.0%	99.3%
% of periods with 1 merger	0%	0%	0.29%	1.57%	1.73%	11.4%	0%	1.03%	0.67%
% of periods with 2 mergers	0%	0%	0.005%	0.08%	0.07%	0.53%	0%	0.005%	0%
% of periods with 3 mergers	0%	0%	0%	0%	0%	0.005%	0%	0%	0%
Mean realized synergy draw (standard deviation)	n/a	n/a	0.133 (0.124)	0.184 (0.073)	0.196 (0.072)	0.217 (0.057)	n/a	0.049 (0.146)	0.051 (0.136)
Mean consumer surplus (standard deviation)	0.893 (0.249)	0.896 (0.244)	0.927 (0.243)	0.955 (0.167)	0.945 (0.178)	0.736 (0.171)	0.461 (0.142)	0.490 (0.117)	1.889 (0.359)
Mean producer surplus (standard deviation)	0.929 (0.112)	0.932 (0.108)	0.920 (0.131)	0.932 (0.178)	0.936 (0.182)	0.939 (0.396)	1.077 (0.123)	1.099 (0.157)	0.402 (0.286)
Mean total surplus (standard deviation)	1.822 (0.304)	1.828 (0.304)	1.847 (0.292)	1.887 (0.289)	1.881 (0.294)	1.674 (0.437)	1.539 (0.259)	1.589 (0.246)	2.292 (0.140)

Table 4b

Production cost differences: results for model with small cost differences.

Ownership structure:	Multiple agents (Markov-Perfect Nash Equilibrium)						Perfect colluder		Social planner
Antitrust max. concentration:	0.0	0.2	0.4	0.6	0.8	1.0	0.0	1.0	1.0
% of periods with 1 firm active	88.8%	90.0%	73.0%	84.2%	83.0%	81.2%	100.0%	99.9%	99.9%
% with 2 firms active	11.2%	10.0%	27.0%	15.7%	16.9%	17.8%	0%	0.09%	0.12%
% with 3 firms active	0%	0%	0%	0.08%	0.09%	1.01%	0%	0%	0%
% with 4 firms active	0%	0%	0%	0.005%	0%	0.01%	0%	0%	0%
% of periods with entry	0.01%	0.006%	0.01%	0.14%	0.20%	16.6%	0.005%	1.06%	0.70%
% of periods with exit	0.01%	0.006%	0.01%	0.02%	0.01%	1.57%	0.005%	0.002%	0.002%
Mean total investment (standard deviation)	0.087 (0.050)	0.090 (0.010)	0.099 (0.058)	0.092 (0.071)	0.092 (0.078)	0.242 (0.426)	0.078 (0.038)	0.076 (0.126)	0.076 (0.102)
Mean firm capital stock (standard deviation)	0.962 (0.231)	0.967 (0.224)	0.912 (0.238)	0.948 (0.227)	0.949 (0.226)	1.390 (0.599)	1.103 (0.214)	1.142 (0.178)	2.041 (0.214)
% of periods with no mergers	100%	100%	100%	99.9%	99.8%	86.1%	100%	98.9%	99.3%
% of periods with 1 merger	0%	0%	0%	0.11%	0.18%	12.9%	0%	1.06%	0.68%
% of periods with 2 mergers	0%	0%	0%	0.007%	0.005%	1.02%	0%	0%	0.009%
% of periods with 3 mergers	0%	0%	0%	0%	0%	0.02%	0%	0%	0%
Mean realized synergy draw (standard deviation)	n/a	n/a	n/a	0.165 (0.090)	0.158 (0.094)	0.172 (0.084)	n/a	0.024 (0.165)	0.044 (0.146)
Mean consumer surplus (standard deviation)	0.526 (0.206)	0.522 (0.198)	0.599 (0.255)	0.549 (0.228)	0.559 (0.232)	0.613 (0.048)	0.474 (0.140)	0.504 (0.111)	1.886 (0.359)
Mean producer surplus (standard deviation)	1.056 (0.142)	1.060 (0.133)	1.029 (0.145)	1.047 (0.149)	1.046 (0.151)	0.985 (0.437)	1.074 (0.134)	1.099 (0.159)	0.405 (0.286)
Mean total surplus (standard deviation)	1.582 (0.285)	1.583 (0.272)	1.629 (0.295)	1.596 (0.291)	1.605 (0.291)	1.598 (0.438)	1.547 (0.268)	1.603 (0.244)	2.291 (0.140)

Table 4c

Production cost differences: results for model with large cost differences.

Ownership structure:	Multiple agents (Markov-Perfect Nash Equilibrium)						Perfect colluder		Social planner
Antitrust max. concentration:	0.0	0.2	0.4	0.6	0.8	1.0	0.0	1.0	1.0
% of periods with 1 firm active	0.21%	0.18%	0.05%	0.31%	3.92%	31.8%	99.0%	99.8%	99.8%
% with 2 firms active	84.8%	86.7%	77.6%	95.8%	91.7%	64.6%	0.96%	0.16%	0.17%
% with 3 firms active	14.9%	12.9%	22.1%	3.74%	4.34%	3.65%	0%	0%	0%
% with 4 firms active	0.07%	0.18%	0.24%	0.11%	0.07%	0.04%	0%	0%	0%
% of periods with entry	0.42%	4.34%	1.12%	3.08%	3.76%	12.2%	0.05%	1.39%	1.10%
% of periods with exit	0.41%	0.43%	0.32%	0.23%	0.36%	1.22%	0.05%	0%	0%
Mean total investment (standard deviation)	0.168 (0.109)	0.168 (0.111)	0.180 (0.153)	0.177 (0.225)	0.182 (0.239)	0.241 (0.370)	0.079 (0.045)	0.079 (0.142)	0.076 (0.125)
Mean firm production (standard deviation)	0.551 (0.149)	0.553 (0.151)	0.540 (0.144)	0.610 (0.117)	0.613 (0.151)	0.664 (0.272)	0.825 (0.189)	0.882 (0.126)	1.757 (0.187)
% of periods with no mergers	100%	99.9%	99.2%	97.3%	96.7%	89.6%	100%	98.6%	98.9%
% of periods with 1 merger	0%	0%	0.79%	2.51%	3.20%	9.87%	0%	1.38%	1.09%
% of periods with 2 mergers	0%	0%	0%	0.17%	0.10%	0.51%	0%	0.002%	0.01%
% of periods with 3 mergers	0%	0%	0%	0%	0%	0.01%	0%	0%	0%
Mean realized synergy draw (standard deviation)	n/a	n/a	0.270 (0.087)	0.318 (0.052)	0.342 (0.039)	0.348 (0.038)	n/a	0.219 (0.104)	0.226 (0.099)
Mean consumer surplus (standard deviation)	0.724 (0.235)	0.718 (0.233)	0.746 (0.237)	0.782 (0.154)	0.764 (0.152)	0.661 (0.151)	0.362 (0.132)	0.398 (0.102)	1.563 (0.303)
Mean producer surplus (standard deviation)	0.751 (0.128)	0.753 (0.129)	0.735 (0.166)	0.755 (0.213)	0.760 (0.231)	0.761 (0.374)	0.861 (0.138)	0.895 (0.171)	0.319 (0.250)
Mean total surplus (standard deviation)	1.475 (0.292)	1.471 (0.296)	1.481 (0.310)	1.536 (0.308)	1.524 (0.310)	1.422 (0.417)	1.224 (0.263)	1.293 (0.243)	1.882 (0.155)

Table 5a

Merger cost/synergy differences: results for model with positive merger synergies.

Ownership structure:	Multiple agents (Markov-Perfect Nash Equilibrium)						Perfect colluder		Social planner
Antitrust max. concentration:	0.0	0.2	0.4	0.6	0.8	1.0	0.0	1.0	1.0
% of periods with 1 firm active	0.21%	0.24%	0.17%	0.24%	1.26%	55.2%	99.0%	99.9%	99.9%
% with 2 firms active	84.8%	84.7%	85.7%	97.7%	95.5%	42.6%	0.96%	0.09%	0.11%
% with 3 firms active	14.9%	14.7%	14.0%	1.95%	3.13%	2.18%	0%	0%	0%
% with 4 firms active	0.07%	0.35%	0.11%	0.12%	0.06%	0.007%	0%	0%	0%
% of periods with entry	0.42%	0.46%	0.63%	1.18%	1.57%	9.65%	0.05%	0.07%	0.4%
% of periods with exit	0.41%	0.45%	0.32%	0.28%	0.43%	2.03%	0.05%	0%	0.001%
Mean total investment (standard deviation)	0.168 (0.109)	0.170 (0.114)	0.169 (0.123)	0.163 (0.153)	0.167 (0.168)	0.202 (0.332)	0.079 (0.045)	0.078 (0.104)	0.074 (0.080)
Mean firm production (standard deviation)	0.551 (0.149)	0.548 (0.153)	0.554 (0.147)	0.593 (0.134)	0.593 (0.154)	0.711 (0.346)	0.825 (0.189)	0.862 (0.141)	1.725 (0.205)
% of periods with no mergers	100%	100%	99.7%	99.2%	98.8%	92.6%	100%	99.3%	99.6%
% of periods with 1 merger	0%	0%	0.29%	0.81%	1.10%	7.24%	0%	0.68%	0.41%
% of periods with 2 mergers	0%	0%	0%	0.04%	0.01%	0.17%	0%	0.002%	0.001%
% of periods with 3 mergers	0%	0%	0%	0%	0%	0%	0%	0%	0%
Mean realized synergy draw (standard deviation)	n/a	n/a	-0.101 (0.068)	-0.065 (0.046)	-0.042 (0.038)	-0.030 (0.020)	n/a	-0.172 (0.099)	-0.161 (0.093)
Mean consumer surplus (standard deviation)	0.724 (0.235)	0.720 (0.238)	0.727 (0.230)	0.731 (0.187)	0.731 (0.181)	0.550 (0.093)	0.362 (0.132)	0.382 (0.113)	1.510 (0.335)
Mean producer surplus (standard deviation)	0.751 (0.128)	0.748 (0.133)	0.750 (0.140)	0.768 (0.152)	0.766 (0.165)	0.788 (0.332)	0.861 (0.138)	0.883 (0.144)	0.358 (0.256)
Mean total surplus (standard deviation)	1.475 (0.292)	1.468 (0.301)	1.477 (0.298)	1.500 (0.287)	1.497 (0.291)	1.338 (0.351)	1.224 (0.263)	1.265 (0.235)	1.868 (0.127)

Table 5b

Merger cost/synergy differences: results for model with merger transaction costs.

Ownership structure:	Multiple agents (Markov-Perfect Nash Equilibrium)						Perfect colluder		Social planner
Antitrust max. concentration:	0.0	0.2	0.4	0.6	0.8	1.0	0.0	1.0	1.0
% of periods with 1 firm active	0.21%	0.27%	0.08%	0.28%	3.33%	58.1%	99.0%	99.9%	99.8%
% with 2 firms active	84.8%	83.6%	84.2%	97.4%	93.2%	39.8%	0.96%	0.12%	0.14%
% with 3 firms active	14.9%	15.5%	15.6%	2.28%	3.39%	0.02%	0%	0%	0%
% with 4 firms active	0.07%	0.60%	0.14%	0.08%	0.06%	0.01%	0%	0%	0%
% of periods with entry	0.42%	0.48%	0.77%	1.77%	2.40%	12.67%	0.05%	0.96%	0.68%
% of periods with exit	0.41%	0.46%	0.31%	0.24%	0.38%	2.07%	0.05%	0%	0.002%
Mean total investment (standard deviation)	0.168 (0.109)	0.170 (0.115)	0.171 (0.132)	0.165 (0.177)	0.171 (0.198)	0.227 (0.373)	0.079 (0.045)	0.078 (0.120)	0.076 (0.101)
Mean firm production (standard deviation)	0.551 (0.149)	0.548 (0.152)	0.552 (0.145)	0.602 (0.132)	0.603 (0.157)	0.726 (0.349)	0.825 (0.189)	0.871 (0.135)	1.727 (0.198)
% of periods with no mergers	100%	99.9%	99.6%	98.5%	98.0%	89.7%	100%	99.0%	99.3%
% of periods with 1 merger	0%	0.002%	0.44%	1.42%	1.91%	9.97%	0%	0.96%	0.67%
% of periods with 2 mergers	0%	0%	0.003%	0.05%	0.05%	0.31%	0%	0.002%	0.003%
% of periods with 3 mergers	0%	0%	0%	0%	0%	0.001%	0%	0%	0%
Mean realized synergy draw (standard deviation)	n/a	0.050 (0.038)	0.080 (0.076)	0.143 (0.038)	0.148 (0.037)	0.162 (0.024)	n/a	0.027 (0.103)	0.034 (0.097)
Mean consumer surplus (standard deviation)	0.724 (0.235)	0.728 (0.236)	0.733 (0.230)	0.753 (0.169)	0.741 (0.170)	0.550 (0.085)	0.362 (0.132)	0.389 (0.109)	1.514 (0.321)
Mean producer surplus (standard deviation)	0.751 (0.128)	0.747 (0.137)	0.747 (0.146)	0.766 (0.170)	0.765 (0.191)	0.784 (0.376)	0.861 (0.138)	0.888 (0.154)	0.356 (0.251)
Mean total surplus (standard deviation)	1.475 (0.292)	1.475 (0.298)	1.480 (0.300)	1.518 (0.284)	1.507 (0.296)	1.334 (0.388)	1.224 (0.263)	1.277 (0.237)	1.870 (0.139)

Table 5c and 6a

Base case results (no barriers to entry and mean-zero merger cost/synergy).

Ownership structure:	Multiple agents (Markov-Perfect Nash Equilibrium)						Perfect colluder		Social planner
Antitrust max. concentration:	0.0	0.2	0.4	0.6	0.8	1.0	0.0	1.0	1.0
% of periods with 1 firm active	66.7%	62.8%	66.0%	59.6%	71.7%	94.9%	99.96%	99.99%	99.99%
% with 2 firms active	33.3%	37.2%	34.0%	40.4%	28.3%	5.12%	0.04%	0.01%	0.01%
% with 3 firms active	0%	0%	0%	0.04%	0.02%	0.01%	0%	0%	0%
% with 4 firms active	0%	0%	0%	0%	0%	0%	0%	0%	0%
% of periods with entry	0.07%	0.08%	0.08%	0.09%	0.08%	3.27%	0.003%	0.25%	0.10%
% of periods with exit	0.07%	0.08%	0.08%	0.07%	0.06%	0.29%	0.003%	0%	0%
Mean total investment (standard deviation)	0.104 (0.078)	0.108 (0.083)	0.106 (0.081)	0.110 (0.085)	0.101 (0.081)	0.129 (0.327)	0.078 (0.036)	0.078 (0.101)	0.075 (0.068)
Mean firm production (standard deviation)	0.713 (0.204)	0.699 (0.204)	0.706 (0.204)	0.691 (0.201)	0.726 (0.205)	0.909 (0.197)	0.835 (0.174)	0.852 (0.153)	1.700 (0.224)
% of periods with no mergers	100%	100%	100%	99.99%	99.98%	97.0%	100%	99.7%	99.9%
% of periods with 1 merger	0%	0%	0%	0.01%	0.02%	2.98%	0%	0.25%	0.10%
% of periods with 2 mergers	0%	0%	0%	0%	0.002%	0%	0%	0%	0.002%
% of periods with 3 mergers	0%	0%	0%	0%	0%	0%	0%	0%	0%
Mean realized synergy draw (standard deviation)	n/a	n/a	n/a	0.104 (0.031)	0.130 (0.051)	0.121 (0.048)	n/a	0.006 (0.111)	0.035 (0.099)
Mean consumer surplus (standard deviation)	0.478 (0.221)	0.487 (0.227)	0.475 (0.223)	0.499 (0.231)	0.459 (0.210)	0.461 (0.080)	0.364 (0.131)	0.375 (0.121)	1.470 (0.367)
Mean producer surplus (standard deviation)	0.840 (0.128)	0.835 (0.133)	0.836 (0.132)	0.832 (0.132)	0.845 (0.130)	0.864 (0.336)	0.865 (0.127)	0.877 (0.148)	0.385 (0.269)
Mean total surplus (standard deviation)	1.318 (0.277)	1.322 (0.284)	1.310 (0.284)	1.331 (0.285)	1.304 (0.272)	1.325 (0.366)	1.228 (0.252)	1.252 (0.248)	1.855 (0.133)

Table 6b

Barriers to entry: results for model with small barriers to entry.

Ownership structure:	Multiple agents (Markov-Perfect Nash Equilibrium)						Perfect colluder		Social planner
Antitrust max. concentration:	0.0	0.2	0.4	0.6	0.8	1.0	0.0	1.0	1.0
% of periods with 1 firm active	100%	100%	100%	100%	100%	94.3%	100%	100%	99.997%
% with 2 firms active	0%	0%	0%	0%	0%	5.67%	0%	0.001%	0.003%
% with 3 firms active	0%	0%	0%	0%	0%	0.02%	0%	0%	0%
% with 4 firms active	0%	0%	0%	0%	0%	0%	0%	0%	0%
% of periods with entry	0.003%	0.003%	0.01%	0.002%	0.004%	3.69%	0.003%	0.02%	0.004%
% of periods with exit	0.003%	0.003%	0.01%	0.002%	0.004%	0.33%	0.003%	0%	0%
Mean total investment (standard deviation)	0.077 (0.038)	0.078 (0.038)	0.078 (0.048)	0.077 (0.037)	0.078 (0.039)	0.173 (0.532)	0.078 (0.038)	0.078 (0.052)	0.075 (0.033)
Mean firm production (standard deviation)	0.839 (0.170)	0.835 (0.172)	0.837 (0.173)	0.840 (0.168)	0.836 (0.171)	0.906 (0.204)	0.835 (0.173)	0.836 (0.170)	1.691 (0.233)
% of periods with no mergers	100%	100%	100%	100%	100%	96.6%	100%	99.98%	99.996%
% of periods with 1 merger	0%	0%	0%	0%	0%	3.35%	0%	0.02%	0.004%
% of periods with 2 mergers	0%	0%	0%	0%	0%	0.001%	0%	0%	0%
% of periods with 3 mergers	0%	0%	0%	0%	0%	0%	0%	0%	0%
Mean realized synergy draw (standard deviation)	n/a	n/a	n/a	n/a	n/a	0.119 (0.048)	n/a	0.014 (0.099)	0.065 (0.091)
Mean consumer surplus (standard deviation)	0.366 (0.129)	0.364 (0.129)	0.365 (0.130)	0.367 (0.128)	0.364 (0.129)	0.463 (0.081)	0.364 (0.130)	0.364 (0.129)	1.456 (0.377)
Mean producer surplus (standard deviation)	0.868 (0.125)	0.865 (0.128)	0.866 (0.134)	0.869 (0.122)	0.866 (0.128)	0.820 (0.539)	0.865 (0.128)	0.866 (0.131)	0.394 (0.268)
Mean total surplus (standard deviation)	1.234 (0.248)	1.229 (0.251)	1.231 (0.256)	1.236 (0.244)	1.230 (0.251)	1.283 (0.559)	1.229 (0.252)	1.230 (0.251)	1.851 (0.124)

Table 6c

Barriers to entry: results for model with large barriers to entry.

