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October 29, 2016
Columbia/Duke/MIT/Northwestern IO Theory Conference
Introduction
Motivation

- Bilateral bargaining in industry is pervasive
  - Health care (over $600B in negotiated US private health care payments)
  - Content distribution deals (e.g., AT&T - Time Warner)
  - Retail - Manufacturer relationships
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Important to understand the determinants of surplus division
- Provides incentives for investment, entry, exit, mergers, integration...
- Enables the evaluation of welfare and competitive effects of:
  - market structure changes (mergers, integration, entry/exit),
  - regulatory interventions (limiting price discrimination, bundling, vertical restraints), ...
Literature on Bilateral Bargaining

Theoretical Approaches: *Focus on analytically tractable models, typically w/ single agent on one side and/or limited externalities. E.g.,*

- Wage Bargaining
  [Davidson 88, Jun 89, Stole Zwiebel 96, Westermark 03, ...]

- Bargaining on Networks
  [Kranton Minehart 01, Corominas-Bosc 04, Manea 11, ...]
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**Empirical:** *Emphasis on “many-to-many” environments & general externalities*

- Motivated by institutional details of various industries, e.g.:
  - Retailer-Manufacturer [Draganska et al. 10, ...]
  - Content Distribution [Crawford Yurukoglu 12, Crawford et al. 15, ...]
  - Health Care [Grennan 13, Gowrisankaran et al. 15, Ho Lee 16]

- Empirical literature often restricts attention to surplus division for a fixed “network” and either linear or lump-sum contracts; has used “simple” solution concepts, which includes the “Nash-in-Nash” bargaining solution

- Related to theory literature on vertical contracting
  [Hart Tirole 90, O’Brien Schaffer 92, McAfee Schwarz 94, Segal 99, Rey Vergé 04, ...]
“Nash-in-Nash” Bargaining Solution: Example

- $D$ needs to contract w/ upstream firms to generate surplus
  - E.g., say $D$ represents DirecTV and $U_1$ and $U_2$ are channels (CNN, Fox News)

- How to determine $p_1$ and $p_2$?
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- How to determine \( p_1 \) and \( p_2 \)?

- Nash-in-Nash solution:

\[
p_1 = p_2 = \arg \max_p [(10 - 8) - p] \times [p] = 1
\]

Total payments are \( p_1 + p_2 = 2 \)
“Nash-in-Nash” Bargaining Solution: Example

Can be used to evaluate what happens if $U_1$ and $U_2$ merge.

$U_M$ $U_M$

$D$ $0$

$D$ $10$

$P_M$
"Nash-in-Nash" Bargaining Solution: Example

Can be used to evaluate what happens if $U_1$ and $U_2$ merge

Nash-in-Nash:

$$p^M = \arg \max_p [(10 - 0) - p] \times [p] = 5$$
Can be used to evaluate what happens if $U_1$ and $U_2$ merge

**Nash-in-Nash:**

$$p^M = \arg\max_p [(10 - 0) - p] \times [p] = 5$$

Can nest take-it-or-leave-it (TIOLI) predictions w/ asymmetric weights:

- If $D$ makes TIOLI offers, payments are 0 both pre- and post-merger
- If upstream firms make TIOLI offers, payments increase from 4 to 10
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  - If $D$ makes TIOLI offers, payments are 0 both pre- and post-merger
  - If upstream firms make TIOLI offers, payments increase from 4 to 10

- Simplicity and tractability has been appealing for applied work
“Nash-in-Nash” Bargaining Solution: Previous Motivations

- “Nash equilibrium in Nash bargains”
  - Each bilateral bargain represented by an agent, and each agent simultaneously maximizes its respective bargain’s Nash product
  - Form of *contract equilibrium* (Cremer Riordan 87)

A Non-Cooperative Interpretation?
Horn Wolinsky 88 proposed simultaneous Nash bargains as a way to examine horizontal merger incentives with exclusive vertical relationships.