Type of Merger Process:	Acquired Firm		Acquiring Firm	
	Ratio of selling price to value at beginning of current period where firm is acquired	Ratio of selling price to value at beginning of previous period to that where firm is acquired	Ratio of value at end of current period merger process to value at beginning of period with acquisition	Ratio of value at beginning of next period to value at beginning of period with acquisition
Auction Process (standard deviation)	1.050 (0.087)	1.156 (0.343)	1.004 (0.006)	0.963 (0.092)
Auction Process, Reverse Order (standard deviation)	1.125 (0.904)	1.321 (1.521)	1.024 (0.026)	0.398 (0.435)
Tioli Process (standard deviation)	1.001 (0.033)	1.198 (0.399)	1.004 (0.009)	0.989 (0.030)

Table 7
Change in stock-market valuations for acquired and acquiring firms.

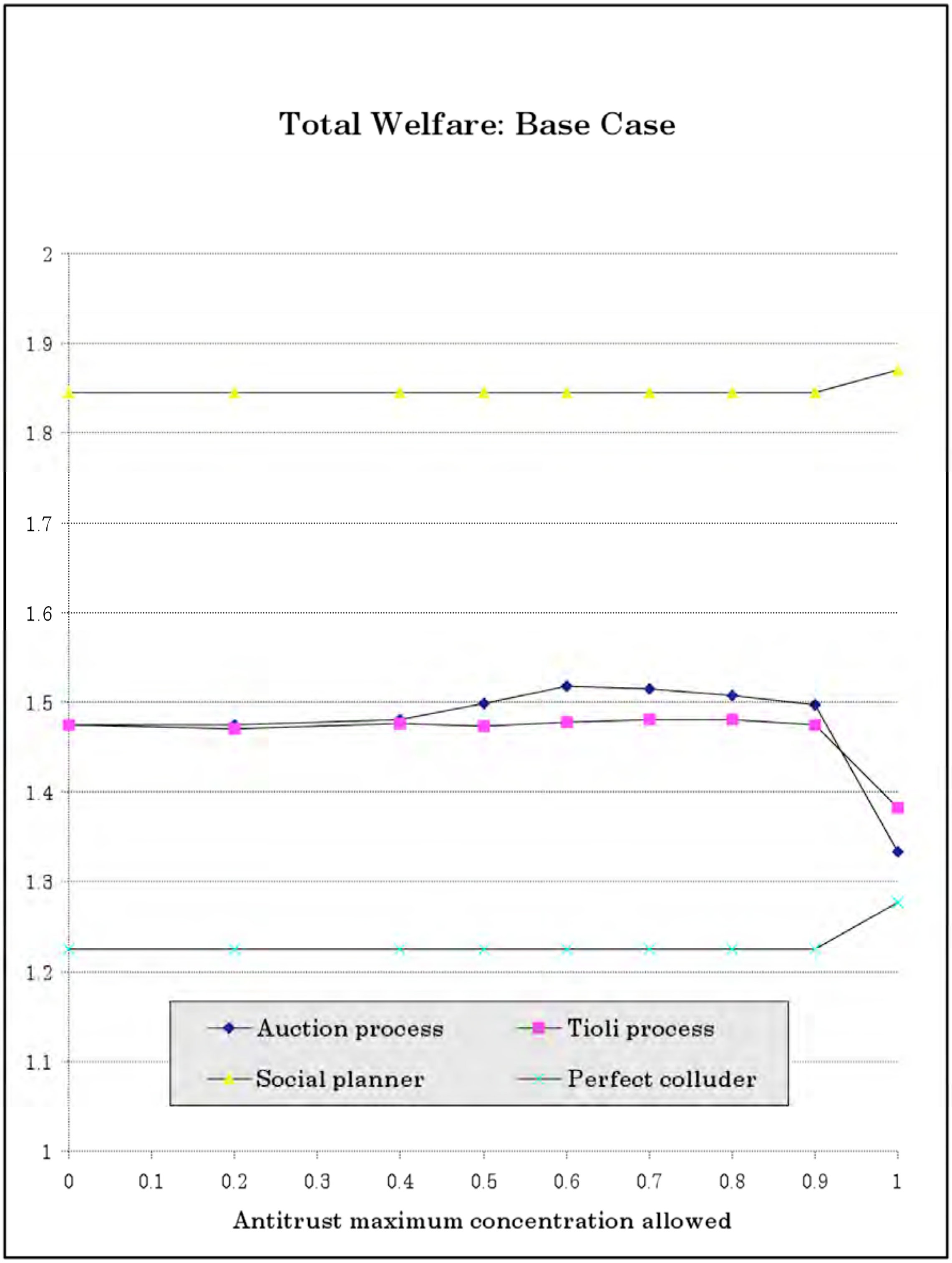

Figure 1
Base-case welfare results with different antitrust policies.

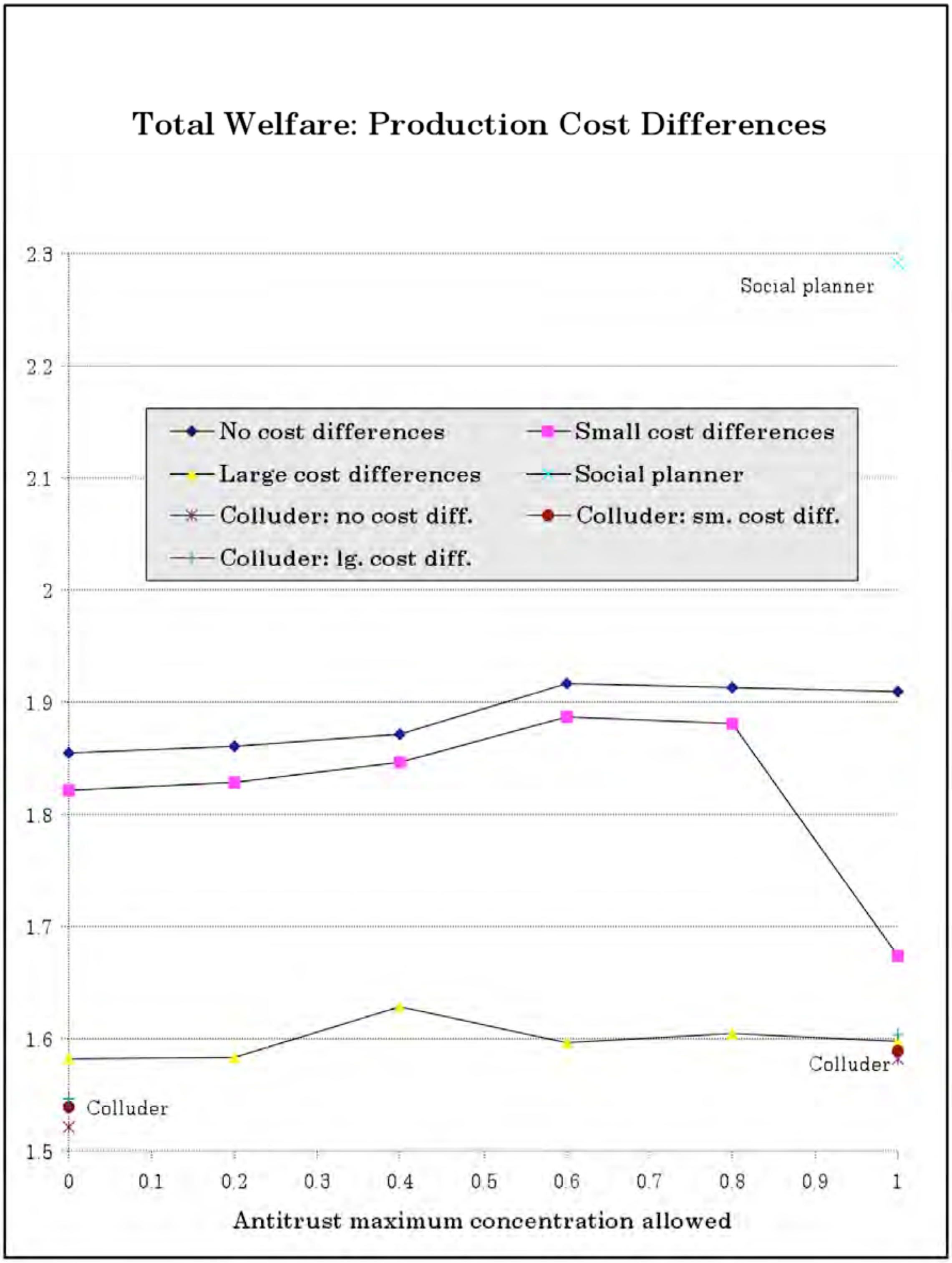

Figure 2
Welfare results for models with production cost differences.

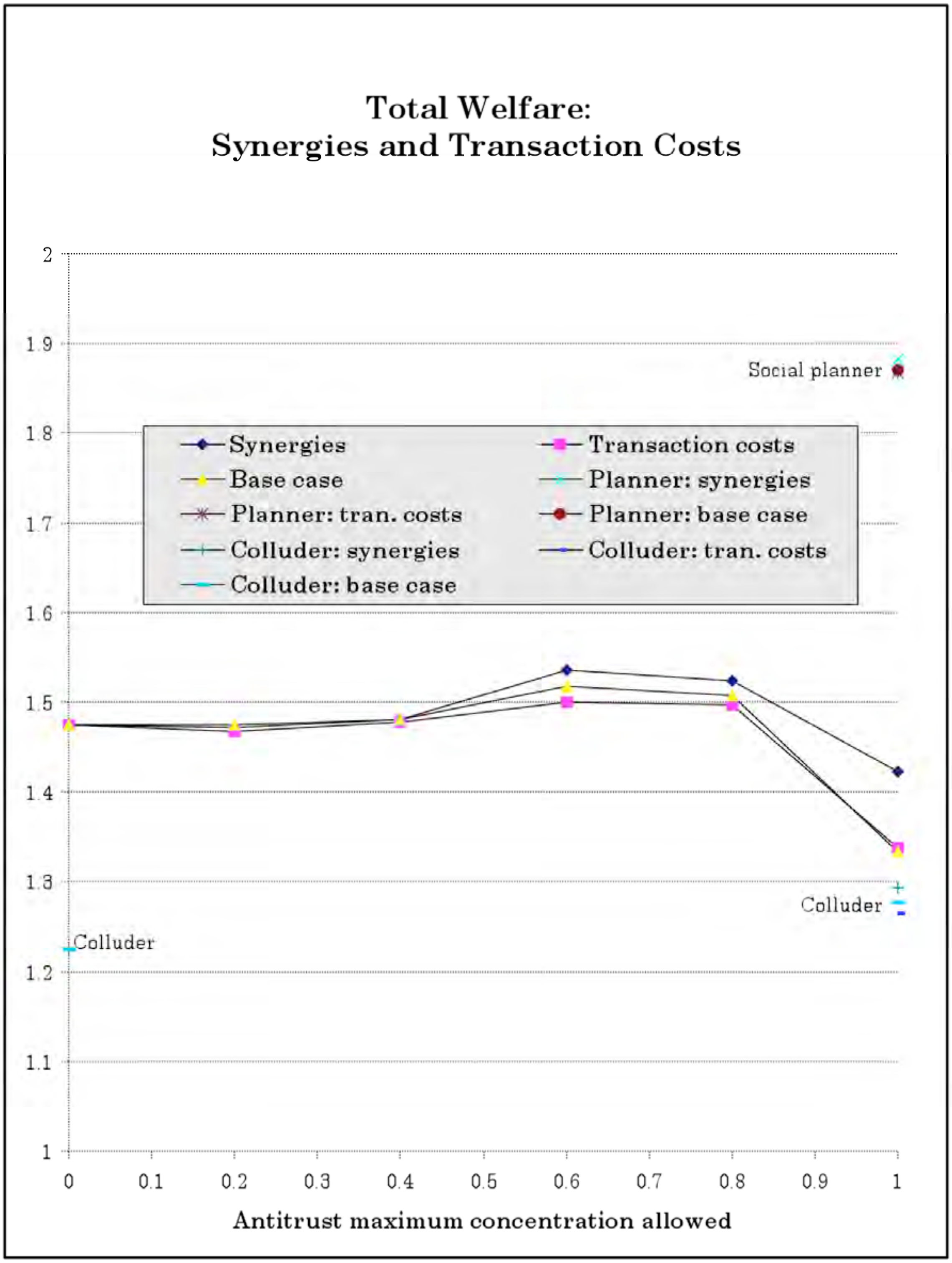

Figure 3

Welfare results for models with merger synergies and transaction costs.

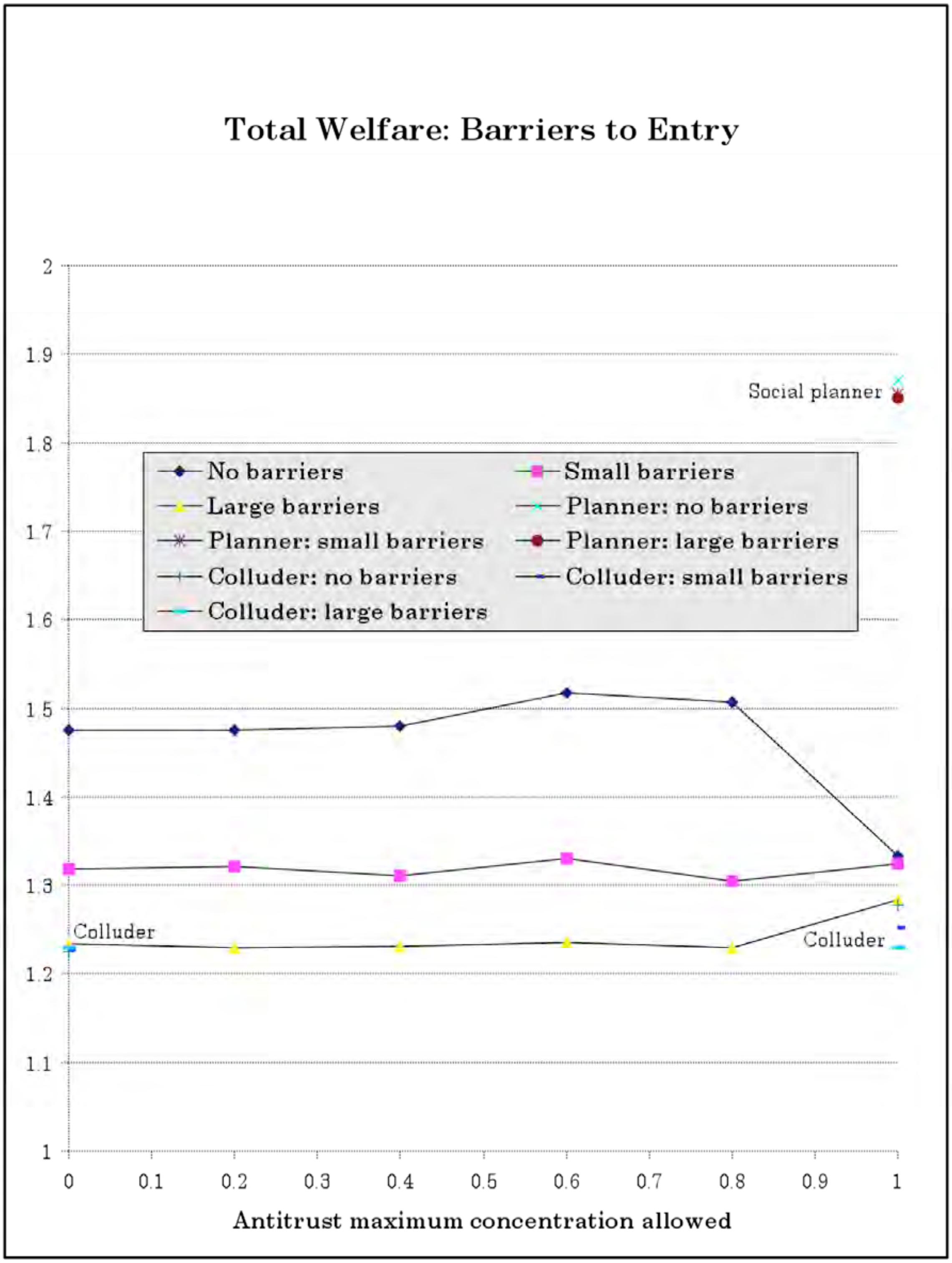

Figure 4

Welfare results for models with barriers to entry.

Chapter 3:

Computing Endogenous Merger Models.

Section 1: Introduction

In Chapters 1 and 2, I discussed a dynamic model of endogenous mergers and examined the implications of this model in different industry settings. As the model incorporates dynamic exit, entry and investment decisions by firms and endogenizes the merger process, I am able to obtain many surprising results that have not been shown in the literature to date. The discussion and results from Chapters 1 and 2 underscore the importance of endogenizing firm choices and using dynamics when evaluating mergers and many other economic phenomena. However, because of the complexity involved in modeling the dynamics, the model can only be solved computationally. While I discussed the model in detail in Chapter 1, I have thus far not discussed the techniques necessary to compute the model. The goal of this chapter is to discuss the technical reasons underlying the choice of model, the issues of computation and the computational methods used.

In order to understand the results of the model presented in the previous chapters, is it only necessary to understand the specifics of the model (as discussed in Chapter 1, Section 2); and not how to compute the model. That is the reason why the computation of the model is not discussed in Chapters 1 and 2. Given that the computational methods are not necessary to grasp the details of the model and evaluate the effects of mergers with the model, the reader might wonder why it is important to understand the computational issues at all. While understanding the model is not one of the reasons to comprehend the computational issues, there are many other reasons; I detail some of them below.

First, a reader may want to understand techniques necessary to compute the model in order to use the computational model. As I have only examined a handful of policies, other economists may want to use the model to investigate antitrust implications in different settings. Many issues that have been previously studied by industrial organization economists, such as appropriate market definitions, multimarket contact and costly antitrust enforcement, can be considered with this model. In addition, this model can serve as a basis for examining the implications of mergers in particular industries, both with calibrated and empirically estimated data. In order to use the model to examine different policies or different industries, it is necessary to change the specification of the model. Thus, one must understand the choice and implementation of the algorithm to do this.

Second, the techniques that are discussed illustrate some of the issues involved in solving models of endogenous mergers. While there is a wide literature that discusses solutions to coalition formation games, the focus of this literature has been on theoretically evaluating possible coalitions and not on predicting the probability of occurrence of particular coalitions. In contrast, the techniques discussed here illustrate when coalition formation games can be solved numerically and what assumptions are necessary to do this. In addition, the processes discussed here can be applied to static merger models in order to compute solutions for these models. This is useful in order to evaluate the effects of mergers in cases where dynamics are too cumbersome or not essential.

Third, the results presented here are of general interest to economists who are interested in computing models of firm interactions in order to characterize many different economic phenomena. In addition to the effects of mergers, computational methods are necessary to analyze many other economic questions, which are too complex to be examined theoretically. In this chapter, I discuss the use of randomness as a method of ensuring convergence to an equilibrium, techniques for solving models with non-differentiabilities and issues of existence and uniqueness of equilibrium. Furthermore, I discuss in detail the computation of auction games with externalities. While auction externalities are often very important theoretically, they have rarely been analyzed because of the difficulty in theoretically examining these externalities. This chapter provides a basis for computationally examining the equilibria of auctions with externalities, including auctions unrelated to a merger process.

In order to answer the above questions, I discuss the choice of merger process and the computational methods necessary to solve the merger process and general industry game. The remainder of this chapter is organized as follows: Section 2 formally describes the industry model including the models of endogenous mergers; Section 3 discusses how to solve models of endogenous mergers; Section 4 illustrates the dynamic programming solution concept that I have used and examines existence of an equilibrium of the model; and Section 5 concludes.

Section 2: Modeling Endogenous Merger Processes

In this section, I discuss the choice of processes for the different components of the model. While the focus of this model is mergers, in order to examine the effects that mergers have on other dynamic industry variables, it is necessary to specify the processes for these other variables. However, as the industry model is not the main focus of this paper, I have taken a version of an existing model of industry dynamics, that of Ericson and Pakes (1995), and added an endogenous merger process to the industry game. The basic framework of the model is that there are infinitely many periods, where a period corresponds to a year. Every period, mergers, exit, investment, entry, and production occur. In this section, I first briefly discuss the industry framework and then detail the choice of merger processes and the specifics of each merger process. The reader should consult Ericson and Pakes for more specifics about the industry model.

<u>Assumptions regarding the static production model:</u>
At any given period:
(a) Every firm is producing the same homogeneous good; demand for the good is linear and the same every period.
(b) Each firm i in the industry has some capacity level u_i, where $u_i = \alpha w_i$, $\alpha \in \mathrm{R}$, $w_i \in \{0,1,2,\ldots\}$. Each firm faces no fixed costs, and a constant marginal cost mc of production up to production level u. At production level u, the marginal cost of additional production is infinite.
(c) Firms set their levels of production simultaneously, and choose these levels in order to form a Cournot-Nash equilibrium.

Thus, the quantities produced will be the Cournot-Nash quantities. Existence and uniqueness of this equilibrium and hence of the quantities is guaranteed by the weakly increasing marginal costs and the linear demand curve.

<u>Definition:</u> A state of the industry is an element $(w_1,\ldots,w_N;n)$ such that $N \in \mathbf{N}$; $w_i \in \{0,1,2,\ldots\}, \forall i$; $w_i \geq w_{i+1}, i = 1,\ldots,N-1$; and $n \in 1,\ldots,N$.

A state here represents the firms active in an industry. The first component of the state represents the capacity of each firm (where capacity is αw_i) and the second component represents the firm that is being considered. A state where some

$R < N$ firms are active can be represented in the above framework, with $w_{R+1} = \cdots = w_N = 0$. In the following discussion, I will use R as the number of active firms. As an example, if $\alpha = 1$, then a state of $(10,9,7,7,0;2)$ represents an industry where there are four firms active, the firm we are considering has a capacity of 9, and its competitors have capacities of 10,7 and 7.

<u>Assumptions regarding entry, exit and investment processes:</u>
This model follows the investment, exit and entry processes specified in Ericson and Pakes, Pakes and McGuire and P-G-M. In this model, the industry exists forever, but firms enter and exit. The reader may consult these works for details. In brief:

(a) At any time period, every firm chooses actions in order to maximize the EDV of its net future profits, conditional on the information that it has available at that time. There is a common discount factor β.

(b) A firm's capacity evolves from period to period as a stochastically increasing function of its level of investment. If a firm's capacity at period t is αk_t, then its capacity at period t + 1 is αk_{t+1}, where $k_{t+1} = \max\{0, k_t + \nu - \tau\}$, where τ is the (random) depreciation of firms current capital that is common to all firms in the industry, and ν is the (random) return to the firm's own investment.[1] Specifically,

$$\tau = \begin{cases} 0, \text{ with prob. } 1-\delta \\ 1, \text{ with prob. } \delta \end{cases}$$

$$\nu = \begin{cases} 0, \text{ with prob. } 1/(1+ax) \\ 1, \text{ with prob. } ax/(1+ax) \end{cases},$$

where x is the amount spent on investment by the firm, and δ and a are constants.[2]

(c) Firms may exit at any period and receive a scrap value of Φ when they exit. Any firm with capacity level 0 will always choose to exit.

(d) At each time t, there is a potential entrant to the industry, who may enter at time t + 1, by paying an entry fee at time t. The entry fee that the new firm must pay is a draw from some uniform distribution $U\left(x^E_{MIN}, x^E_{MAX}\right)$, known to the potential entrant before it has to make the decision. The entrant enters with a

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[1] The maximum is specified because it does not make sense for firms to have negative capacities.

[2] In order to make the algorithm less computationally burdensome, I imposed a maximum capacity $\bar{k}$ on the model. Thus, in the computation, firms cannot obtain a capacity higher than $\bar{k}$; the capacity increase from mergers or investment will be truncated at $\bar{k}$. The reader will see in Section 4 that there is a theoretical maximum capacity; however, for the parameters used, this theoretical maximum proved to be too high to allow for computation.

capacity $\alpha(k_E - \tau)$ (depending on the industry-wide realization of τ at time t), for some fixed value $k_E \in \{0,1,2,\ldots\}$

Assumptions regarding the timing of industry processes:

At every time period t, the following happens, in order:

(a) Firms merge, according to the merger process;

(b) Firms simultaneously decide whether or not to exit immediately;

(c) Firms simultaneously choose levels of investment for this period;

(d) A potential entrant decides whether or not to enter;

(e) The incumbents simultaneously decide on levels of production and production takes place.

From the assumptions, one can see that the endogenous merger process occurs every period in this industry. The idea of this section is to discuss possible ways of modeling this merger process. From a game theoretic point of view, a merger is the same as a coalition that cannot be broken. Hence, the endogenous merger process is often called a coalition formation process and I use these terms interchangeably. In designing the merger formation process, the methodology that I use is to consider future payoffs from any final coalition that is present at the end of the merger process to be a fixed value. Since these future payoffs depend upon actions in future coalition games, the payoffs are actually not fixed. However, because of the dynamic programming solution concept, one can think of them as fixed - this is proved formally in Section 4. Thus, although I am solving the coalition formation process for a dynamic game, the analysis of this section and Section 3 is static, and would apply equally well to static endogenous merger models.

As I am computing the model numerically, the design of the coalition formation process for this model is influenced by the need for the model to generate a unique solution that calculates which mergers occur at any state. Because there are often many possible profitable coalitions, previous papers on coalition formation have not been able to uniquely predict the occurrence of mergers. In general, the difficulty occurs in trying to define what is a profitable merger, as the reservation price from not merging depends on other merger decisions. Thus far, these papers have tackled this problem using one of two basic approaches, reduced-form and structural.

An example of the reduced-form approach to coalition formation is shown in Hart and Kurz (1981). This work uses a variant of Shapley values to examine which mergers have the potential to increase the surplus accruing to participants. Another example is tried by Bernheim, Peleg and Whinston (1984) who use a refinement of Nash equilibrium that recursively defines and prohibits unprofitable joint deviations. The strategy profiles that are not eliminated by this solution concept form the set of potential mergers in this model. With both of these models, the equilibrium is very unlikely to be unique, and with Bernheim, Peleg and Whinston's model, equilibrium may not exist. Thus, although these solution concepts are useful in many cases, the lack of uniqueness or the ability to discern between different equilibria make them unusable for this model.

The structural approaches to endogenous coalition formation have generally used an ordering scheme to enumerate how coalitions can form. Structural approaches have been used in the literature because they can eliminate some of the many equilibria that occur in a reduced-form solution, and because they can mimic reality more accurately. For these reasons, a structural approach is also preferable for this model. An example of a structural model of coalition formation is given in Bloch (1995 and 1995). His model uses a multi-player variant of the Rubinstein bargaining model together with a Markov-Perfect solution concept. In his model, there is some fixed order of players; each player in order proposes a coalition and then other players agree or disagree with the coalition. Bloch is able to obtain unique solutions to the coalition formation process for some special cases, such as symmetric Cournot-Nash equilibrium. While the structural approach of Bloch is appropriate for this model, the fact that symmetry is required to obtain meaningful results and that the division of payoffs is fixed and not a choice variable of the firms are troublesome for using his model in this context. Another possible structural model of the coalition formation process is a random token model. Here, each pair of firms would receive a token randomly, and would merge if the firms in the pair both wanted to; the modeler can again use a Markov-Perfect solution concept. Although this model is also appealing, there may not be a well-defined end to this process. In addition, as I will show in Section 4, in order to be able to prove the existence of and compute the dynamic equilibria for these models, I need for the reduced-form before-merger values V^{BM} of the firms to be continuous in the inputted post-merger V^{PM} values. Without some type of randomness neither of these models will satisfy this condition. But, adding this randomness destroys the easy solution that the Markov-Perfect solution concept yields for the types of games used above, as these games then do not have a definite stopping time. This suggests the need for a well-defined end to the process.

Because of the problems in using one of the previous coalition formation processes for my model, I develop my own process. For my coalition formation process, I use a structural approach and split the problem into three parts. First, I impose structure on the model, and define an acquisition process where firms can make offers to buy smaller firms, in descending order by size. This ordering is similar to the process used by Bloch, but less general. Second, within each offer set, I use bidding processes to determine which merger occurs. As discussed in Chapter 1, I present results for two different bidding processes, an auction and a take-it-or-leave-it (tioli) process. Third, I add randomness to the model, in order to ensure existence of the dynamic equilibrium. In my model, recall that there is some ordering of firms (generally by capacity level). I now formally define the overall merger process in any period.

Assumption regarding the structure of a merged firm:
When firms $(w_1,\ldots,w_N;i)$ and $(w_1,\ldots,w_N;j)$ merge, the resulting state, before reordering, is $(w_1,\ldots,w_{i-1},w_i+w_j,w_{i+1},\ldots,w_{j-1},0,w_{j+1},\ldots,w_N;i)$. Immediately after the merger occurs, the merged firm pays/receives a one-time cost/synergy s, which is drawn from some uniform distribution $U(S^{MIN},S^{MAX})$.

The assumption states that when two firms merge, they combine their capacities, but have to pay/receive some cost/synergy. Because firms are differentiated by their level of capacity, there is a natural conception of a merged firm, which is that it has the combined capacity levels of the two individual firms. It is because of this conception that I chose to examine capacity-based models. For any state $(w_1,\ldots,w_N;k), k \neq j$, one can define the state resulting from the merger as (w',k'); I use this notation later. Note that if the kth element of w' before resorting is the same as some other element, there is a potential ambiguity as to what k' should be. I resolve this ambiguity by allowing the sorting to be done without displacing the relative position of like elements. I now define the overall merger process.

Definition: Consider any state $\boxed{w = (w_1, \ldots, w_N)}$ with R firms active. For this state, I define the merger process $MM(w, i, M)$, where $i \in \{1, \ldots, R-1\}$, $M = \{i+1, \ldots, R\}$, to be a process whereby w_i 'the potential buyer' can acquire any one firm w_j 'potential sellers' such that $j \in M$. As there is one cost/synergy variable for each potential seller, in each MM game there are $o(M)$ different cost/synergy variables, where $o(\cdot)$ refers to the order or number of elements in a set. I label these synergy draws $s_{i+1}, \ldots, s_R$ and assume that each of them is an independent draw from the above distribution and that the values of these draws for any merger game MM are known to the potential buying firm at the start of this merger game MM.

From the definition of the merger game MM, one can see that the potential buying firm can buy any one firm that is smaller than it. Thus, it is not necessary to list M as one of the parameters of $MM(w, i, M)$, since $M = \{i+1, \ldots, R\}$. However, I list M in order to explicitly state the set of potential selling firms. In addition, as I discuss later, the outcome of the MM merger game (in terms of which merger occurs and what price is paid for the merger) will be random and will depend on whether I use a tioli or auction process for the merger.

I now define the full merger game, illustrating when firms are sellers and when they are buyers. I am treating the merger process MM as a black box for the time being. Figure 1 of Chapter 1 provides a graphical representation of the merger game for the case with three firms active with capacities A, B and C, where $A \geq B \geq C$ and B + C > A.

Assumptions regarding the merger game: For any state w, with R firms active, the merger game proceeds as follows:

(a) The first firm can buy any other firms. Thus, the first merger process to occur is $MM(w, 1, \{2, \ldots, R\})$. The synergy draws between w_1 and every other firm are unknown until the start of $MM(w, 1, \{2, \ldots, R\})$; at this point, these synergy draws are all known to the potential buying firm.

(b) If no firm was bought, then the next biggest firm can buy any firm that is smaller then it. Thus, the next merger game would be $MM(w, 2, \{3, \ldots, R\})$. At the start of this MM, the synergy draws between w_2 and all smaller firms are known. This process will repeat itself with the buying firm being 3, 4, …, R - 1, unless some merger occurs. If no merger occurs, the merger process will be finished.

(c) If some firm w_j was bought then the merger process starts all over again, with the new biggest firm being the buyer. Thus, the merger game is now $MM(w',1,\{2,\ldots,R-1\})$, as there is one less firm active, and I then repeat (b) for the new MM game. At the start of this MM, the synergy draws between w_1' and every other firm are known to the potential buying firm.

One important property about this merger process is that, given any starting state, the industry can end the merger process with any arbitrary partition of the original firms into merged firms, provided that this is the merger chosen by the firms at appropriate MM games. The intuitive reason for this property is that firms are given a chance to merge with any other firm, in order. I prove this property below.

<u>Proposition 2.1:</u> Given any state w, if I can assign any outcome that I want to each MM merger game, then it is possible to achieve any arbitrary partition of the firms into merged firms.

<u>Sketch of Proof:</u> Consider the state w as a set $W = \{w_1,\ldots,w_N\}$. Then a merger partition of w can be written as a collection $\{v^i\}_{1\le i\le I}$, where $v^i \subseteq W$, and each v^i represents a merged firm. Then, I can construct $\{v^i\}$ as follows: find the largest firm that has merged (i.e. find w_k such that the v^i that satisfies that $w_k \in v^i$ has more than one element, and for any other w_l that satisfies this property, $l > k$. Then, define the outcome of MM for this state to be the merger between w_k and the next largest element of v^i with probability one; define the outcomes of all previous MM games to be no merger with probability one. Now, reconsider the problem with v^i redefined to have one less element, with one element representing the newly merged firm. Repeat the process of finding the new largest firm which is involved in a

merger and allowing that merger to occur. As I am always allowing the merger with the largest firm to occur and the merger process ended, all the mergers must have occurred: if any one did not transpire, it would be the largest firm that was involved in a merger at some point in this process, and so this merger must have occurred at that point. As this is a contradiction, any feasible partition of existing firms into merged firms can be achieved given the process specified.■

All of the merger models that I have used incorporate the basic structure above. The differences between the models are in terms of the underlying structure of the MM games. Within this framework, I have computed two different mechanisms for the MM game: a take-it-or-leave-it mechanisms and an auction mechanism. In Sections 2.1 and 3.1, I discuss the tioli model, and in Sections 2.2 and 3.2, I discuss the auction model. In addition, recall that in Chapter 1, I discussed quantitative results for both of these models, and discussed how these change depending on the size of the synergies, and the mechanism and ordering of the firms.

Before examining the auction and tioli structures separately, I detail some of the notation needed to examine these structures. There are two types of variables that I define: value functions (parts (a) - (c) below) and reservation prices (parts (d) - (f) below). Value functions represent the EDV of future profits at different stages, while reservation prices represent the option value from a particular action. Thus, in general, reservation prices can be expressed as weighted sums of different value function elements.

<u>Definition:</u>

(a) Let $V^{BM}(w,i;n)$ be the EDV of firm w_n's net future profits when $MM(w,i,M)$ is the merger game that is about to begin play; these values are evaluated before the synergy draws of the players are revealed to them.

(b) Let $V^{PM}(w,n)$ be the EDV of firm w_n's net future profits when the merger process has just finished.

(c) Let $V(w,n)$ be the EDV of firm w_n's net future profits at the start of the period (i.e. before the merger process has begun); again these values are evaluated before the synergy draws of the players are revealed to them.

(d) Let $C(w,i,j), j \in \{i+1,\ldots,R\}$ be the combined EDV of net future profits to firms w_i and w_j not counting their cost/synergy payment, if they choose to merge, when $MM(w,i,M)$ is the merger game that is being played.

(e) Let $B(w,i)$ be the EDV of net future profits to firm w_i if firm w_i does not acquire any firm, when $MM(w,i,M)$ is the merger game that is being played.

(f) Let $S(w,i,j,k), j \in \{i+1,\ldots,R\}, j \neq k \neq i$ be the EDV of net future profits to firm w_k if firms w_i and w_j decide to merge, when $MM(w,i,M)$ is the merger game that is being played. This notation can also be used with j = 0, in which case, $S(w,i,0,k)$ refers to the value from no merger occurring.

In Section 4, I enumerate the values for V^{BM} and V^{PM} and tie them in with the overall value function. For now, the reader may think of V^{PM} as being fixed values, and V^{BM} as being derived from these. Thus, the reader may think of the V^{PM} values used here as being analogous to the reduced-form static profits from some Cournot game. In addition, note that, by definition of the merger process,

$$(2.1)\ V(w,n) = V^{BM}(w,1;n);$$

this equality is used when evaluating the V value function in Section 4.

I now explicitly define the manner in which reservation prices can be expressed as weighted sums of value function elements.

<u>Lemma 2.1:</u> Let (w',k') be the state the results from a merger between i and j given that the starting state is (w,k). Then, the reservation prices defined above satisfy:

$$C(w,i,j) = \begin{cases} V^{BM}(w',1;i'), \text{ if } R > 2 \\ V^{PM}(w',1), \text{ if } R = 2 \end{cases}$$

$$B(w,i) = \begin{cases} V^{BM}(w,i+1;i), \text{ if } i < R-1 \\ V^{PM}(w,i), \text{ if } i = R-1 \end{cases} \tag{2.2}$$

$$S(w,i,j,k) = \begin{cases} V^{BM}(w',1;k'), \text{ if } j \neq 0 \\ V^{BM}(w,i+1;k), \text{ if } j = 0 \text{ and } i < R-1 \\ V^{PM}(w,k), \text{ if } j = 0 \text{ and } i = R-1 \end{cases}$$

Proof: Given the assumptions regarding the merger process, if a merger between w_i and w_j occurs and there are more than two firms in the industry, the merger process MM that is about to begin play is that with the new largest firm as the potential buyer and all the smaller firms as the potential sellers. If there are only two firms in the industry, the merger process will be over. Thus, by the assumption regarding the merger process and because V^{BM} and V^{PM} are defined as being the EDV of profits in these cases, the expression for the combined value C is correct. For the same reasons, the expressions for B and S are correct as well.■

I will now discuss both of the structural models for the merger game $MM(w,i,M)$ separately.

2.1 A Take-it-or-Leave-it Coalition Formation Process

The simplest possible model for the merger game $MM^{tioli}(w,i,M)$ is to have w_i pick one firm in M and make a take-it-or-leave-it (tioli) offer to that firm. The specific details of the MM^{tioli} game are given below:

Assumptions regarding the MM^{tioli} Game:
(a) The synergy draws $s_{i+1},\ldots,s_R$ are announced to w_i, the potential buyer.
(b) Firm w_i then chooses any one firm $w_{i+1},\ldots,w_R$ and price p and offers to buy w_j for p.
(c) Firm w_j then decides whether or not to accept the offer from w_i. If it accepts the offer, w_i receives/pays the cost/synergy draw and acquires w_j for price p. Either way, the merger game $MM^{tioli}(w,i,M)$ is over.