“We model the outcomes...by using the formula of a Nash bargaining solution...although this will not be part of the formal model, it will sometimes be useful to think of the static model...as the reduced form of an appropriate dynamic bargaining model.”

Delegated Agent Representations
[Chipty Snyder 99, Björnerstedt Stennek 07, Inderst Montez 14]

E.g., Crawford Yurukoglu 12 motivate approach as:
“Each distributor and each conglomerate sends separate representatives to each meeting. Once negotiations start, representatives of the same firm do not coordinate with each other. We view this absence of informational asymmetries as a weakness of the bargaining model.”
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This Paper: Provides Support for Nash-in-Nash

- Proposes an extensive form bargaining game that extends Rubinstein (1982) to bilateral oligopoly settings
  - Allow firms to engage in and coordinate across multiple bargains at once
- Provides conditions under which:
  - There exists equilibria that result in Nash-in-Nash outcomes
  - Necessary & Sufficient conditions for "Rubinstein" prices
  - Sufficient conditions for prices that still "converge to" Nash-in-Nash
  - Conditions limit extent of complementarities across agreements
- Equilibrium outcomes are unique
- No-Delay Equilibria
  - Add'l Sufficient Conditions (also limit negative externalities across agreements)
- Provides guidance for when Nash-in-Nash may be appropriate
- Contributes to Nash program of cooperative/non-cooperative analysis of bargaining games
- Simple, tractable solution concept useful for applied work [E.g., Möllers Normann Snyder (2016)]
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- Provides conditions under which:
  - There **exists** equilibria that result in Nash-in-Nash outcomes
    - Necessary & Sufficient conditions for “Rubinstein” prices
    - Sufficient conditions for prices that still “converge to” Nash-in-Nash
    - *Conditions limit extent of complementarities across agreements*

- Equilibrium outcomes are **unique**
  - No-Delay Equilibria
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- Provides guidance for when Nash-in-Nash may be appropriate
  - Contributes to *Nash program* of cooperative/non-cooperative analyses of bargaining games
  - *Simple, tractable solution concept useful for applied work*
    [E.g., Möllers Normann Snyder (2016)]
Organization of talk

1. Model

2. II. Existence

3. III. Uniqueness

4. IV. Discussion and Conclusion
I. Model
Primitives:

- **Firms**: Upstream $U_1, \ldots, U_N$, Downstream $D_1, \ldots, D_M$

- **Feasible Agreements/Network**: $G \subseteq \{0, 1\}^{N \times M}$ [Exogenous]

- **Profits**: $\{\pi_i, U(A)\}_{i=1,\ldots,N; A \subseteq G}$ and $\{\pi_j, D(A)\}_{j=1,\ldots,M; A \subseteq G}$

- **Discounting**: $\delta \equiv \exp(-r \Lambda)$ and $\Lambda$ is period length 
  [Paper has firm-specific discount factors & asymmetric “bargaining weights”]
I. Model: Primitives and Notation

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- Firms: Upstream $U_1, \ldots, U_N$, Downstream $D_1, \ldots, D_M$
- Feasible Agreements/Network $G \subseteq \{0, 1\}^{N \times M}$ [Exogenous]
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  [Paper has firm-specific discount factors & asymmetric “bargaining weights”]

Notation:
- $\Delta \pi_{i, U}(A, B) \equiv \pi_i, U(A) - \pi_i, U(A \setminus B)$ for $B \subseteq A$
  [“Marginal contribution to $U_i$ of $B$ at $A$”]
- $A_{i, U}$ are agreements in $A$ involving $U_i$ (Similarly for $A_{j, D}$)
I. Model: Timing

Infinite horizon game with discrete periods $t = 1, 2, \ldots$.

Let $\mathcal{A}^{t-1} \subseteq \mathcal{G}$ denote agreements formed by period $t$. 
I. Model: Timing

Infinite horizon game with discrete periods $t = 1, 2, \ldots$.

Let $A^{t-1} \subseteq G$ denote agreements formed by period $t$.

In each period $t$:
- If $t$ is odd:
  - Each downstream firm $D_j$ simultaneously makes private offers $p_{ij}$ (lump-sum) to all upstream firms $U_i$ s.t. $ij \notin A^{t-1}$.
  - Each upstream firm $U_i$ accepts any subset of received offers.
- If $t$ is even:
  - Upstream firms make offers and downstream firms accept/reject.

For each agreement $ij \in A^t$, $D_j$ pays $U_i$ the amount $p_{ij}$.
Each $D_j$ receives $(1-\delta)\pi_j$, $D(A^t)$ and each $U_i$ receives $(1-\delta)\pi_i$, $U(A^t)$.

All offers and acceptances are revealed.
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  - Each upstream firm $U_i$ accepts any subset of received offers.
- If $t$ is even:
  - Upstream firms make offers and downstream firms accept/reject.
  - $\mathcal{A}^t \equiv \mathcal{A}^{t-1} \cup a^t$, where $a^t$ are the set of agreements accepted at $t$.
  - For each agreement $ij \in a^t$, $D_j$ pays $U_i$ the amount $p_{ij}$
  - Each $D_j$ receives $(1 - \delta)\pi_{j,D}(\mathcal{A}^t)$ and each $U_i$ receives $(1 - \delta)\pi_{i,U}(\mathcal{A}^t)$.
- All offers and acceptances are revealed.
I. Model: Timing & Payoffs Example

\[
\begin{align*}
&\text{Total payoffs to } D \text{ (in period } t = 1 \text{ units):} \\
&\quad (1 - \delta) \pi_D(\emptyset) + \delta \left[ (1 - \delta) \pi_D\left(\{1\}\right) - p_1 \right] + \delta^2 \left[ (1 - \delta) \pi_D\left(\{1, 2\}\right) - p_2 \right] + \ldots
\end{align*}
\]

\[
\begin{align*}
&\text{Total payoffs to } U_1 \text{ (in period } t = 1 \text{ units):} \\
&\quad (1 - \delta) \pi_1(U_1, U_2)(\emptyset) + \delta \left[ (1 - \delta) \pi_1(U_1, U_2)\left(\{1\}\right) + p_1 \right] + \delta^2 \left[ \pi_1(U_1, U_2)\left(\{1, 2\}\right) \right] + \ldots
\end{align*}
\]
I. Model: Timing & Payoffs Example

Total payoffs to $D$ (in period $t = 1$ units):

$$(1 - \delta)\pi_D(\emptyset) + \delta \left[ (1 - \delta)\pi_D(\{1\}) - p_1 \right] + \delta^2 \left[ (1 - \delta)\pi_D(\{1, 2\}) - p_2 \right]$$

$$+ \delta^3 \left[ (1 - \delta)\pi_D(\{1, 2\}) \right] + \ldots$$
I. Model: Timing & Payoffs Example

Total payoffs to $D$ (in period $t = 1$ units):

$$(1 - \delta)\pi_D(\emptyset) + \delta [(1 - \delta)\pi_D(\{1\}) - p_1] + \delta^2 [(1 - \delta)\pi_D(\{1, 2\}) - p_2]$$

$$+ \delta^3 [(1 - \delta)\pi_D(\{1, 2\})] + \ldots$$

$$= (1 - \delta)\pi_D(\emptyset) + \delta [(1 - \delta)\pi_D(\{1\}) - p_1] + \delta^2 [\pi_D(\{1, 2\}) - p_2]$$
I. Model: Timing & Payoffs Example

Total payoffs to $D$ (in period $t = 1$ units):

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$$= (1 - \delta)\pi_D(\emptyset) + \delta \left[ (1 - \delta)\pi_D(\{1\}) - p_1 \right] + \delta^2 \left[ \pi_D(\{1, 2\}) - p_2 \right]$$