Because of the simple nature of the take-it-or-leave-it model, it is possible to determine its effects quite easily. In the remainder of this subsection, I define the equilibrium strategies of the players, prove that there exists a unique equilibrium to this merger game conditional on V^{PM} values, and prove that the process satisfies the continuity properties necessary to ensure the existence of a dynamic equilibrium for the overall model.

Proposition 2.2:
The Bayesian Perfect Equilibrium solution to the $MM^{tioli}(w,i,M)$ merger is as follows:
Let

$$(2.3)\quad j' = \underset{j\in\{i+1,\ldots,R\}}{\arg\max}\left\{C(w,i,j)-S(w,i,0,j)+s_j\right\}$$

If $C(w,i,j')-S(w,i,0,j')+s_{j'} \geq B(w,i,j')$ then w_i will make an offer of $S(w,i,0,j')$ to j' which will be accepted.
If $C(w,i,j')-S(w,i,0,j')+s_{j'} < B(w,i,j')$ then w_i will not make an offer that is accepted, so no merger will happen.

Proof: Working backwards, for any firm w_j (in the potential selling set) that has received an offer, if it turns down an offer, then no other merger will occur in the $MM^{tioli}(w,i,M)$ merger process. By definition of S then, if w_j turns down an offer, it will receive a payoff of $S(w,i,0,j)$. Firm w_i, then, must offer at least $S(w,i,0,j)$ to any firm w_j to get it to sell. By the tioli offer scheme, it is never optimal to offer more. As the total value of the merged combination of w_i and w_j is $C(w,i,j)+s_j$, the net value of the merger to w_i, after paying for w_j, is $C(w,i,j) - S(w,i,0,j) + s_j$. As the value of not acquiring a firm is $B(w,i)$, w_i will make an acceptable offer if and only if the conditions stated in the Proposition hold.■

Corollary 2.1:

(a) With probability one, there will be a unique equilibrium payoff for each player from this merger game $MM^{tioli}(w,i,M)$.

(b) The structure of knowledge of the current synergies is unimportant: so long as w_i knows all the current synergies, the same equilibria will hold.

Proof:

(a) Because the synergies are iid draws from a uniform distribution, the probability that two different firms will satisfy the condition of j' is zero. Thus, with probability one there will be a unique reduced form payoff to this game.

(b) If additional parties know the current synergies, this would not change the reaction function of any firm w_j in the potential selling set, as the different values of the synergy do not affect w_j's payoff from that point. As this knowledge does not affect w_j's decision, it will not affect w_i's offer structure.■

Proposition 2.3:

Given post-merger values V^{PM} for every state, there exists a unique pre-merger V value for each state, where the merger game being played at each stage is $MM^{tioli}(w,i,M)$ and where the solution that I am using is the Bayesian Perfect Equilibrium.

Proof: I can prove this by showing that there exists a unique V^{BM} value conditional on the V^{PM} value. Then, V values are just particular elements of V^{BM} values, so it will be true for them as well. To prove this property for V^{BM} values, I use double induction on the state space, in the number of active firms R and within that, on r, the number of firms in the set M. The order of the first few elements in the induction are: R = 2 and r = 1; R = 3 and r = 1; R = 3 and r = 2; R = 4 and r = 1; R = 4 and r = 2; R = 4 and r = 3; R = 5 and r = 1; etc.

First, for any state w in which R = 2 and r = 1, for the buyer:

$$V^{BM}(w,1;1) = \int \{\text{Merger between 1 and 2 occurs} | s_2\}\left(V^{PM}(w_1 + w_2, 1) + s_2\right) + \{\text{No merger}\}V(w,1)\,dP(s_{i+1},\ldots,s_R), \tag{2.4}$$

while for the seller,

$$V^{BM}(w,1;2) = \int \{\text{Merger between 1 and 2 occurs} | s_2\}V^{PM}(w_1 + w_2, 2) + \{\text{No merger}\}V(w,1)\,dP(s_{i+1},\ldots,s_R). \tag{2.5}$$

Now by the Corollary, the probability (over the synergy space) that both having the merger and not having the merger will form a Bayesian Perfect Equilibrium is zero. So, for R = 2 and r = 1, V^{BM} are uniquely defined in equilibrium.

Now, assume that there is a unique equilibrium value $V^{BM}(w,i;n)$ for all states with less than R firms in, and for all states with R firms active but less than r firms in the potential selling set M. Then, for any state (w,n) and merger process $MM^{tioli}(w,i,M)$ with R firms active and r firms in the potential selling set, the pre-merger value can be written as:

$$V^{BM}(w,i;n) = \int \sum_{j=i+1}^{R} \Big[\{\text{Merger between i and j occurs} | s_{i+1},\ldots,s_R\} \left(\text{Future value from merger} + \{i = n\}s_j\right) + \{\text{No merger}\}(\text{Value from no merger})\Big]dP(s_{i+1},\ldots,s_R) \tag{2.6}$$.[3]

As future values from merging depend on games with less than R firms active and future values from not merging depend on games with less than r firms in the potential selling set M, each of these values is well-defined by the inductive

[3] I define the exact values to each of the possible players (buyer, seller and non-participant) in Section 4, when discussing computation. For now, it is sufficient to know that these values are well-defined for each of these cases.

hypothesis. Again, by the Corollary, the probability (over the synergy space) that any two mergers will form a Bayesian Perfect Equilibrium is zero. So, elements of V^{BM} for this R and r are uniquely defined in equilibrium. By induction, all elements are uniquely defined in equilibrium. As shown in (2.1), V is just equal to particular elements of V^{BM}, and so it too is uniquely defined.■

The following corollary is needed in Section 4, to ensure existence of the dynamic equilibrium of the model.

Corollary 2.2: The values resulting from the equilibrium strategies of the merger game $\mathrm{MM}^{\mathrm{tioli}}(\mathrm{w},\mathrm{i},\mathrm{M})$ are continuous in the reservation prices C, B, and S.

Proof: By Proposition 2.2, a small change in one of the reservation prices C, B, or S will only change which acquisition the buying firm decides on for a small measured space of the cost/synergy draw distribution. Additionally, a change in the reservation price $\mathrm{S}(\mathrm{w},\mathrm{i},0,\mathrm{j})$ will cause the buying firm to change its bid to its chosen partner to match the new value of $\mathrm{S}(\mathrm{w},\mathrm{i},0,\mathrm{j})$. Thus, the merger that occurs, the prices that are paid, and the values to non-participants will only change a small amount.■

2.2 An Auction Coalition Formation Process

For reasons discussed in Chapter 1, I developed the auction model as an alternative to the tioli model, in order to address some of its deficiencies. In general, the easiest auction model for game theorists to solve is the second-price sealed-bid auction. With a standard symmetric private independent valuations framework it is a dominant strategy to bid one's true valuation for this auction mechanism. Unfortunately, the MM model does not conform to the symmetric private independent valuations framework. In particular, there is a public component to the valuations (namely the C, B and S prices) and some private component (namely the synergies). Thus, even though the valuations might have overlapping distributions, they would not necessarily be symmetric. While the non-symmetry might lead to some obstacles in computing equilibria (because of the difficulty in computing one's true valuation), even more daunting for computation is the fact that the valuations are not independent, because of the externality mentioned above. In the standard model, a firm's reservation price from not bidding does not depend on other firms' bids, because given that the firm does not win, it does not matter to the firm who wins. In this model, a firm's reservation price depends on other bids, because of the externality. If the reservation value for a firm depended on other firms' bids but not on its own bid, then conditional on other firms' bids, it can be shown that it would still be optimal to bid one's true valuation. This is the basis behind the result shown in Milgrom and Weber (1982) where with correlations of tastes, bidding one's valuation is still optimal in a second-price sealed-bid auction. In this case, optimal reaction functions for the second-price auction could be found quite easily, though finding the equilibrium bids would perhaps be more difficult.

Unfortunately, in the merger model, though, a firm's reservation price depends not only on other firms' bids, but also on its own bid. This is because, in this model, for different bids, the distribution of the bid for the winning firm, conditional on the firm that is being considered not winning, will be different. Thus, suppose that the algorithm is able to find a 'true valuation' for a firm, i.e. a bid where the conditional distribution of values if the firm loses generates a reservation price that leads to this bid as the true valuation. Then, in this model, if the firm bids differently from this supposed 'true valuation', that will change its reservation price and through that its 'true valuation'. As changing one's bid will change one's marginal reservation price, it is no longer a dominant strategy to bid one's true valuation.

To illustrate this algebraically, note that in the auction model for the merger process, the valuation to any firm from bidding β_j given that other firms are bidding β_{-j} is:

$$\begin{aligned} &V(\beta_j) = \Pr(\text{Win} \mid \beta_j, \beta_{-j}) \cdot \\ (2.7)\quad &(\text{Value of good} - E[\text{Second highest bid} \mid \beta_j \text{ being highest bid}, \beta_j, \beta_{-j}]) \cdot \\ &+ \sum_{k \neq j} \Pr(k \text{ wins} \mid \beta_j, \beta_{-j})(\text{Value if k wins}) \end{aligned}$$

In the private independent valuation model, or even in the model of Milgrom and Robert's, where there are externalities but one's reservation prices do not depend on one's own bid, the expression in (2.7) is still valid, but it simplifies, as the value if some $k \neq j$ wins is the same across k. Thus, in these models, the valuation is:

$$\begin{aligned} &V(\beta_j) = \Pr(\text{Win} \mid \beta_j, \beta_{-j}) \cdot \\ (2.8)\quad &(\text{Value of good} - E[\text{Second highest bid} \mid \beta_j \text{ being highest bid}, \beta_j, \beta_{-j}]) \\ &+ \Pr(j \text{ does not win} \mid \beta_j, \beta_{-j})(\text{Reservation price}) \end{aligned}$$

As there are several extra terms in (2.7) the analytical solution for the dominant strategy equilibrium that applies in (2.8), of every firm bidding its true valuation, does not hold.

Because of the externalities, the second-price auction is no longer easier to solve than the first-price auction, for the coalition formation game. For this model, the only way to solve the optimal policies for either auction is with numerical methods. Using a gradient search method with the second price auction, one must keep track not only of the derivative of the winning bid with respect to the bid price, but also of the derivative of the second highest bid. Thus, a second price auction is actually harder to compute than a first price auction for this model. In addition, there are two other reasons why a second price auction is not appropriate for this model. First, there are often only two firms (one buyer and one seller) and a second-price auction does not give appropriate answers unless there are more than two parties, as the trading price would be zero always without another party. Second, in this model, there are multiple sellers and one buyer, which is the reverse of normal auctions. Thus, the price that each seller is stating is the price that it is willing to sell itself for. However, different sellers may be worth different amounts to the buyer; for instance a firm with a large amount of capacity would probably be worth more than one with a small amount. Thus, in the context of a second-price auction, one may want to look at the difference between the price and the value (in some sense) rather than just the price. As this may not be possible to do, the second-price auction mechanism may not make any sense for this model.

In spite of the difficulties in using a second-price auction, using a first-price sealed-bid auction is relatively straightforward. Here, there is no necessity to have more than one bidder for the results to make sense. In addition, there is no need to compare across bids in order to determine the winning bid and amount to be paid to the seller; the buyer can just choose the bid that it wants. In spite of the simplicity of the first-price auction relative to the second-price auction, it is still necessary to appropriately specify the information for the synergy draws with the first-price auction. There are several possible assumptions that can be made regarding the information structure: one can assume that the synergy draws are known to both the potential buyer and seller prior to the merger, to either party, or to neither. If the draws are not known to the potential buying firm before the merger decision is made, then the merger decisions will not be stochastic, and the reaction functions will not be smooth in the post-merger value functions. If the draws are known to both parties, then computation becomes very difficult. The reason for this is that although the equilibrium actions may be computable conditional on a realization of synergy draws, the algorithm would potentially have to store actions for the selling firms for each realization of the synergy draws, which would be impossible. However, if the synergy draws are known only to the potential buying firm, then the selling firm chooses only one bid, but the reaction functions will be smooth in the post-merger values, because the buying firm will choose different mergers depending on the draws. Accordingly, I choose this information structure for the auction merger model. The specific details of the $MM^{auction}$ game are detailed below:

Assumptions regarding the $MM^{auction}$ Game:

(a) The synergy draws $s_{i+1},\dots,s_R$ are announced to w_i, the potential buyer.
(b) Every firm $w_{i+1},\dots,w_R$ then simultaneously announces prices $\beta_{i+1},\dots,\beta_R$ which they would be willing to sell themselves for.
(c) Firm w_i then decides which one, if any, of the offers to pick. If it accepts an offer, w_i receives/pays the cost/synergy draw and acquires w_j for price β_j. Either way, the merger game $MM^{auction}(w,i,M)$ is over.

In general, the solution concept used to solve first-price auctions has been to express the first-order condition for the firm's maximization problem as a differential equation and then to solve this differential equation, parametrically.[4]

[4] See, for instance, Fudenberg and Tirole (1992), Chapter 6.

Again because of the externality, standard differential equation techniques do not work - in fact, the valuations, as defined in (2.9) are not even differentiable everywhere. However, it is possible to evaluate first-order conditions and solve the model numerically; this is the technique that I adopt. In the remainder of this subsection, I enumerate the optimal strategies, given some equilibrium vector of bids. Because the bids are made simultaneously, this is not a sufficient characterization of the solution; I must instead discuss the properties of an equilibrium vector of bids. Accordingly, I provide conditions under which reaction functions exist and are continuous in competitors' valuations, and discuss when this would ensure existence of an equilibrium vector of bids. I also provide counterexamples showing why an equilibrium might not exist or even if one exists, why it might not be unique. Finally, I provide sufficient conditions for the model to satisfy the continuity properties necessary to ensure the existence of a dynamic equilibrium for the overall model.

Proposition 2.4: The Bayesian Perfect Equilibrium solution to the $\mathrm{MM}^{\mathrm{auction}}(\mathrm{w},\mathrm{i},\mathrm{M})$ merger game is as follows:
(a) For the buyer, given a vector of bids $(\beta_{i+1},\ldots,\beta_R)$ submitted by the sellers, let

$$\text{(2.9)}\quad j' = \underset{j=i+1,\ldots,R}{\arg\max}\left\{C(w,i,j) - \beta_j + s_j\right\}$$

Then, if $C(w,i,j') - \beta_{j'} + s_{j'} \geq B(w,i,j')$ then w_i will accept the offer of firm $w_{j'}$.
If $C(w,i,j') - \beta_{j'} + s_{j'} < B(w,i,j')$ then w_i will not accept any offer.

Before stating when any bid will be accepted, I need to introduce some notation.

Definition:

(a) Let the normalized bid $\alpha_1 = C(w,i,1) - \beta_1 - B(w,i)$.
(b) Let the value-bid function W_l be the potential seller's value from bidding some bid, given that the potential buyer plays its reduced-form equilibrium strategy, as in (2.9). Because the normalized bid is just a one-to-one mapping of the actual bid, I

can redefine strategies to be in terms of the normalized bids. Accordingly, W_l can be written as:

$$
\begin{aligned}
&W_l\left(\alpha_l \middle| \alpha_{-l}\right) = \\
(2.9)\quad &\sum_{j\neq l=i+1}^{R} \Pr\left\{\alpha_j \geq \max\left\{0, \max_{k\neq j\in\{i+1,\ldots,R\}}\{\alpha_k + s_k\}\right\}\right\} \cdot S(w,i,j,l) \\
&+\Pr\left\{\alpha_l + s_l \geq \max\left\{0, \max_{k\neq j\in\{i+1,\ldots,R\}}\{\alpha_k + s_k\}\right\}\right\} \cdot \left(C(w,i,l) - \alpha_l - B(w,i)\right) \\
&+\Pr\left\{\max_{k\in\{i+1,\ldots,R\}}\{\alpha_k\} < 0\right\}\left(S(w,i,0,l)\right)
\end{aligned}
$$

Now, I proceed to part (b) of the Proposition.

Proposition 2.4: The Bayesian Perfect Equilibrium solution to the $MM^{auction}(w,i,M)$ merger game is as follows:

(b) For the sellers, the vector of bids $(\overline{\alpha}_{i+1},\ldots,\overline{\alpha}_N)$ submitted will satisfy:

$$(2.10)\quad \overline{\alpha}_j = \arg\max_{\alpha}\left\{W_j\left(\alpha \middle| \overline{\alpha}_{-j}\right)\right\},$$

given the definitions in (2.9).

Proof: I work backwards from the second stage. At the second stage, the buyer must pay β_j in order to acquire firm w_j. By the same logic as shown in Proposition 2.2 for the tioli game, the buyer's strategies will be as stated above. For the sellers, the strategies are defined in terms of the bids. By backward induction, the winning probabilities used will be correct, and by definition of S, I am using the correct value for the seller if another firm merges with w_i. Thus, (2.9) is the true valuation from any bid. Finally, (2.10) is just the definition of Bayesian Perfect Equilibrium, given that the valuations are correct.■

Now that I have enumerated the conditions necessary for strategies to form an equilibrium, the next step is to try to show that an equilibrium exists for this model. My general strategy of proof is to solve for the buyer's strategy, conditional on bids and synergies, and treat this in its reduced form. Using this reduced form, the model becomes a simple simultaneous game. A Nash Equilibrium of this simple game, using the correct reduced-form buyer's choices, will be a Bayesian Perfect Equilibrium of the auction game. The problem with this strategy of proof is that an equilibrium to the model does not always exist, because, with the externalities the optimal bid correspondence is not always convex-valued. Accordingly, I provide a counterexample that illustrates than an equilibrium might not exist and sufficient conditions for an equilibrium to exist.

To show existence, the first step is to show that a firm's value function is continuous in the firm's bid, conditional on the other bids.

Lemma 2.2: The value-bid function W_l is continuous in α_l.

Proof: All of the probabilities in (2.11) are evaluated over uniform distributions. As I will show in Section 3, each of the expressions on the left side of the probabilities are independent random variables. Increasing a bid by a small amount will change the distribution of the random variable only by a small amount, because of the properties of uniform random variables. Hence, the probabilities are continuous in α_l. The expression is composed of constants multiplied by these probabilities, and hence is also continuous in α_l.

I would like to be able to prove existence for the model, by proving that the model satisfies all the assumptions required to apply Kakutani's Fixed Point Theorem, and by then applying this theorem. However, in general, the model does not satisfy all the assumptions of Kakutani's Fixed Point Theorem. In particular, it is not possible to prove that the optimal bid correspondence is convex-valued. The following counterexample shows that it is not possible to prove the existence of equilibrium for every set of reservation prices.

Example of non-existence of equilibrium:
Suppose that there are three firms, i, X and Y, and i is the potential buying firm and X and Y are the potential selling firms. Suppose that there is a large negative externality to X if Y merges, and a large positive externality to X if no one merges, while to Y there is a large positive externality if X merges and a large negative externality if no one merges. Accordingly, let the seller's reservation prices be:

$$S(w,i,0,X)=1000;\ S(w,i,0,Y)=-1000;$$

$$S(w,i,Y,X)=-1000;\ S(w,i,Y,X)=1000.$$

Let the buyer's mean value be 0, i.e. let

$$C(w,i,X)-B(w,i)=C(w,i,Y)-B(w,i)=0.$$

Finally, let the synergies be distributed $U(-0.5,0.5)$.