Total payoffs to $U_1$ (in period $t = 1$ units):

$$(1 - \delta)\pi_{1,U}(\emptyset) + \delta \left[ (1 - \delta)\pi_{1,U}(\{1\}) + p_1 \right] + \delta^2 \left[ \pi_{1,U}(\{1, 2\}) \right]$$
I. Model: Equilibrium and Maintained Assumptions

- Strategies: offers (when proposing); acceptances (when receiving)
  - No stationarity restriction on strategies: can condition on full history $h^t$

- Equilibrium Concept: PBE w/ passive beliefs
  
  *When a firm receives an out-of-equilibrium price offer following any history of play $h^t$, the firm does not update its beliefs over any unobserved actions in the current period.*

  [c.f. Hart Tirole 90, McAfee Schwarz 94, Rey Vergé 04]
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  [c.f. Hart Tirole 90, McAfee Schwarz 94, Rey Vergé 04]

- Maintained assumptions:
  - Payments are non-contingent lump-sum transfers
  - Assumption Gains From Trade (A.GFT):
    \[
    \Delta \pi_{j,D}(G, \{ij\}) + \Delta \pi_{i,U}(G, \{ij\}) > 0 \quad \forall ij \in G
    \]
    Often agreements in $G$ have been observed to or are expected to form
    (e.g., announced in a prior network formation stage as in Lee Fong 13)
1. Model: Nash-in-Nash Prices

Nash-in-Nash prices $p^{Nash} \equiv \{p_{ij}\}_{i=1,...,N;j=1,...,M}$

\[
p_{ij}^{Nash} = \arg \max_p [\Delta \pi_j, D(G, \{ij\}) - p]^{b_j, D} \times [\Delta \pi_i, U(G, \{ij\}) + p]^{b_i, U}
\]

\[
= \frac{\Delta \pi_j, D(G, \{ij\}) - \Delta \pi_i, U(G, \{ij\})}{2} \quad \text{when } b_i, U = b_j, D
\]

- Adaptation of Horn Wolinsky (1988) to $N \times M$ case w/ lump-sum transfers
- Payment based on “marginal contributions of $ij$ at $G$”: i.e., firms split their “gains-from-trade” given all other agreements form
I. Model: Rubinstein Prices

"Rubinstein" prices $p^R$ (with common discount factor)

Odd period:

$$p^R_{ij,D} = \frac{\delta \Delta \pi^D_j(G, \{ij\}) - \Delta \pi_{i,U}(G, \{ij\})}{1 + \delta}$$

Even period:

$$p^R_{ij,U} = \frac{\Delta \pi^D_j(G, \{ij\}) - \delta \Delta \pi_{i,U}(G, \{ij\})}{1 + \delta}$$

- Corresponds to Rubinstein (1982) SPE prices for $N = M = 1$
- $p^R_{ij,D} < p^Nash_{ij} < p^R_{ij,U}$
- **Lemma 2.2:** Prices converge to Nash-in-Nash prices as $\Lambda \to 0$.
  [Binmore, Rubinstein, and Wolinsky (1986)]
II. Existence
II. Existence Results: Overview

First set of results concerns the existence of a *Nash-in-Nash limit equilibrium*:

For any $\varepsilon > 0$, there exists $\bar{\Lambda} > 0$ s.t. $\forall \Lambda \in (0, \bar{\Lambda}]$, there is an equilibrium with complete agreements at prices within $\varepsilon$ of Nash-in-Nash prices.

We provide:

1. A necessary and sufficient condition on *decreasing marginal contributions of agreements* for there to exist an equilibrium where all open agreements immediately form at Rubinstein prices.

2. Sufficient conditions for there to exist a Nash-in-Nash limit equilibrium
   - Admits complementarities on one “side” of the market
II. Existence (1): Necessary and Sufficient Condition

A.WCDMC: Weak Conditional Decreasing Marginal Contribution

\[ \Delta \pi_{j,D}(G, A) \geq \sum_{hj \in A} \Delta \pi_{j,D}(G, \{hj\}) \quad \forall j = 1, \ldots, M; \forall A \subseteq G_{j,D} \]

(and the same holds for upstream firms)

[Similar to conditions in Stole Zwiebel 96, Westermark 03, Bloch Jackson 07, Hellman 13]

- Marginal contribution of any set of agreements is weakly greater than the sum of the marginal contributions of individual agreements within the set when \( G \) is formed

- Generally satisfied if firms on same side of market are substitutes
  - A.WCDMC is also implied by assuming that the value of an agreement to a firm is lower as additional agreements are signed
II. Existence (1): A.WCDMC Example

- A.WCDMC is satisfied here iff $a \geq 5$:

\[
\pi_{1,D}(\{1, 2\}) - \pi_{1,D}(\emptyset) \geq \Delta \pi_{1,D}(\{1, 2\}, \{1\}) + \Delta \pi_{1,D}(\{1, 2\}, \{2\}) \\
10 \geq (10 - a) + (10 - a)
\]
Theorem 3.2: A.WCDMC is necessary and sufficient for there to exist an equilibrium where at every period $t$ and history $h^t$, all open agreements are immediately formed at Rubinstein prices.

Intuition: eq. strategies are always offer & accept Rubinstein prices, reject worse than Rubinstein prices; these ensure that agents do not wish to delay agreement.

Consider $D_j$'s gains from accepting $A$ agreements vs. rejecting them (and forming them next period given candidate strategies) in an even period:

$$\text{(1} - \delta)\Delta\pi_j, D(G, A) + \sum_{hj \in A} [\pi_h, U + \delta\pi_h, D]$$

change in $D_j$'s profits from accepting vs. rejecting offers in $A \geq \sum_{hj \in A} [(1 - \delta)\Delta\pi_j, D(G, \{hj\}) - \pi_h, U + \delta\pi_h, D] = 0$

where inequality follows by A.WCDMC, and equality by Rubinstein prices.
II. Existence (1): Result and Intuition

**Theorem 3.2:** A.WCDMC is necessary and sufficient for there to exist an equilibrium where at every period $t$ and history $h^t$, all open agreements are immediately formed at Rubinstein prices.

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- Consider $D_j$'s gains from accepting $A$ agreements vs. rejecting them (and forming them next period given candidate strategies) in an even period:

\[
(1 - \delta)\Delta \pi_{j,D}(G, A) + \sum_{h_j \in A} [-p_{h_j,U} + \delta p_{h_j,D}]
\]

change in $D_j$'s profits from accepting vs. rejecting offers in $A$.
II. Existence (1): Result and Intuition

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- Consider $D_j$’s gains from accepting $A$ agreements vs. rejecting them (and forming them next period given candidate strategies) in an even period:

$$(1 - \delta)\Delta \pi_{j,D}(G, A) + \sum_{hj \in A} [-p_{hj,U}^R + \delta p_{hj,D}^R]$$

change in $D_j$’s profits from accepting vs. rejecting offers in $A$

$$\geq \sum_{hj \in A} [(1 - \delta)\Delta \pi_{j,D}(G, \{hj\}) - p_{hj,U}^R + \delta p_{hj,D}^R] = 0$$

where inequality follows by A.WCDMC, and equality by Rubinstein prices.
II. Existence (1): A.WCDMC Example (Cont’d)

A.WCDMC is violated if \( a < 5 \):

- If \( a < 5 \), complementarities imply that in an even period, downstream firm will prefer rejecting both offers if Rubinstein and coming to agreement next period.

- E.g., if \( a = 0 \), \( p_{i,U}^R > 5 \), and \( D \) will earn negative profits at these prices.
A.WCDMC is violated if $a < 5$:

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- E.g., if $a = 0$, $p_{i,U}^R > 5$, and $D$ will earn negative profits at these prices

- However, there will exist an equilibrium at prices that need not be Rubinstein in each period, but still converge to Nash-in-Nash prices as $\Lambda \to 0$. [Example related to setting in Jun 89]
Moving away from Rubinstein prices expands cases where a Nash-in-Nash outcome can still emerge in equilibrium. Consider alternative set of assumptions:

\[ \Delta \pi_j, D(G, A) \geq \sum_{ij \in A} p_{Nash}ij \quad \forall j = 1, \ldots, M; \quad \forall A \subseteq G \]

Ensures no firm would wish to drop any subset of agreements at Nash-in-Nash prices.