Lemma 2.3: For this example, there is no equilibrium.

Proof: Consider the following:

Claim: In any equilibrium, it is not possible for a normalized bid range for either X or Y (defined by $\left[\alpha_j+S^{MIN},\alpha_j+S^{MAX}\right]$) to always be negative.

Reason: If X's normalized bid range is always negative and Y's normalized bid range is not always positive, Y will find it optimal to switch to an always positive bid range, so this is not an equilibrium. If X's normalized bid range is always negative and Y's normalized bid range is always positive, X will find it optimal to switch to an always positive bid range. Now, if Y's normalized bid range is always negative and X's normalized bid range is not always negative, X will find it optimal to switch to an always negative bid range. But, if Y's normalized bid range is always negative and

X's normalized bid range is always negative, Y will find it optimal to switch to an always positive bid range. Thus, none of the possible cases are equilibria.

<u>Claim:</u> In any equilibrium, it is not possible for a normalized bid range for either X or Y to always be positive.

<u>Reason:</u> Suppose X's normalized bid range is always positive. Then, Y will find it optimal to have its highest normalized bid no greater than X's lowest normalized bid (i.e. Y will set α_Y such that $\alpha_Y + S^{MAX} \leq \alpha_X + S^{MIN}$), thus any possible equilibria must be of this form. But, if X's lowest normalized bid is higher than Y's highest normalized bid, then X will find it optimal to lower its bid range to overlap with Y's at least a little: if X's bid is $\left[\alpha_Y + S^{MAX}, \alpha_Y + S^{MAX} + S^{MAX} - S^{MIN}\right]$, then by lowering its normalized bid by some small ε, it loses with probability ε^2 (and receives -1000 in this case) but gains ε with probability almost one from its higher real bid. Thus, if X's lowest normalized bid is higher than Y's highest normalized bid, this is not an equilibrium either. Now suppose Y's normalized bid range is always positive. Then, if X's normalized bid range is strictly positive and not completely higher than Y's bid range, Y will lower its bid, or if X's normalized bid range is strictly positive but completely higher than Y's normalized bid range, then it is optimal for X to lower its bid range, as discussed earlier. Thus, X's normalized bid range is not strictly positive, in which case, Y's normalized bid range must be no higher than touching X's. In this case, X will gain by increasing its bid range so it is the same as Y's.

The only possible remaining equilibrium is for both X and Y to have normalized bid ranges that are overlapping 0. Now, it must be the case that X, Y and 0 each have very close to one-third probability of winning, or else either X or Y is earning much less than 0, and it will increase its normalized bid so it will always win, and achieve a payoff very close to 0. Thus, the normalized bid ranges for both X and Y must be approximately $\left[-1/\sqrt{3}, 1-1/\sqrt{3}\right]$. But, if these are the normalized bid ranges, then if Y chooses to increase its normalized bid by ε, it will increase its probability of winning by ε, and decrease the probability of no one winning by $1/\sqrt{3}$ and decrease the probability of X winning by $1-1/\sqrt{3}$. Thus, Y will stand to gain $\left(1/\sqrt{3}\cdot 1000-\left(1-1/\sqrt{3}\right)\cdot 1000-1\right)\varepsilon > 0$ from a small ε increase in its normalized bid range, and so the proposed equilibrium is not optimal. (In order to have satisfied the criterion that a marginal change by either party does not increase its payoff, the normalized bid ranges would have to be approximately $\left[-1/2, 1-1/2\right]$.) Since it was the only possible equilibrium, there is no equilibrium for this model.■

While the above counterexample shows that an equilibrium does not always exists, I can prove that an equilibrium exists if the size of the seller's externality is sufficiently small. The reason for this result is that as the size of the externality becomes smaller, the model approaches a standard auction model without externalities.

<u>Proposition 2.5:</u> Provided that the difference between sellers' reservation prices $\left\|S(w,i,k,j,M)-S(w,i,l,j,M)\right\|$ is sufficiently small, an equilibrium to the $MM^{auction}(w,i,M)$ merger game will exist.

<u>Proof:</u> I would like to apply Kakutani's Fixed Point Theorem. In order to do this, I must show that the optimal bid correspondence is upper semi-continuous, convex-valued, non-empty-valued, and defined on a compact, convex and non-empty set. If

the bid correspondence satisfies all of these conditions, then I can apply Kakutani's Fixed Point Theorem to the vector of optimal value-bid functions W_l. As the model has been reduced to a simple simultaneous game, a fixed point of this function will be an equilibrium of the model.

Because the value/bid function is continuous, and the bid correspondence is defined implicitly from this, a standard theorem establishes that the bid correspondence is upper semi-continuous, and non-empty-valued. It is easy to define the normalized bid function to lie in a compact space. In particular, if some normalized bid has a probability one of winning, then it would never be optimal to bid higher than it. Similarly, if a normalized bid will never win, then decreasing it will not change the value. Thus, every property except for being convex-valued holds trivially.

A sufficient condition for the bid correspondence to be convex-valued is for the value/bid function to be strictly quasi-concave. Now, for a model without externalities (i.e. where it does not matter to the selling firm which firm, if any, is acquired, if it is not itself), then restricting the strategy space so that the lowest possible normalized bid is the highest bid which has no probability of winning, the value/bid function is strictly quasi-concave, since a standard result establishes that there is a unique optimal strategy in that case.

I illustrate the implications of quasi-concavity for the model without externalities as follows: define the mean seller's reservation price to be $\overline{S}(w,i,j) = \sum_{k=0,k\neq j\in M} S(w,i,k,j)/(R-i)$. Now, consider some firm w_j. and assume that instead of receiving a different value S depending on who wins, the firm always receives $\overline{S}(w,i,j)$ if it does not win. Then, the strict quasi-concavity of this model implies that $\forall\alpha_j^1,\alpha_j^2,\forall\lambda\in(0,1)$, if $\alpha_j^3=\lambda\alpha_j^1+(1-\lambda)\alpha_j^2$,

$$(2.11)\quad W_j\left(\alpha_j^3\middle|\alpha_{-j}\right)>\min\left\{W_j\left(\alpha_j^1\middle|\alpha_{-j}\right)W_j\left(\alpha_j^2\middle|\alpha_{-j}\right)\right\}.$$

Thus, for the model without externalities, it follows that:

$$\begin{aligned}
&\alpha_j^3\Pr\left(j\text{ wins}\middle|\alpha_j^3,\alpha_{-j}\right)+\sum_{k=0,k\neq j\in M}\Pr\left(k\text{ wins}\middle|\alpha_j^3,\alpha_{-j}\right)\overline{S}(w,i,j)>\\
(2.12)\quad \min&\left\{\alpha_j^1\Pr\left(j\text{ wins}\middle|\alpha_j^1,\alpha_{-j}\right)+\sum_{k=0,k\neq j\in M}\Pr\left(k\text{ wins}\middle|\alpha_j^1,\alpha_{-j}\right)\overline{S}(w,i,j),\right.\\
&\left.\alpha_j^2\Pr\left(j\text{ wins}\middle|\alpha_j^2,\alpha_{-j}\right)+\sum_{k=0,k\neq j\in M}\Pr\left(k\text{ wins}\middle|\alpha_j^2,\alpha_{-j}\right)\overline{S}(w,i,j)\right\}.
\end{aligned}$$

Since this property holds for all possible normalized bids α_j and these normalized bids lie in a compact set, it follows that there is some $\varepsilon>0$ such that the left side of (2.12) is always at least ε more than the right side.

For the regular model (with externalities), quasiconcavity holds if and only if:

$$\begin{aligned}
&\alpha_j^3\Pr\left(j\text{ wins}\middle|\alpha_j^3,\alpha_{-j}\right)+\sum_{k=0,k\neq j\in\{i+1,\ldots,R\}}\Pr\left(k\text{ wins}\middle|\alpha_j^3,\alpha_{-j}\right)S(w,i,k,j)\overset{?}{\geq}\\
(2.13)\quad \min&\left\{\alpha_j^1\Pr\left(j\text{ wins}\middle|\alpha_j^1,\alpha_{-j}\right)+\sum_{k=0,k\neq j\in\{i+1,\ldots,R\}}\Pr\left(k\text{ wins}\middle|\alpha_j^1,\alpha_{-j}\right)S(w,i,k,j),\right.\\
&\left.\alpha_j^2\Pr\left(j\text{ wins}\middle|\alpha_j^2,\alpha_{-j}\right)+\sum_{k=0,k\neq j\in\{i+1,\ldots,R\}}\Pr\left(k\text{ wins}\middle|\alpha_j^2,\alpha_{-j}\right)S(w,i,k,j)\right\}.
\end{aligned}$$

Differencing (2.12) with (2.13), it follows that a sufficient condition for quasiconcavity for the model with externalities to hold is that

$$\left| \sum_{k=0, k \neq j \in \{i+1,\ldots,R\}} \Pr\left(k \text{ wins} \middle| \alpha_j^3, \alpha_{-j}\right) S(w,i,k,j) - \right.$$

$$(2.14) \qquad \left. \sum_{k=0, k \neq j \in \{i+1,\ldots,R\}} \Pr\left(k \text{ wins} \middle| \alpha_j^3, \alpha_{-j}\right) \overline{S}(w,i,j) \right|$$

$$\leq \varepsilon/2.$$

Thus, provided the externalities are small enough, an equilibrium to the model exists, by Kakutani's Fixed Point Theorem.■

Although I have shown that an equilibrium to this model exists, if the size of the externalities is small enough, this is not a particularly useful result, since the sizes of the externalities are not primitives of the model. In the example given that showed that an equilibrium might not exist, it is not clear if these reservation prices can even be generated by primitives of the model. More useful would be to characterize whether an equilibrium exists depending on the primitives of the model. However, it is not possible to answer this question: because of the complexity of the model, one cannot solve it algebraically, or characterize its solution or lack thereof. Thus, it is possible that an equilibrium exists for all parameters of the model, but it is not possible to prove it. However, in the numerical algorithm, I know if I have reached an equilibrium, so, it is not vital to prove the existence of one in general, because the algorithm proves the existence for specific cases. Furthermore, I can characterize existence depending on the size of the cost/synergy random variable, which is a primitive of the algorithm.

<u>Corollary 2.3:</u> There exists $\hat{S}$ such that if the size of the cost/synergy distribution (i.e. $S^{MAX} - S^{MIN}$) is at least $\hat{S}$, then an equilibrium to the $MM^{auction}(w,i,M)$ merger game will exist.

Proof: As long as bids are bounded, as $\hat{S}$ grows large, the probability of each merger happening will converge to some fixed value that is independent of the bid. But even as $\hat{S}$ grows large, the bids will be bounded, because the reservation values are fixed. Now, let $\overline{S}(w,i,j)$ be defined as above, but with the limit probabilities taken as $\hat{S}$ grows large. Then, the expression in (2.13) will approach the expression in (2.12) as $\hat{S}$ becomes larger. For large enough $\hat{S}$, the difference will be below the $\varepsilon/2$ limit.■

The following corollary is needed in Section 4, to ensure existence of the dynamic equilibrium of the model.

Corollary 2.4:
For the merger game $MM^{auction}(w,i,M)$, the optimal bids and resulting values, taking the other sellers' bids as fixed, are continuous in the reservation prices C, B, and S and in the other sellers' bids, provided that the externalities are small enough to satisfy the conditions of Proposition 2.5.

Proof: As shown in Proposition 2.5, if the externalities are small enough, the value/bid function is strictly quasiconcave and the optimal bids are upper semi-continuous and convex-valued in the other bids. As changing the reservation prices has exactly the same effect as changing other bids, the value-bid function is upper semi-continuous and convex-valued in the reservation prices. These properties together ensure that the optimal bid is a continuous function of the reservation prices and other bids given. In addition, note that by Proposition 2.4, small changes in the bids will only change the probability that the buyer buys any firm over a small measured space of the cost/synergy distribution, and thus they will only change the probability a small amount; the change in the price paid, of course, will be exactly the change in the bid. Because of this, the values to the buyer, sellers and non-participants of the merger resulting from an optimal bid will be continuous in the reservation prices and competitors' bids.■

Finally, it would be useful to prove that given that an equilibrium exists, it has a unique outcome to each firm. However, this is not necessarily the case. Because of the externalities in the model, it might be an equilibrium for two firms not to merge, or for them to merge. This is so because not merging for one firm might yield a much higher value to the non-participant than if the firm merged. Thus, it is possible to have a "tough" equilibrium of low bids and a "nice" equilibrium of high bids. The following example illustrates this fact.

<u>Example of non-uniqueness of equilibrium:</u>

Suppose that there are three firms, i, X and Y, and that i is the potential buying firms and that X and Y are the potential selling firms and are identical. Suppose that there is a negative externality to a firm if the other firm merges. Accordingly, let the seller's reservation prices be:

$$S(w,i,0,X) = 10; S(w,i,0,Y) = 10;$$
$$S(w,i,Y,X) = 4; S(w,i,Y,X) = 4.$$

Allow the buyer's mean value to be somewhere in between the high and low reservation prices for the sellers. So, let

$$C(w,i,X) - B(w,i) = C(w,i,Y) - B(w,i) = 7.$$

Finally, let the synergies be distributed $U(-0.5, 0.5)$.

For this example, there are two possible equilibrium outcomes. One possible equilibrium outcome (the "nice" one) occurs when the bids are given by $\beta_X = \beta_Y = 8$. Then, even with the highest synergy draw, the net value to the buyer from buying either firm will be no greater than 0, so no merger will happen. By raising its bid, a firm will not change what happens. By lowering its bid, a firm can only be bought with some positive probability and would then receive less than its reservation price of 10, so that would never be optimal. Another possible equilibrium outcome (the "tough" one) occurs when the bids are given by $\beta_X = \beta_Y = 5$. For this strategy profile, the value of acquisition to the buying firm will always be greater than 0, thus the buying firm will always buy the selling firm which has the higher synergy draw. Thus, if a firm chooses to raise its bid by a small $\varepsilon > 0$, it will lower its probability of winning by ε and receive 4 instead of 5, with probability ε more, which translates to a first-order loss of ε. However, it will receive an increased value of ε from its higher bid with probability $1-\varepsilon$, which translates to a first-order gain of ε. Therefore, the first derivative is zero for this bid, which implies that bidding 5 is the optimal bid given that the other firm is bidding 5, and so $\beta_X = \beta_Y = 5$ forms an equilibrium as well.

Section 3: Solving Endogenous Merger Games

I have now characterized two possible structural models for the MM merger game. Given these characterizations, I discuss how to compute the equilibria for these two models. The solution techniques that I describe involve taking post-merger V^{PM} values as given and finding the pre-merger V^{BM} values that would ensue given that firms are choosing equilibrium strategies. Thus, just as in Section 2, the computational solution concepts that I discuss can be applied as mechanisms for solving static computational models of coalition formation, by allowing the V^{PM} values to be the static reduced-form Cournot profits. In addition, in Section 3.2, I discuss the computation of the auction process with externalities, which can be of use to economists who are interested in adding externalities to auction models. In Section 4, I discuss the mechanism used to solve the overall dynamic equilibrium of the model, which is a modified form of dynamic programming.

In order to compute the V^{BM} values from the V^{PM} values, I use backward induction on the MM merger games. The ordering for the backward induction is chosen in order to ensure that values that are currently being computed will depend only on values that have been previously computed. One can determine the dependence of the values by noting that for both the auction and tioli models, every merger game $MM(w,i,M)$ depends for its values on the reservation prices C, B and S. From the definitions of these reservation prices, given in (2.1), it follows that they depend only on post-merger V^{PM} values or pre-merger V^{BM} values for games with a lesser number of active firms or a lesser number of firms in the potential selling set, which is true precisely because the process ensures that merger games that follow the current merger game must have less firms active or less firms in the potential selling set. Let R be the number of active firms and let r be the number of firms in the potential selling set (i.e. for a merger game $MM(w,i,M)$, let $r = R - i \equiv o(M)$). Then, because of this dependence structure I can use backward induction to solve for the V^{BM} values, with the following ordering: first, I solve the V^{BM} values for all MM games with $R = 2$ and $r = 1$, then for $R = 3$ and $r = 1$; then for $R = 3$ and $r = 2$, etc. Given this ordering, the V^{BM} values that I compute at each time will be the true valuation (i.e. the actual EDV of profits), given that the V^{PM} values are correct.

The computer algorithm solves for the optimal decision rules and reduced-form valuations for each state, using the backward induction ordering structure listed above. With this ordering, the reservation prices C, B, and S are weighted sums of value functions elements that have already been solved, because of the order of the backward induction process. Thus, the algorithm computes these values by simply adding and weighting the appropriate value function elements. Apart from the reservation prices, the particular solution mechanism for V^{BM} within each merger games depends on which merger game I am using. Accordingly, I split the rest of this discussion into two subsections, one for the tioli model and one for the auction model.

3.1 Solving the Take-it-or-Leave-it Model

Thus far, I have discussed the dependence structure for the $MM(w,i,M)$ merger games and the computation of the reservation prices C, B and S within each merger game. I would like to compute the solution for the $MM^{tioli}(w,i,M)$ merger game, which can be encapsulated by the V^{BM} value for this game. As the V^{BM} value is the ex-ante payoff from the game, it is the value to firms when the cost/synergy draws are still a random distribution and have not yet been realized. Thus, computing the V^{BM} values involves integrating over the cost/synergy draws joint distribution.

In order to illustrate the derivation of V^{BM}, recall that Proposition 2.2 discusses the structure of the solution that occurs in the $MM^{tioli}(w,i)$ game in terms of these reservation prices. Let us examine the V^{BM} values for the buyer, seller and non-participants in each merger. First, for the potential buyer, define $netgain_j = C(w,i,j) - S(w,i,0,j) - B(w,i)$. Then, using Proposition 2.2 and noting that the synergies are iid, the value function can be written as:

$$
\begin{aligned}
V^{BM}(w,i;i) &= \int \sum_{j=i+1}^{R} \Big[\{\text{Merger between i and j occurs} \,|\, s_{i+1},\ldots,s_R\} \\
&\qquad (\text{Future value from merger}) \\
&\qquad + \{\text{No merger}\}(\text{Value from no merger}) \Big] dP(s_{i+1},\ldots,s_R) \\
&= \int \sum_{j=i+1}^{R} \left[\left\{ \text{netgain}_j + s_j \geq \max\left\{0, \max_{k\neq j\in\{i+1,\ldots,R\}} \{\text{netgain}_k + s_k\}\right\}\right\} \cdot \right. \\
&\qquad (\text{netgain}_j + B(w,i)) \\
&\qquad \left. + \left\{ \max_{k\in\in\{i+1,\ldots,R\}} \{\text{netgain}_k\} < 0 \right\} (B(w,i)) \right] dP(s_{i+1}) \cdots dP(s_R)
\end{aligned} \tag{3.1}
$$

Converting the integrals of expectations into probabilities, I get

$$
\begin{aligned}
V^{BM}(w,i;i) &= \sum_{j=i+1}^{R} \Pr\left\{ \text{netgain}_j + s_j \geq \max\left\{0, \max_{k\neq j\in\{i+1,\ldots,R\}} \{\text{netgain}_k + s_k\}\right\}\right\} \cdot \\
&\left[E\left(\text{netgain}_j + s_j \,\middle|\, \text{netgain}_j + s_j \geq \max\left\{0, \max_{k\neq j\in\{i+1,\ldots,R\}} \{\text{netgain}_k + s_k\}\right\}\right) + B(w,i) \right] \\
&\qquad + \Pr\left\{ \max_{k\in\in\{i+1,\ldots,R\}} \{\text{netgain}_k\} < 0 \right\} (B(w,i))
\end{aligned} \tag{3.2}
$$

Note that netgain_j are all fixed values. Thus, $\text{netgain}_j + s_j$ is a uniform random variable of the same length as s_j, but with different starting and ending points. Furthermore, note that $\text{netgain}_j + s_j$ will be independent, although not identical, across j. The distribution of these random variables $\text{netgain}_j + s_j$ can be represented by horizontal bars over $[\text{netgain}_j + S^{MIN}, \text{netgain}_j + S^{MAX}]$ that are potentially overlapping. As shown in Section 2, the highest one of these $\text{netgain}_j + s_j$, if it is greater than zero, will be the merger chosen. Figure 2 from Chapter 1 represents a particular case of these random variables graphically. In Figure 2 from Chapter 1, any firm and no firm can potentially be bought, with the largest probability event being that firm 4 is bought.