**Theorem 3.4:** A.FEAS is necessary for there to exist a Nash-in-Nash limit equilibrium where all agreements in $G$ immediately form.

A.WCDMC implies A.FEAS.
II. Existence (2): Existence w/ Complementarities

Moving away from Rubinstein prices expands cases where a Nash-in-Nash outcome can still emerge in equilibrium. Consider alternative set of assumptions:

A.FEAS: Feasibility [Related to condition in Stole Zwiebel 96]

\[
\Delta \pi_{j,D}(G,A) \geq \sum_{ij \in A} p_{ij}^{Nash} \quad \forall j = 1, \ldots, M; \forall A \subseteq G_{j,D}
\]

(and the same holds for upstream firms)

- Ensures no firm would wish to drop any subset of agreements at Nash-in-Nash prices
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- Ensures no firm would wish to drop any subset of agreements at Nash-in-Nash prices
- **Theorem 3.4:** A.FEAS is necessary for there to exist a Nash-in-Nash limit equilibrium where all agreements in $G$ immediately form.
- A.WCDMC implies A.FEAS
II. Existence (2): A.FEAS Example #1

- A.FEAS is satisfied here if $a \geq 0$, as will imply that $p_i^{Nash} \leq 5$ and:

$$10 - p_1^{Nash} - p_2^{Nash} \geq 0$$
Now imagine that $D$ needs all 3 upstream firms to generate any surplus.
Now imagine that $D$ needs all 3 upstream firms to generate any surplus.

$D_A$ would prefer no agreement than paying Nash-in-Nash prices to all upstream firms as $p_{1A}^{Nash} = p_{2A}^{Nash} = p_{3A}^{Nash} = 5$.

However, if $D_A$ generated at least a surplus of $10/3$ with 2 upstream firms, then A.FEAS would be satisfied (as then $p_{iA}^{Nash} \leq 10/3$ for all $i$).
II. Existence (2): Existence w/ Complementarities

Moving away from Rubinstein prices expands cases where a Nash-in-Nash outcome can still emerge in equilibrium. Consider alternative set of assumptions:

A.FEAS: Feasibility

A.SCDMC(b): Strong Conditional Decreasing Marginal Contribution

\[
\pi_{j,D}(A \cup B \cup \{ij\}) - \pi_{j,D}(A' \cup B) \geq \Delta \pi_{j,D}(G, \{ij\})
\]

\[
\forall ij \in G; B \subseteq G_{-i,u}; A, A' \subseteq G_{i,u} \setminus \{ij\}
\]

[A.SCDMC(a) is same assumption for upstream firms]

- Requires that inframarginal contribution to a firm of a link is greater than marginal contribution at full network, even if counterparty adjusts
- Stronger than A.WCDMC (on one side)
II. Existence (2): Existence w/ Complementarities

**Theorem 3.7:** A.FEAS and either A.SCDMC(a) or A.SCDMC(b) is sufficient for there to exist a Nash-in-Nash limit equilibrium.
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**Intuition:** Assume that upstream firms’ profits satisfy A.SCDMC and downstream firms’ profits satisfy A.FEAS.

- Odd periods (downstream proposing): Equilibrium offers are Rubinstein
- Even periods (upstream proposing): Equilibrium offers are lower than Rubinstein, as upstream firms need to insure that downstream firms do not wish to reject multiple offers.
III. Uniqueness
II. Uniqueness Results: Overview

Second set of results concerns the uniqueness of *Nash-in-Nash* outcomes.

1. We prove that all equilibria where all agreements are formed without delay have equilibrium prices arbitrarily close to Nash-in-Nash prices as $\Lambda \to 0$.

2. We provide sufficient conditions for any equilibrium to result in all agreements being formed immediately at Rubinstein prices.
Theorem 4.1: For any $\varepsilon > 0$, there exists $\bar{\Lambda} > 0$ s.t. $\forall \Lambda \in (0, \bar{\Lambda}]$, any no-delay equilibrium—i.e., an equilibrium in which all open agreements at any history immediately form—has prices at every period $t$ and history $h^t$ that are within $\varepsilon$ of Nash-in-Nash prices.
III. Uniqueness (1): No-Delay Result

**Theorem 4.1:** For any $\varepsilon > 0$, there exists $\bar{\Lambda} > 0$ s.t. $\forall \Lambda \in (0, \bar{\Lambda}]$, any *no-delay equilibrium*—i.e., an equilibrium in which all open agreements at any history immediately form—has prices at every period $t$ and history $h^t$ that are within $\varepsilon$ of Nash-in-Nash prices.

- Does not require *any* assumptions on underlying payoffs
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- Does not require *any* assumptions on underlying payoffs
- Similar to restrictions used in literature:
  E.g., Ray Vohra (2015): in complete information games, delays are “more artificial [than in incomplete information games] and stem from two possible sources...a typical folk theorem-like reason...[and in] protocols that are sensitive to the identity of previous rejectors.”
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- Intuition for result:
  - Receiving firm cannot obtain a worse price: it is better off rejecting a single offer and coming to agreement next period.
  - Proposing firm cannot obtain a worse price: if time periods are sufficiently small, it is better off “withdrawing” (potentially induce multiple rejections) and coming to agreement next period.
With stronger conditions, all equilibria are no-delay (and yield Rubinstein prices).
III. Uniqueness (2): Stronger Conditions

With stronger conditions, all equilibria are no-delay (and yield Rubinstein prices).

A. LNEXT: Limited Negative Externalities

For all non-empty $\mathcal{A} \subseteq \mathcal{G}$, there exists $ij \in \mathcal{A}$ s.t.

$$\Delta \pi_i, U(\mathcal{G}, \mathcal{A}) \geq \sum_{ik \in \mathcal{A}_{i,u}} \Delta \pi_i, U(\mathcal{G}, \{ik\}) \text{ and}$$

$$\Delta \pi_j, D(\mathcal{G}, \mathcal{A}) \geq \sum_{hj \in \mathcal{A}_{j,d}} \Delta \pi_j, D(\mathcal{G}, \{hj\})$$

- Requires for any subset of agreements $\mathcal{A}$ that may not yet have formed, there is some pair $ij \in \mathcal{A}$ such that for each firm in the pair, having all remaining agreements form is sufficiently beneficial.
- Paired with A.SCDMC, limits the degree of negative externalities imposed on at least one pair of firms by other agreements not involving them

[Implied by Bloch Jackson 07 nonnegative externalities condition]
Assume that $G = \{1A, 2B\}$ and $\pi_{1,D}(\emptyset) = \pi_{2,D}(\emptyset) = 0$

III. Uniqueness (2): Violation of A.LNEXT

Nash-in-Nash prices are $\frac{1}{2}$.

A.LNEXT is violated for $A = G$; e.g., $\pi_{A,D}(\{1A, 2B\}) - \pi_{A,D}(\emptyset) \nless -9 -0 \nless 1$.

Thus, there exists an equilibrium (starting at empty network) where no agreements are ever formed. [prisoners' dilemma]
Assume that $\mathcal{G} = \{1A, 2B\}$ and $\pi_{1,D}(\emptyset) = \pi_{2,D}(\emptyset) = 0$

- Nash-in-Nash prices are $1/2$.
- A.LNEXT is violated for $\mathcal{A} = \mathcal{G}$; e.g.,

$$\pi_{A,D}(\{1A, 2B\}) - \pi_{A,D}(\emptyset) \neq \Delta \pi_{A,D}(\{1A, 2B\}, \{1A\})$$

$$-9 - 0 \neq 1$$

Thus, there exists an equilibrium (starting at empty network) where no agreements are ever formed. [prisoners’ dilemma]
III. Uniqueness (2): Result

**Theorem 4.3:** Assume A.SCDMC(a)+(b) and A.LNEXT. For any $\Lambda > 0$, every common tie-breaking equilibrium results in all open agreements at every period $t$ and history $h^t$ immediately forming at Rubinstein prices.