As shown from (3.2), in order to evaluate V^{BM} for the seller, the algorithm requires the probabilities that these uniform random variables $\text{netgain}_j + s_j$ are the highest and greater than zero, and the conditional expectation for each one, given that it is the highest. Because these variables are overlapping and not identical, analytic expressions for these values are very difficult to obtain, but numerical evaluation of these values is possible. I compute these values by separating the probability space into regions, corresponding to the lower and upper limit of each of the random variables $\text{netgain}_j + s_j$ and 0. Then, the algorithm cycles through all of the potential regions for each random variable, in order to calculate these values.

Within each region, I can easily calculate both the probability of winning and the conditional expectation given that a firm wins, because, conditional on being in a region, the variables are identical. Then, the overall probability of winning and conditional expectation are the sum over all the regions of the values within each region. I illustrate the calculation of the conditional expectation; calculating the probability of winning is similar but easier. Using the regions approach, the conditional expectation from (3.2) is equal to:

$$\begin{gathered}
E\left(\text{netgain}_j + s_j \middle| \text{being highest and } > 0\right) = \\
\sum_{\text{regions } r_{i+1}} \cdots \sum_{\text{regions } r_R} \Pr\left(\text{netgain}_{i+1} + s_{i+1} \in \text{region } r_{i+1}\right) \cdots \\
\Pr\left(\text{netgain}_R + s_R \in \text{region } r_R\right) \cdot \\
\Pr\left(\text{netgain}_j + s_j \text{ being highest and } > 0 \middle| \text{netgain}_k \in \text{region } r_k, \forall k\right) \cdot \\
E\left(\text{netgain}_j + s_j \middle| \text{netgain}_j \text{ being highest and } > 0 \text{ and netgain}_k \in \text{region } r_k, \forall k\right)
\end{gathered} \tag{3.3}$$

The problem now has been reduced to finding out the probability of each random variable being at each region, and of finding the conditional probability of winning and the conditional expectation of the value of the winner given each region. The probability of any variable being in a particular region is simply the length of that region, if the random variable is active in a given region. The conditional probability and expectation can take one of three forms, depending on three events: first, if the highest region is less than 0, the conditional probability is 0. Second, if there is one firm in the highest region, the conditional probability is 1, and the conditional expectation is the midpoint of the region. Third, if there is more than one firm in the highest region, then note that conditional on restricting these random variables to regions, they are still independent and now they are also identical. Thus, I can use the standard properties of uniform random variables. In particular, if there are γ random variables in the winning region, then the probability of winning for any one is $1/\gamma$. In addition, defining the boundaries of the region to be $\underline{r}$ and $\bar{r}$, the conditional expectation is just

$$\begin{aligned} &E\left(\text{netgain}_j + s_j \middle| \text{being highest and netgain}_k \in \text{region } r_k, \forall k\right) \\ &= \frac{1}{\gamma}\underline{r} + \frac{\gamma - 1}{\gamma}\bar{r}. \end{aligned} \quad (3.4)$$

With this algorithm for computing the probabilities and conditional expectations of these random variables, I have reduced an infinite expectation problem into a finite one. This allows for easy computation of the buyer's V^{BM} value function.[5]

In the tioli model, the seller's and non participant's problems are similar to the buyer's problem, except that their values depend only on the probability of any

[5] As with all computer algorithms, it is important to check that this algorithm is free of programming errors. For this algorithm, it is easy to do this: the above probability and conditional expectation can be easily approximated by simulation and thus checked against the results from this algorithm.

merger taking place, and not on the synergy conditional on that merger taking place. Recall that if a potential selling firm is acquired by the buying firm, the expected value for this firm is the same as if no merger happens: it is paid $S(w,i,0,1)$. Using this fact, I can write the seller's or non-participant's ex-ante value as:

(3.5)

$$
\begin{aligned}
V^{BM}(w,i;1) = \int \sum_{j=i+1}^{R} \Big[& \{\text{Merger between i and j occurs} \mid s_{i+1},\ldots,s_R\} \\
& (\text{Future value from merger}) \\
& + \{\text{No merger}\}(\text{Value from no merger})\Big] dP(s_{i+1},\ldots,s_R) \\
= \int \sum_{j\neq l=i+1}^{R} \Bigg[& \left\{ \text{netgain}_j + s_j \geq \max\left\{0, \max_{k\in\{i+1,\ldots,R\}} \{\text{netgain}_k + s_k\}\right\}\right\} \cdot S(w,i,j,1) \\
+ & \left\{ \max_{k\in\{i+1,\ldots,R\}} \{\text{netgain}_k\} < 0 \text{ or } \left\{\text{netgain}_l + s_l \geq \max_{k\in\{i+1,\ldots,R\}} \{\text{netgain}_k + s_k\}\right\}\right\}\big(S(w,i,0,1)\big)\Bigg] \cdot \\
& dP(s_{i+1}) \cdots dP(s_R)
\end{aligned}
$$

Again converting the integrals of expectations into probabilities, I get

$$
\begin{aligned}
& V^{BM}(w,i;1) = \\
& \sum_{k\neq l=i+1}^{R} \Pr\left\{ \text{netgain}_j + s_j \geq \max\left\{0, \max_{k\neq j\in\{i+1,\ldots,R\}} \{\text{netgain}_k + s_k\}\right\}\right\} \cdot S(w,i,j,1) \\
& + \Pr\left\{ \max_{k\in M} \{\text{netgain}_k\} < 0 \text{ or } \text{netgain}_j + s_j \geq \max_{k\neq j\in\{i+1,\ldots,R\}} \{\text{netgain}_k + s_k\}\right\} \cdot \\
& \big(S(w,i,0,1)\big)
\end{aligned} \tag{3.6}
$$

In order to compute the V^{BM} values, the algorithm potentially has to store all of the V^{PM} values and the V^{BM} values for states that have already been computed. As the functions C, B, and S are defined in terms of these V^{BM} and V^{PM} values, the algorithm does not have to store them. In addition, note that for any state w, the reservation prices C, B and S (and hence the V^{BM} value) depends only on V^{BM} values for other merger games in w and for games where i = 1 (i.e. where the new largest firm is the potential buying firm). Thus, by computing all the values for a given w at once, it is only necessary to store the V^{BM} values for when i = 1, which is the same as the V values, as shown in (2.1). This saves a great deal of the storage space otherwise required.

3.2 Solving the Auction Model

The computation of the auction model is similar to that of the tioli model, although more complicated because the auction mechanism involves bids, which occur simultaneously. In order to compute an equilibrium for the $MM^{auction}(w, i, M)$ model, the mechanism I use is to fix a bid for each of the potential sellers, and to find out what the buyer would do conditional on these bids. Applying backward induction, this then implies a value to each seller for the vector of bids. I use the optimal strategies for the buyer conditional on the sellers' bids (as defined in Proposition 2.4) in order to examine how changing its bid would affect a seller's value. I then find new optimal bids for each of the sellers conditional on the vector of previous bids. The process, of finding new optimal bids for each seller conditional on the other sellers' bids, can iterate until convergence.

In order to use the above process, the algorithm needs to evaluate the buyer's policies conditional on a vector of bids. Given a vector of bids, the policy for the buyer is very similar to that of the tioli model. The main difference in policies is that here the buyer must pay the seller its bid, rather than the reservation price $S(w, i, 0, j)$. Given some vector of bids $(\beta_{i+1}, \ldots, \beta_N)$, I use again the normalized bid, defined as

$\alpha_j = C(w,i,j) - \beta_j - B(w,i)$.[6] Then, I can write the V^{BM} value for the potential buyer in exactly the same way as in the tioli model:

$$\begin{aligned} &V^{BM}(w,i;i|\alpha_{i+1},\ldots,\alpha_R) = \sum_{j=i+1}^{R} \Pr\left\{\alpha_j + s_j \geq \max\left\{0, \max_{k\neq j\in\{i+1,\ldots,R\}}\{\alpha_k + s_k\}\right\}\right\} \cdot \\ (3.7)\qquad &\left[E\left(\alpha_j + s_j \middle| \alpha_j + s_j \geq \max\left\{0, \max_{k\neq j\in\{i+1,\ldots,R\}}\{\alpha_k + s_k\}\right\}\right) + B(w,i)\right] \\ &+ \Pr\left\{\max_{k\in\{i+1,\ldots,R\}}\{\alpha_k\} < 0\right\}(B(w,i)) \end{aligned}$$

Given the sellers' bids, the choice of the buyer will be the same as in the tioli model. Thus, I can again represent the bids by overlapping uniform random variables, and examine the probabilities of each one being the highest one and greater than 0, using the same methods.

I now examine the potential seller's bid, which is more difficult to compute than the buyer's strategy because the bids are made simultaneously. Recall that, in equilibrium, the seller's bid must maximize its value, conditioning on all the other bids. The seller's value function V^{BM} is just the EDV of future profits, given that the firm is choosing maximizing behavior, conditional on the other firms. Using the definitions of maximizing bids given in (2.11), and using once again the value-bid function W_l, it follows that:

$$\begin{aligned} &V^{BM}(w,i;l) = \\ &\sum_{j\neq l=i+1}^{R} \Pr\left\{\overline{\alpha}_j + s_j \geq \max\left\{0, \max_{k\neq j\in\{i+1,\ldots,R\}}\{\overline{\alpha}_k + s_k\}\right\}\right\} S(w,i,j,l) \\ (3.8)\qquad &+ \Pr\left\{\overline{\alpha}_l + s_l \geq \max\left\{0, \max_{k\neq l\in\{i+1,\ldots,R\}}\{\overline{\alpha}_k + s_k\}\right\}\right\} \cdot \left(C(w,i,l) - \overline{\alpha}_l - B(w,i)\right) \\ &+ \Pr\left\{\max_{k\in\{i+1,\ldots,R\}}\{\overline{\alpha}_k\} < 0\right\}(S(w,i,0,l)). \end{aligned}$$

---

[6] Note that the normalized bid is similar to netgain, as defined for the tioli model, in that it enumerates the likelihood of any bid being successful.

where $(\overline{\alpha}_{i+1},\ldots,\overline{\alpha}_R)$ satisfy the equilibrium condition

$$\overline{\alpha}_j = \max_{\alpha_j}\left\{W_j\left(\alpha_j|\alpha_{-j}\right)\right\}\forall j = \{i+1,\ldots,R\}.$$

The expressions in (3.8) describe the equations that the sellers' bids must satisfy, but they do not describe how to compute them, because of the simultaneous nature of the bids; I discuss the computation shortly.

First, I note that the valuation for the non-participant is similar to that of the seller, except that there is no special value if the non-participant wins. The value to the non-participant is:

$$\text{(3.9)}\quad \begin{aligned} V^{BM}(w,i;1) = &\sum_{j=i+1}^{R} \Pr\left\{\overline{\alpha}_j + s_j \geq \max\left\{0, \max_{k\neq j\in\{i+1,\ldots,R\}}\{\overline{\alpha}_k + s_k\}\right\}\right\} S(w,i,j,1) \\ &+ \Pr\left\{\max_{k\in\{i+1,\ldots,R\}}\{\overline{\alpha}_k\} < 0\right\}\left(S(w,i,0,1)\right) \end{aligned}$$

where $(\overline{\alpha}_{i+1},\ldots,\overline{\alpha}_R)$ satisfy the equilibrium condition

$$\overline{\alpha}_j = \max_{\alpha_j}\left\{W_j\left(\alpha_j|\alpha_{-j}\right)\right\}\forall j = \{i+1,\ldots,R\}.$$

I have now discussed how to compute the V^{BM} values given that the value/bid function W_l satisfies the equilibrium conditions. The remaining part of the discussion focuses on the computation of this value/bid function. Since analytical solutions for W_l do not exist, even conditioning on other firms' strategies, the strategy used is to find the equilibrium solution by solving the first-order conditions for all the firms simultaneously and then applying a Newton step method to iterate to an equilibrium.

The Newton step method is a gradient search method designed to find the root of a function, that is based on the first-order Taylor expansion of the function. In one dimension, the problem that the Newton step method can attempt to solve is to find $x \text{ s.t. } f(x)=0$ starting with some x'. Taylor's theorem ensures that $f(x)=f(x')+(x-x')f'(x'')$ where $x''\in[x,x']$. Solving for x, this becomes

$$\text{(3.10)} \quad x=-f(x')/f'(x'').$$

The Newton step method attempts to find the root of the function by successively solving (3.10) for x given x' and approximating x'' by x'. Once a value of x is obtained, the algorithm replaces x' with this x and repeats the process until convergence. For the purposes of this algorithm, the function that I attempt to solve is $f(\beta)=dW/d\beta$. As this function is the vector of first-order conditions, a root of this function is an equilibrium of the model, provided that second-order conditions are met. Given this function, I then want to approximate $f'(\beta'')$ by $f'(\beta')$, and iterate until finding the equilibrium β vector.

As the Newton step method requires $f(\beta)$ and $f'(\beta)$ to obtain a solution, one can see that in order to use the method for this problem, I require first and second derivatives of the value/bid function. While simple in principle, applying a Newton step algorithm to this auction problem is complicated somewhat by potential non-differentiability of the value/bid function and other problems. I will discuss these problems and the solutions I used to fix them shortly. For now, however, I note that, as I showed in Section 2, the value/bid function is continuous everywhere. Accordingly, left and right derivatives for this function exist everywhere. I can write the right derivative of the value/bid function in the following first-order condition:

$$\text{(3.11)} \quad \begin{aligned} \frac{dW_1(\alpha_1|\alpha_{-1})^+}{d\alpha_1} &= \sum_{j=0, j\neq l=i+1}^{R} \frac{d\Pr\{j \text{ wins}\}}{d\alpha_1} S(w,i,j,l) \\ &+ \frac{d\Pr\{l \text{ wins}\}^+}{d\alpha_1} \cdot \left(C(w,i,l)-\alpha_1-B(w,i)\right) - \Pr\{l \text{ wins}\} = 0. \end{aligned}$$

The left derivative is the same, except for the derivative of the probability of winning, which is taken in the left direction. This first-order condition in (3.11) can be evaluated analytically. As all of the values except for the derivatives of the probability of winning, namely $\mathrm{d}\Pr(\mathrm{j\ wins})/\mathrm{d}\alpha_1$, are fixed, these are the only values which I must evaluate. I compute these derivatives using the same method of regions that I illustrated in the Section 3.1, there in order to compute the conditional expectation. Within each region, the value of the derivative can be found by the properties of the uniform distribution.

In addition to the first-order condition, I can compute the second-order condition for this problem. Similarly to the first-order conditions, I must compute separate left and right second-order conditions. The right second-order condition can be expressed as:

$$\begin{aligned}\frac{\mathrm{d}^2 V^{BM}(w,i;l|\alpha_{i+1},\ldots,\alpha_R)^+}{\mathrm{d}\alpha_1^2} &= \sum_{j=0,j\neq l=i+1}^{R} \frac{\mathrm{d}^2 \Pr\{\mathrm{j\ wins}\}^+}{\mathrm{d}\alpha_1^2} S(w,i,j,l) \\ &+ \frac{\mathrm{d}^2 \Pr\{\mathrm{l\ wins}\}^+}{\mathrm{d}\alpha_1^2} \cdot \left(C(w,i,l) - \alpha_1 - B(w,i)\right) - 2\frac{\mathrm{d}\Pr\{\mathrm{l\ wins}\}^+}{\mathrm{d}\alpha_1}.\end{aligned} \tag{3.12}$$

The second-order condition is computable using similar methods to the first order condition. Again, the only value which must be computed is the derivative of the probability of winning, and this can be computed using the same method of regions.[7]

Now that I have discussed the basic application of the Newton step method to solving the auction game, I detail some of the problems that I encountered in using this method and the associated solutions below.

The first problem with evaluating the equilibrium numerically is that, as discussed above, the value/bid function is not differentiable everywhere. In

[7] I omit the details of evaluating these expressions within each region. While the expressions are complex, their computations are easily checked using simulation methods, similarly to the conditional expectation from Section 3.1.

particular, the value/bid function may not differentiable at either of two points, when $\alpha_1 \in \{-S^{MAX}, -S^{MIN}\}$. The following example illustrates this:

Example of non-differentiability of value-bid function:
There is one selling firm w_j, $S(w,i,0,j) = 0$, and $C(w,i,j)$ and $B(w,i)$ are such that when $\alpha = 0$, $\beta > 0$.

With this example, if the firm's bid is $\alpha_1 = -S^{MAX}$, then by decreasing α_1, the firm will win with the same zero probability, and so it will get $S(w,i,0,j) = 0$. By increasing α_1 to some ε, the firm will win with the probability $\varepsilon / (S^{MAX} - S^{MIN})$, and receive its bid $\beta > 0$ with this probability. Thus, at $\alpha_1 = -S^{MAX}$, the right derivative of the value-bid function with respect to α_1 is $(C(w,i,1) - B(w,i) - \alpha_1) / (S^{MAX} - S^{MIN})$, while the left derivative is 0. Similarly, at $\alpha_1 = -S^{MIN}$, the left derivative of the value-bid function will be $(C(w,i,1) - B(w,i) - \alpha_1) / (S^{MAX} - S^{MIN})$, while the right derivative will be -1. Because of the randomness imposed by the cost/synergy, the value-bid function is always differentiable for every $\alpha_1 \notin \{-S^{MIN}, -S^{MAX}\}$ which can be seen by the derivative expression in (3.11).

Although this non-differentiability might seem trivial, since the function is differentiable almost everywhere, it is not trivial: exactly because the function has a kink at each of these points, the optimal value will be one of the kinks for a non-measure zero set of reservation prices. At the kinks, the derivative will not exist, and at points close to the kinks, the derivative will not necessarily equal 0.

The second problem is that while the first-derivative of the value/bid function (from (3.11)) is differentiable almost everywhere, it is likely non-differentiable (but still continuous) at some points. These points are the stopping and starting points for other firms normalized bids, i.e. whenever

$$\alpha_l \in \left\{\alpha_j, \alpha_j + S^{MIN} - S^{MAX}, \alpha_j + S^{MAX} - S^{MIN}\right\}_{j \neq l \in M}.$$[8]

Although this might also seem like a measure zero event for one bid to line up in exactly this manner with another bid, this is not true, because some firms are symmetric, and would naturally bid the same amount.

The third problem is that if a normalized bid α is low enough that it does not have any probability of winning given other bids, then the value/bid function will be flat at this point. Because of this, the first derivative of the value/bid function will be 0 at this point. This would lead the Newton step algorithm to (correctly) assume that it has reached a local maximum, but (potentially incorrectly) assume that this is a global maximum.

In order to solve these problems, I use the following techniques.

First, although the first derivative of the value/bid function is undefined at two points, the left and right derivatives at these points are defined, as stated earlier. Thus, in the Newton step iterative process, I compute the left and right derivatives to decide in which direction the normalized bid should move. There are three possibilities: either the left derivative is negative, in which case the normalized bid should decrease; the right derivative is positive, in which case the normalized bid should increase; or the right derivative is negative and the left derivative is positive, in which case the current bid marks a maximum of the value/bid function and a kink of this function.

Second, in order to deal with the second derivative, I use the same approach that I do for the first derivative: I compute right and left second derivatives, and use these in deciding how much to move each bid in the Newton step algorithm

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[8] I do not prove the above assertion. However, it follows from the properties of the second derivative of the probability of winning given the uniform distribution.

depending on the direction to move in, as indicated by the first derivative. More importantly, because the second derivative can change discretely when moving to a different bid range, I do not allow the normalized bid to change past the end of a given bid range, but instead fix the maximum increment so that the new normalized bid is at the changing point. Additionally, in order to have more control over the movement of each bid in this manner, I do not do a Newton step search on the whole vector of FOCs. Instead, I individually iterate on each FOC taking the others as fixed.

Third, in order to deal with the fact that the value/bid function is flat for normalized bids below the amount that never wins, I ensure at every iteration that each normalized bid is at least high enough that increasing the normalized bid by any ε would make the probability of winning greater than 0. Thus, given some normalized bid α_l, this bid will have a 0 probability of winning if the highest net value that this bid could give to the buying firm is less than 0, or if it is less than the lowest net value that some other potential selling firm could give to the buying firm. Accordingly, I set:

$$\alpha_1 = \max\left\{\alpha_1, -\mathrm{S}^{\mathrm{MAX}}, \max_{j \neq 1}\left\{\mathrm{S}^{\mathrm{MIN}} - \mathrm{S}^{\mathrm{MAX}} + \alpha_j\right\} + \varepsilon\right\}.$$

Then, at this point, the value/bid function will have a non-zero right derivative, even if it may have a zero left derivative. Note that, if the normalized bid is sufficiently high that the selling firm always wins, then the value/bid function will have a derivative of -1, as the firm simply gets less for each increase in bid. Thus, the computer algorithm does not need to ensure that the normalized bid is below a certain amount, as the firms will never find it optimal to increase their normalized bid beyond a certain point.

In the discussion in this section, I have explained the computation the auction model as though it were the equilibrium of a static game. For my purposes, I compute the equilibrium of this model with constantly changing V^{PM} values, as these are derived from the dynamic Markov-Perfect equilibrium. There are only two differences between the computation of the merger game for the static and dynamic cases. The first difference is that in the static game, I would use the Newton step method to solve for the equilibrium in bids of every $MM^{auction}(w, i, M)$ merger game. In my model, I do not solve for the optimal solution at every iteration. Instead, I take one Newton step for every bid at every iteration. The rationale for this procedure is that it saves time: it makes each iteration proceed much faster, and has proven to be quicker than solving for the optimal bids at every iteration. Second, because I am not solving for the optimal bid vector at every iteration, it is necessary to store the bids of all potential selling firms at every state, in order to refer to them in subsequent iterations. Thus, in computing the auction model, there is more storage required than in the tioli model. However, similarly to the tioli model, I do not have to keep track of the V^{BM} values for merger games where $i > 1$.

Finally, I discuss two general points regarding the computation of the auction model. First, note that the information structure of the cost/synergy draws is important in being able to solve the equilibrium for this auction game numerically. If the cost/synergy draws were known by the selling firms, then any algorithm to compute the equilibria of the auction game would have to keep track of the equilibrium actions conditional on each realized vector of synergy draws, which would render computation much more difficult. Thus, the algorithm presented in this subsection shows that while introducing randomness may be necessary to allow for the computation of the model, allowing the proper type of randomness is equally important. Second, note that the problems encountered in computing the auction model exist because there are externalities present in this model, and are not directly related to the fact that the auction process is used to determine the occurrence of mergers. As significant externalities are present in auctions in many different settings, the problems and solutions discussed here are of use in computing auctions with externalities for different economic models.

Section 4: Solving the Dynamic Equilibrium

I have developed a process which attempts to find a MPNE of this model for fixed parameter values, using a computational algorithm with discrete time dynamic programming as a foundation. The technique used is based on the algorithms developed by Pakes-McGuire (1994) and extends them to examine mergers.. In this section, I enumerate the value functions of firms, define a MPNE for this model, prove that such an equilibrium exists, and briefly outline the computational strategy used, which is based on the proofs given. The reader may consult P-G-M for details on the computational strategies used to solve for MPNE of dynamic games with entry and exit. In the interest of brevity, I am not proving the ergodic properties (such as that there is some unique recurring class of states and a fixed ergodic distribution over this class) about this model that Ericson and Pakes (1995) show for their model. Although I have not proved these properties, I strongly suspect that they are also true for my model.

In Section 2, I defined the value functions, V^{BM}, V^{PM}, and V as EDVs of net future profits at various stages, while in Section 3, I discussed V^{BM} and V in terms of V^{PM}. Here, I detail V^{PM}, and tie together these variables, in their dynamic context. Value functions are the central tool used in this solution algorithm. The premise of the algorithm is to repeatedly iterate on the value functions and policies until a set of equilibrium value functions is achieved.

The V^{PM} value function is dependent on firms' perceptions about competitors' entry and exit decisions and investment levels, all of which affect a firm's competitors' efficiency levels at the start of next period, given that the algorithm is at the point in a period just after the merger process has ended. I define x to be the investment level and Π to be the reduced form static Cournot profit functions for each state. Then, given that V is the true EDV of net future profits at the start of any period, I can write V^{PM} as

$$V^{PM}(w,n) = \max\left\{\Phi, \sup_{x\geq 0}\left\{\Pi(w,n) - x + \beta \right.\right. \tag{4.1}$$
$$\left.\left.\sum_{(w',n')} \Pr\begin{pmatrix}\text{State next period being } (w',n') \text{ given x and percep -} \\ \text{tions about competitors' investment, entry and exit}\end{pmatrix} V(w',n')\right\}\right\}.$$

The reader may consult P-G-M for details about (4.1).[9] In particular, note that the use of the reduced-form Cournot profits Π follows from the Markov-Perfect assumption. The V^{PM} value function is the same as the value function described in P-G-M as it is a function only of entry, exit and investment.

Recall that the V^{BM} values are defined for the different merger models (auction and tioli) and different states in Section 3. Additionally, the value function

[9] Note that in the case where a firm's position is tied next period, there is an ambiguity as to how to reorder the firm's position. I resolve this ambiguity by saying that firms which have the same capacity levels next period have an equal probability of being at any of these tied positions.

V is just defined as particular elements of V^{BM}. In Section 3, I defined V^{BM} for arbitrary continuation values V^{PM}. Here, I am explicitly defining the continuation values to be the equilibrium values from the dynamic equilibrium game.

Now that I have enumerated the value functions, I want to define a MPNE for the model. First, I need a couple of preliminary definitions.

Definition:

(a) Let $EP(w)$ be the probability that a potential entrant has of entering when the firms currently in the industry are given by w.

(b) Let $XP(w,n)$ be the probability of exit for the incumbent firms (w,n).

(c) Let $RR(w,i)$ be a set of actions for the merger game $MM(w,i,M)$ given some set of reservation prices. For the $MM^{tioli}(w,i,M)$ model, these actions specify a buyer's offer and sellers' responses for each realization of the cost/synergy vector, while for the $MM^{auction}(w,i,M)$ model, these actions specify bids from each seller and a buyer's response for each realization of the cost/synergy vector. With the set of actions, the probability of occurrence for each potential merger is then defined.

(d) Let the partial transition matrix Q be the transition matrix describing the probabilities of any state occurring, given any state structure that exists after the merger process has just finished (i.e. at the V^{PM} stage).

(e) Let Π_{MAX} be the maximum one-firm profits that can be achieved by a firm in the industry. Because of the nature of the static problem, this profit level will be realized by a monopolist in the industry with a sufficiently high capacity to satisfy all the demand.

Recall from Section 2 that an entrant signals its intention to enter after receiving an amount x_E that it must pay to enter; the amount is drawn from a uniform distribution. Thus, EP represents the probability of entry before this draw is received and can be written as

$$(4.2)\quad EP(w)=\int_{x_E^{MIN}}^{x_E^{MAX}}\{-x_E+\text{EDV of profits from entering}>0\}dPx_E\ .$$

Also as illustrated in Section 2, a firm signals its intention to leave at any period and receives a scrap value Φ. Thus, XP is a binary variable. However, when I prove the existence of equilibrium, I prove it for a model where each firm receives a simultaneous draw $\Phi \sim U\left(\Phi^{MIN}, \Phi^{MAX}\right)$, as otherwise I cannot guarantee existence due to discontinuities. For this variant, then, I can define XP in a similar fashion to EP as

$$(4.3)\ \ XP(w,n) = \int_{\Phi^{MIN}}^{\Phi^{MAX}} \left\{\text{EDV of remaining active} < \Phi\right\} dP\Phi .$$

Note that the partial transition matrix Q describes the probabilities of different states occurring given a state that exists at the conclusion of the merger process. From the fact that the merger process occurs at the beginning of the period, one can see that Q is dependent on firms' investment, entry and exit decisions, but <u>not</u> on merger decisions, except indirectly through their effect on the above variables.

I now define an equilibrium of the model. The definition treats the number of firms N as fixed. As I will show in Lemma 4.1, there is a maximum N that will ever be reached. Thus, N is really an endogenous function of the other parameters below. However, I prove that an equilibrium exists for arbitrary N, from which is follows that one exists for the maximum N which occurs in equilibrium as a function of the other parameters.

<u>Definition:</u> Given a set of parameters

$$\left(S^{MIN}, S^{MAX}, a, \delta, \beta, \alpha, N, k_E, \Phi^{MIN}, \Phi^{MAX}, x_E^{MIN}, x_E^{MAX}, mc, \text{Demand}\right),$$

a MPNE for the merger model is a 11-tuple:

$$\left(V(w,n), X(w,n), C(w,i,j), B(w,i), S(w,i,j,k), RR(w,i)\right.$$
$$\left. XP(w,n), EP(w), V^{BM}(w,i,j;n), V^{PM}(w,n), Q(w)\right) \text{for}$$
all states (w,n) and all merger games (w,i) such that:

(1) $V^{PM}(w,n)$ is the EDV of net future profits of firms at the point in the period just after the merger process has ended, given that:
  - (a) $(X(w,n), XP(w,n))$ are the chosen investment and exit decisions, respectively;
  - (b) Q is the partial state transition matrix; and
  - (c) $RR(w,i)$ are the parameters that will be used by all future merger games MM.

(2) $EP(w)$ is the optimal probability of entry (i.e. it satisfies (4.2)) given (b) and (c) above.

(3) $X(w,n)$ is the optimal choice of investment (i.e. it satisfies (4.1)), given (b) and (c) above.

(4) $XP(w,n)$ is the optimal choice of exit (i.e. it satisfies (4.3)), given (b) and (c) above.

(5) $Q(w)$ is the true partial state transition matrix implied by $(X(w,n), XP(w,n), EP(w))$.

(6) $C(w,i,j)$ is the true EDV of net future profits from merger, as specified in its definition, given (b) and (c) above.

(7) $B(w,i)$ is the true EDV of net future profits from no merger to the buyer, as specified in its definition, given (b) and (c) above.

(8) $S(w,i,k,j)$ is the true EDV of net future profits from each merger to non-participants, as specified in its definition, given (b) and (c) above.

(9) $RR(w,i)$ form equilibrium strategies for the appropriate underlying MM game, given that C, B and S represent the true EDV of net future profits as defined.

(10) $V^{BM}(w,i;n)$ is the EDV of net future profits at the stage in the merger game indicated by the variable, given (b) and (c) above.

(11) $V(w,n)$ is the EDV of net future profits at the beginning of the period, given (b) and (c) above.

Finally, I provide a set of assumptions on the entry, exit and merger processes which are sufficient to ensure the existence of an equilibrium.

<u>Assumptions necessary to ensure existence of equilibrium:</u>

(a) The distribution of entry and exit costs is not degenerate (i.e. it is not a fixed value).

(b) For each firm, holding competitors' simultaneous strategies fixed, the values that result given the optimal strategy for the merger game RR are continuous in the reservation prices (C,B,S) and in the competitors' simultaneous strategies.

Recall that in Section 2, I discussed conditions under which assumption (b) below will hold for both the tioli and auction models, in Corollary 2.2 and Corollary 2.4 respectively. Additionally, the parameters I choose ensure that (a) holds.

Given this definitions and assumptions above, I now prove that an equilibrium to the model exists, provided that the entry and exit costs are not a degenerate distribution and provided that each underlying merger game is well-behaved. I proceed to prove this as follows: First, I show that the investment decisions lie in a compact space provided that the value functions do. Second, I show that I can rewrite the definition of equilibrium using dynamic programming methodology and functional equations. Third, I define a function which maps from the above 11-tuple to itself, and show that this function is continuous on a compact, convex and non-empty set. Fourth, I show that a fixed point of the function is an equilibrium of the model, and that as such a fixed point exists, the model has an equilibrium.

Lemma 4.1: The following functions can be bounded:

(a) If there is a fixed bound $[\Phi,\overline{v}]$ within which the equilibrium value function $V(w,n)$ lies, for every (w,n), then there is a fixed bound, $[0,\overline{x}]$ within which the equilibrium investment function $X(w,n)$ lies, for every (w,n).

(b) For any number of firms N, if $\Phi > S^{MAX}$, then there is a fixed maximum capacity that any firm will ever choose to reach, no matter what are its perceptions of other firms' strategies. Let $\overline{w}$ be this maximum capacity and let $\overline{W}$ be the space which includes all the combinations of N firms with capacities from 0 to $\overline{w}$.

(c) Provided the minimum entry cost is greater than the scrap value (i.e. $x_E^{MIN} > \Phi$), there exists a number N, such that whenever there are N firms active in the industry, no additional firms would want to enter the industry, no matter what are the potential entrant's perceptions of other firms' strategies.

Proof:

(a) First, investment can never be negative, by assumption. Now, let v_1 be the EDV of profits to a firm starting next period if its investment is successful (which leads to some distribution of competitors' capacity levels), and let v_2 be the firm's EDV of profits starting next period if its investment is unsuccessful. Then, as detailed in P-G-M, the optimal investment level for the firm can be written as $xx(v_1 - v_2)$, where xx is an increasing function. Then, as I am assuming that V is bounded, there are some maximum and minimum EDV of profits. Thus, the highest value of xx will be obtained if every state in v_1 has value $\bar{v}$ and every state in v_2 has value Φ. Therefore, the optimal investment level can be bounded by $\bar{x} \leq xx(\bar{v} - \Phi)$.

(b) Note that the total EDV of profits that a firm can achieve is bounded by $\frac{\Pi_{MAX}}{1-\beta}$, because of discounting and the maximum static profits. Thus, for any $\varepsilon > 0$, no matter what are one's competitors' capacities there exists a capacity level k high enough such that the difference in values between the value from not investing (i.e. v_2 (from (a)) and the maximum value achievable is less than ε. Now, as can be seen in P-G-M, if the difference between v_1 and v_2 which is less than the above difference is sufficiently small, then a firm will choose to invest nothing. Thus, at capacity level k, a firm's capacity will never rise from investment, because of the choice of investment function together with the fact that it is investing nothing. The other possibility is that the firm will acquire another firm and increase its capacity that way. However, for $\varepsilon < \Phi - S^{MAX}$, the firm will not choose to acquire another firm for the following reason: the possible gain in EDV of profits not counting the cost/synergy payment is ε as detailed above. The maximum gain from the synergy payment is S^{MAX}. Now, the potential acquired firm can receive a value of at least Φ if it is not acquired, no matter who else is acquired, which means that it will not be willing to be acquired for less than Φ. Thus, the price paid to acquire the firm must be at least Φ and so the additional profits from the merger are at most $S^{MAX} - \Phi + \varepsilon$. By the assumption then, for this capacity level, acquiring another firm will result in lower profits than acquiring no firm regardless of the synergy draw, and so no merger will take place.

(c) This argument is the same is in Ericson and Pakes, so I will only briefly outline it. The argument is as follows: the total EDV to firms in the industry is no higher than the maximum EDV of profits that a firm can achieve (i.e. $\frac{\Pi_{MAX}}{1-\beta}$). Thus, as the number of firms in the industry increases, the value to each firm drops, and asymptotically approaches zero. When the value to the firm is less than $x_E^{MIN} - \Phi$, firms will no longer enter, as they will not earn a positive EDV from entering.■

Part (a) of Lemma 4.1 shows that the MPNE X matrix will be bounded provided that V is bounded. Parts (b) and (c) of Lemma 4.1, by showing that the number of firms and each firm's highest capacity are bounded shows that the MPNE value functions and policy functions can be defined on a finite-dimensional space. Proving that these variables are finite-dimensional is important for two reasons: first, it shows that numerical evaluation of the equilibrium by examining the value at each point is possible; second, it shows that in order to examine the compactness of the value and policy variables within each space (which is necessary to apply the fixed-point theorem), it is only necessary to check for closedness and boundedness.

I can express the value function $V(w,n)$ as the fixed point of a functional equation. In order to do this, I define an operator, T, which maps a value function for the state space onto itself, conditional on the vector (RR,Q) of beliefs about the merger process and the investment, entry and exit processes. I can write $T:[\Phi,\bar{v}]^{\overline{W}\times\{1,\ldots,N\}} \to [\Phi,\bar{v}]^{\overline{W}\times\{1,\ldots,N\}}$, and then define T by:

$$(4.4)\quad T[V|M,Q] = \left(V^{PM} \circ V^{BM}{}_{1\text{ potential acquirer}} \circ \cdots \circ V^{BM}{}_{N-1\text{ potential acquirers}}\right)(V).$$

In this definition of T, I am implicitly using the definitions of all these value functions in Section 3 for the appropriate static merger model.

Lemma 4.2: Part (11) of the definition of MPNE is equivalent to (4.4). In other words, given that (RR,Q) satisfy the equilibrium conditions, V is the EDV of net future profits (as above) if and only if V satisfies (4.4).

Sketch of Proof: Part (11) specifies V as a maximization problem given the parameters (RR,Q), where firms choose an optimal path of investment and exit, conditional on their perceptions, and V is the value of this optimal path. This is equivalent to V satisfying the functional equation conditioning on perceptions; see Stokey, Lucas and Prescott (1989) Theorem 9.2 for details.[10] Now, (4.4) specifies the expected value of future returns as a function of V, and the present returns as the maximizing choice over investment and exit of the value that will accrue to the firm

[10] For this results to be true, Stokey, Lucas and Prescott (1989) require measurability of the one-period return function, the stochastic state transition function and the investment function. By the discreteness of the state space in my model, these requirements all hold.

this period. Thus, (4.4) is the functional equation for the above maximization problem, and so the solution to (4.4) solves the maximization problem.■

Now I define an operator f which generates the equilibrium. I will show that f is a continuous function on a compact, convex, non-empty set whose fixed point is an equilibrium of the merger model.

Definition: Let the MPNE generation operator f be a function from the 11-tuple of actions/values to itself, satisfying

$$\left(V_1, X_1, C_1, B_1, S_1, XP_1, EP_1, V_1^{BM}, V_1^{PM}, RR_1, Q_1\right) =$$
$$f\left[V, X, C, B, S, XP, EP, V^{BM}, V^{PM}, RR, Q\right] = \left(V^{PM} \circ V^{BM}{}_{1\text{ potential acquirer}} \circ\right.$$
$$\left.\cdots \circ V^{BM}{}_{N-1\text{ potential acquirers}}\right)\left(V_0, X_0, C_0, B_0, S_0, XP_0, EP_0, V_0^{BM}, V_0^{PM}, RR_0, Q_0\right)$$

Although their definitions look similar, the MPNE generation function f differs from the earlier operator T. To evaluate T, the value function is computed for fixed values of the perceptions of competitors. In f, these perceptions are not held fixed, but rather are updated, so that f is a function from the 11-tuple of actions/values to itself, instead of just from the value function to itself. The perceptions are updated in the following manner: first, $\left(V_1^{PM}, X_1, EP_1, XP_1\right)$ are computed using $\left(Q_0, V_0\right)$ and (4.1). Then, $\left(V_1^{PM}, X_1, EP_1, XP_1\right)$ in turn imply a new Q_1, namely the partial state transition matrix that is generated by their actions. As in T, V_1^{BM} is computed by applying the merger game strategies RR together with the mechanism of the game form of the chosen process. The difference lies in the values that are used for the perceptions in computing these variables. Here, for all merger games with $R = 2$, $\left(S_1, B_1, C_1\right)$ are computed using V_1^{PM}. Then, again for all the merger games with R = 2, RR_1 and V_1^{BM} are computed using (S_1, B_1, C_1). For all merger games with R = 3 and $r \equiv o(M) = 1$, (S_1, B_1, C_1) are computed using V_1^{BM} and V_1^{PM} values. Recall that these V_1^{BM} values are for games with R = 2 and hence because of the order of computation, they will already have been computed. Then, RR_1 is computed as the optimal strategies using RR_0 for competitors' simultaneous strategies, whenever the decision process is simultaneous. This process is then repeated for merger games with R = 3 and r = 2, R = 4 and r = 1, etc. Note that at each stage, the V_1^{BM} values used will already have been computed. Finally, V_1 is evaluated as the appropriate V_1^{BM} values.

Thus, as the reader can see from the definition of the MPNE generation function f, the function is defined inductively over the merger process. The order of the induction is for R = 2 and r = 1, R = 3 and r = 1, R = 3 and r = 2, R = 4 and r = 1, R = 4 and r = 2, R = 4 and r = 3, etc. It is necessary to choose this order because of the structure of the acquisition process. With this ordering, the (S_1, B_1, C_1) values are function only of elements that have already been calculated. The following proofs of continuity, compactness, and finally existence of equilibrium all use the same inductive structure, as well.

I now prove that f is continuous on a compact, convex, non-empty set, and apply Brouwer's Fixed Point Theorem to prove existence of equilibrium.

Lemma 4.3: Given the assumptions and definitions above, I can define f to be a continuous function on a compact, convex, non-empty set.

Proof: I split the proof into two parts. In order to avoid endless notation, I simply discuss each point informally.

Claim: f is continuous.

First, note that in the definition of f, $\left(X_0, EP_0, XP_0, V_0^{BM}, V_0^{PM}, B_0, C_0, S_0\right)$ are not actually used. I include these variables as arguments to f in order to define f as a mapping from some space to the same space. As these values do not affect the value of f, f is automatically continuous in them. Thus, I must only examine whether f is continuous in the variables $\left(V_0, RR_0, Q_0\right)$. I will first examine the continuity of $\left(X_1, XP_1, EP_1, V_1^{PM}, Q_1\right)$ in the above variables. These values do not depend on RR_0 so it suffices to look at their continuity as functions of $\left(V_0, Q_0\right)$. These variables are defined so that they are continuous functions of some weighted sum of different elements of V_0, where the weights are specified by Q_0. To illustrate how these values change with a change in V_0, consider the case of EP_1. For EP_1, one can see from (4.2) that a change in the value will only affect the indicator function on a small interval, and thus will affect the integral only a small amount. In particular, if the value changes by ε, then

$$\left|\Delta EP_1\right| \leq \frac{\varepsilon}{x_E^{MAX} - x_E^{MIN}} .$$

Continuity of XP_1 follows in a similar fashion from (4.3), and continuity of X_1 follows from P-G-M.[11] Thus, a small change in $\left(V_0, Q_0\right)$ entails a small change in the weights and a small change in the values associated with each weight and hence

[11] I use this same logic again when I prove the continuity of the variables that involve the costs/synergies. Essentially, I am stating if I define f(a) by $f(a) \equiv \Pr(a + u \geq 0)$ where $u \sim U(b, c)$, then f(a) continuous in a.

(X_1, XP_1, EP_1) is continuous in (V_0, Q_0). Now, recall that V_1^{PM} is defined as a weighted sum over V_0, where the weights are given by (X_1, XP_1, EP_1). As a small change in (V_0, Q_0) entails a small change in (X_1, XP_1, EP_1) and hence a small change in V_1^{PM}, V_1^{PM} is continuous in (V_0, Q_0). Similarly, a small change in (X_1, XP_1, EP_1) entails a small change in the partial state transition matrix Q_1, as (X_1, XP_1, EP_1) maps continuously onto the probabilities of investment, entry and exit that can cause a change in the matrix. Thus, Q_1 is continuous in (V_0, Q_0).

I now want to examine the continuity of $(C_1, B_1, S_1, RR_1, V_1^{BM})$ in (V_0, RR_0, Q_0). I use induction to prove continuity for these variables. As outlined above, induction is done for the basis case of $R = 2$ and $r = 1$, then for $R = 3$ and $r = 1$, $R = 3$ and $r = 2$, etc. First, the basis case: I prove that those elements where $R = 2$ are continuous in (V_0, RR_0, Q_0). For these elements, (C_1, B_1, S_1) are fixed elements of V_1^{PM}, and thus are continuous in (V_0, RR_0, Q_0). Now, by assumption $(C_1, B_1, S_1, V_1^{PM})$ cause small changes in the decisions in RR_1 and small changes in the probability of the (unique) merger happening, and small changes in the payments to the acquired firm conditional on it happening; thus for $R = 2$, (RR_1, V_1^{BM}) is continuous in (V_0, RR_0, Q_0).

Now, the inductive step: consider the continuity of $(C_1, B_1, S_1, RR_1, V_1^{BM})$ for games where $R > 2$ and some r, assuming that the same variables are continuous in (V_0, T_0, Q_0) for all games where $o(M) = r - 1$ and for all games where the number of active firms is R - 1. Recall that (C_1, B_1, S_1) are defined as weighted sums over elements of V_1^{BM} that have already been shown to be continuous in (V_0, RR_0, Q_0) and that the simultaneous components of RR_1 are calculated for each individual using RR_0 as the strategy for its competitors. By assumption, RR_1 and thus V_1^{BM}

are continuous in (V_0, RR_0, Q_0). By induction, then, $(C_1, B_1, S_1, RR_1, V_1^{BM})$ is continuous in (V_0, RR_0, Q_0) for any r and R.

Finally, V_1 is just defined to be a particular element of V_1^{BM}, and so is continuous in (V_0, RR_0, Q_0) as every element of V_1^{BM} is continuous in (V_0, RR_0, Q_0).

Claim: I can define the domain and range of f to be the same compact, convex and non-empty space.

I will show that I can find some compact space such that for any element in the space, f evaluated at that element will also always be in the space. I examine whether each component of the space is compact. First, Q is compact, because it is a simplex, XP and EP are also compact because each element is a probability and RR is compact because, as discussed in Section 3, the strategy space of the MM merger game can be defined to be compact. Additionally, if I can limit V to be in a compact interval, then X will also be in a compact interval, by Lemma 4.1.

Now, I claim that if I limit V_0 to be in the interval $[a, b]$, where

$$a = \Phi, b = \frac{\frac{N(N-1)}{2} S^{MAX} + \Pi_{MAX}}{1-\beta},$$

then $(V_1, C_1, B_1, S_1, V_1^{BM}, V_1^{PM})$ will all be in this interval. The reason is: suppose V_0 is in some interval $[a, b]$. Then, V_1^{PM} is the maximum of the scrap value Φ and the value of not exiting. The value of not exiting is given by β times some distribution over different V values plus whatever is earned (which is at most Π_{MAX}). Thus, V_1^{PM} will be in the interval $[\Phi, \beta b + \Pi_{MAX}]$. Then, I proceed by induction on the number of active firms R and the number of potential acquired firms r, in the same order as before. If R = 2 and r = 1, then (C_1, B_1, S_1) must be in $[\Phi, \beta b + \Pi_{MAX}]$, as they are weighted sums of various V_1^{PM} values. Additionally, for R = 2 and r = 1, the potential acquirer gets at most some value in V_1^{PM} plus the maximum synergy draw, even assuming that it could pay the seller nothing. Similarly, the potential acquired firm gets at most some value in V_1^{PM} plus the maximum synergy draw if it sells itself, even assuming that the seller gains nothing from the merger. Neither the seller nor the buyer can get less than Φ, by definition of the merger game. Thus, for R = 2 and r = 1, I can limit $(V_1^{BM}, C_1, B_1, S_1)$ to be in the interval $[\Phi, \beta b + S_{MAX} + \Pi_{MAX}]$. Now, for R = 3 and r = 1, (C_1, B_1, S_1) must be in $[\Phi, \beta b + S_{MAX} + \Pi_{MAX}]$ as they are now weighted sums of values in V_1^{BM} for R = 2 and r = 1 and for V_1^{PM} (even though for the buyer the weights are determined by S_0). Again similarly to the R = 2 and r = 1 case, for R = 3 and r = 1, an acquiring or acquired firm can get at most something in V_1^{BM} for R = 2 and r = 1 and for V_1^{PM}, plus the maximum synergy draw. Thus, I can bound V_1^{BM} for R = 3 and r = 1 to be in $[\Phi, \beta b + 2S_{MAX} + \Pi_{MAX}]$. Thus, for the last stage (which includes $\frac{N(N-1)}{2}$ inductive steps if R = N), I can bound $(V_1, V_1^{BM}, C_1, B_1, S_1)$ to be in $[\Phi, \beta b + (N-1)S_{MAX} + \Pi_{MAX}]$. But, by definition of b,

$$
\begin{aligned}
&\beta b + \frac{N(N-1)}{2} S_{MAX} + \Pi_{MAX} \\
&= \beta \left[\frac{\frac{N(N-1)}{2} S_{MAX} + \Pi_{MAX}}{1-\beta} \right] + \frac{N(N-1)}{2} S_{MAX} + \Pi_{MAX} \\
&= \frac{\frac{N(N-1)}{2} S_{MAX} + \Pi_{MAX}}{1-\beta} = b.
\end{aligned}
$$

Thus, if the domain for each component of $(V_1, S_1, V_1^{BM}, V_1^{PM}, C_1, B_1)$ is $[a, b]$, the range will be in $[a, b]$.

The above arguments show that I can define f as a function:

$$f:[a,b]^{A_1} \times [0,1]^{A_2} \times \Delta^{A_3} \times [0,\bar{x}]^{A_4} \rightarrow [a,b]^{A_1} \times [0,1]^{A_2} \times \Delta^{A_3} \times [0,\bar{x}]^{A_4},$$

where Δ is the simplex, and A_1, A_2, A_3, A_4 are positive integers indicating the number of elements of each type. As this space is a finite Cartesian product of compact, convex and non-empty finite-dimensional spaces, it is compact, convex and non-empty.■

With Lemma 4.3, I can apply Brouwer's Fixed Point Theorem to show that the function f has a fixed point. All that remains is to show that this fixed point is in fact an equilibrium of the merger model. This is done in Proposition 4.1.

<u>Proposition 4.1:</u> Given the necessary assumptions above, a MPNE for the merger model exists.

<u>Proof:</u> By Lemma 4.3, the function f is a continuous function on a compact, convex and non-empty space. Using Brouwer's Fixed Point Theorem, I find that f has a fixed point $\left(\overline{V},\overline{X},\overline{C},\overline{B},\overline{S},\overline{XP},\overline{EP},\overline{V}^{BM},\overline{V}^{PM},\overline{RR},\overline{Q}\right)$. I want to show that the fixed point of f is an equilibrium of the model.

First, note that $\overline{V}$ is a solution to the functional equation given the fixed point. In other words,

$$\overline{V} = T\left[\overline{V}\middle|\left(\overline{X},\overline{C},\overline{B},\overline{S},\overline{XP},\overline{EP},\overline{V}^{BM},\overline{V}^{PM},\overline{RR},\overline{Q}\right)\right].$$

The reason for this is because f is the same function as T, except that in T perceptions are held fixed, while in f, perceptions are updated. But, at the fixed point, the perceptions used to generate V_1 in f are the same perceptions that are specified in the function call. Thus, at the fixed point, the same perceptions are used to generate the values in f and in T, and thus their values will be the same, so $T[\overline{V}] = \overline{V}$.

Now, by my choice of Q that I have used in f, $\overline{Q}$ is the true partial transition matrix, implied by $(\overline{X}, \overline{XP}, \overline{EP})$, so part (5) of the equilibrium definition holds. In addition, by definition, $(\overline{C}, \overline{B}, \overline{S})$ will characterize future play of the game, and $\overline{RR}$ will form an equilibrium of every MM merger game given $(\overline{C}, \overline{B}, \overline{S})$, so part (9) holds. By Lemma 4.2, part (11) holds, and the value function $\overline{V}$ that I have computed is the true value function. Given that part (11) holds, it follows that parts (2), (3) and (4) hold, because investment, entry and exit are calculated in such a manner that they are true if the value function used in computing them (i.e. $\overline{V}$) is the true EDV of net future profits, which it is, by part (11). Since parts (11), (2), (3) and (4) hold, (1) holds, since $\overline{V}^{PM}$ is computed as the EDV of profits after the merger process has completed given that the computation is done using the true values, which it is, as the above parts hold.

To prove that the remaining assumptions hold, I again use induction on N and r in the same order as above. For games with R = 2 and r = 1, parts (6), (7) and (8) hold, as $(\overline{C}, \overline{B}, \overline{S})$ are all computed to be the true values, given that $\overline{V}^{PM}$ is correct, which it is, by (1), and part (9) holds, as stated above. Additionally, part (10) holds for games with R = 2 and r = 1, because $\overline{V}^{BM}$ is chosen as the correct value by f, given that parts (6) through (8) hold. Now, assume that parts (6) through (10) hold for games with less than R active firms and for games with R active firms and less than r potential acquired firms. I want to examine whether they hold for games with R potential sellers and r potential acquired firms. By the same logic, parts (6) through (10) will hold for all these merger games. Thus, for all R, assumptions (6) through (10) hold.■

As discussed earlier, I cannot analytically solve for an equilibrium for this model, conditional on parameter values. Instead, I can numerically solve for an equilibrium for fixed parameter values, using computational methods. The algorithms are very computationally intensive.[12] The process by which I solve for the equilibrium is essentially to repeatedly apply the function f to some initial starting value, until the function converges to a fixed point. Thus, the algorithm stores value functions, the partial state transition matrix and strategies. Since f is not a contraction mapping, f is not guaranteed to converge to a fixed point. However, since f is similar to contractions used in dynamic programming, it generally converges, for most of the parameter values that I have tried. With the equilibrium policies, I simulate the industry for a large number of time periods, feeding the firms the random draws to their investment and entry processes and their random cost/synergy draws. The program keeps track of the industry structure at every period, as well as other variables of interest, such as welfare indications, concentration measures, and investment levels. The mean equilibrium values of these variables were reported and discussed in Chapter 1.

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[12] For instance, the C version of this program on a Sun Sparcstation 10 takes about 5 hours to converge for the 5 firm equilibrium.

Section 5: Conclusions

In the results presented here, I have defined the merger model and illustrated how to solve the model computationally. The results show that endogenous merger processes that use either the take-it-or-leave-it (tioli) or auction processes can be defined and computed. For the tioli model, it is possible to prove uniqueness and existence of the static equilibrium. For the auction process, the choice of information structure and type of auction is important, as the model is too time- and memory-intensive to compute for many different information structures and auction types. For the specification chosen, which is a simple first-price auction where only the buyer knows the realization of the synergy draws, computation is possible, and using some techniques that I discussed in Section 3, it is not much more cumbersome than for the tioli model. However, counterexamples show that neither uniqueness nor existence is assured, unless the size of the randomness is sufficiently large. Because of the randomness that I have introduced, the results show that, provided an equilibrium to the static merger model exists for every state, a dynamic equilibrium to the overall model will exist. However, as is common with this type of model, uniqueness of the dynamic equilibrium is not guaranteed. As uniqueness is required in order for the results to be sensible but uniqueness cannot be proven, this is a shortcoming of this model and other papers in the literature.

In addition to the above conclusions, the results presented in this chapter demonstrate a few general properties. First, they show that even with a dynamic model such as this one, most of the computational techniques used can be static in nature. The reason for this is that with dynamic programming methods, future events are encapsulated as value functions and thus optimal decisions are computed using these value functions, which can be treated as analogous to reduced-form static quantities. While the dynamics largely do not affect the algorithm solution, they indirectly affect the choice of algorithm and solution method: because of the large amount of memory and time needed to compute the dynamic equilibrium, the choice of static processes and number of agents is constrained. Second, the results demonstrate that introducing a source of randomness may be necessary in order to ensure the existence of equilibrium and allow for computation. In this model, random entry and exit costs, investment realizations and merger synergies allow for the computation of the equilibrium. Many models have not been computable or solvable because the lack of randomness makes computation impossible or results in the lack of existence of an equilibrium. The results here show that appropriate structural randomness may be a solution to this problem, however, as discussed earlier, the choice of randomness is crucial to being able to compute the model. Third, the results here show that it is possible to examine the effects of auctions with externalities. While previous auction models have focused largely on auctions with identical bidders and no externalities, many real-world auctions involve substantial externalities and asymmetries among the bidders. The results presented here examine the development of an auction model with externalities and differentiated bidders and the computation of such a model. Both of these discussions can be used to theoretically and empirically examine auctions with externalities and differentiated bidders. This can in turn allow economists to develop a richer framework with which to empirically analyze auctions in the future.

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