- Common tie-breaking equilibrium: at any history of play $h^t$, if any receiving firm has the same set of best responses for two sets of offers, it chooses the same best response. [Can be relaxed if A.LNEXT is strengthened]
III. Uniqueness (2): Intuition for Proof

Induction on the set of open agreements $C$:

*if any subgame that begins with fewer open agreements than $C$ results in immediate agreement at Rubinstein prices, then any subgame with $C$ open agreements also results in immediate agreement at Rubinstein prices.*
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- Simultaneity of Agreements at Rubinstein Prices:
  - A.SCDMC: if any subset of \( C \) is formed, all agreements in \( C \) are formed.
  - Rubinstein (82) / Shaked Sutton (84) type arguments used to show agreement must be at Rubinstein prices.
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- Immediacy of Agreement:
  - A.LNEXT rules out delay: always one pair of firms that wish to receive / pay Nash-in-Nash prices for their remaining agreements even if other agreements not involving them form.
IV. Discussion and Conclusion
Discussion

Connections to Applied Literatures:

- Wage bargaining: typically has assumed a single firm bargaining w/ multiple workers, profits accruing only to firm, lump-sum payments, & no externalities
  - W/ substitutable workers, our uniqueness + existence results typically apply
  - Extends results to M x N settings (w/ differentiated firms)
Connections to Applied Literatures:

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- IO / Vertical Market Settings:
  - Declining Marginal Contributions often satisfied when upstream firms are substitutable (though also depends on downstream firm competition)
  - Limited Negative Externalities satisfied when markets are separable and served by monopolists (e.g., Chipty Snyder 99), upstream firms earn zero profits (as above), or upstream firms are reimbursed marginal costs alongside lump sum payments (as in Capps et. al 03)
  - Hospital-insurer and content distribution bargaining are recent prominent applications, but insights are also relevant to many B2B interactions
Other Bargaining Protocols

- Allowing for renegotiation or contingent contracts implies that surplus division is no longer a function of *marginal contributions* alone
  [Stole Zweibel 96, Raskovich 03, de Fontenay Gans 14,...]
Discussion

Other Bargaining Protocols

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  [Stole Zweibel 96, Raskovich 03, de Fontenay Gans 14,...]

Richer Contract Spaces

- Different contract spaces (e.g., linear fees) can imply profits depend on contracts that have been signed
  
  - Contract equilibrium (Cremer Riordan 87)
  - Delegated agent representation in applied work
  - Open Q: do similar results extend to these other environments?
“Nash-in-Nash” surplus division can emerge as an equilibrium outcome from a generalization of Rubinstein (1982)’s alternating offers bargaining game

1. Predicts surplus division for a fixed & given network
2. Establish connections b/w vertical contracting and bargaining literatures (e.g., passive beliefs / alternating simultaneous offers)

Results highlight importance of restrictions on marginal contributions

Uniqueness results demonstrate robustness along certain dimensions

- Does not require restriction to stationary strategies
Conclusions & Takeaways

1. “Nash-in-Nash” surplus division can emerge as an equilibrium outcome from a generalization of Rubinstein (1982)’s alternating offers bargaining game
   - Predicts surplus division for a fixed & given network
   - Establish connections b/w vertical contracting and bargaining literatures (e.g., passive beliefs / alternating simultaneous offers)

2. Results highlight importance of restrictions on marginal contributions

3. Uniqueness results demonstrate robustness along certain dimensions
   - Does not require restriction to stationary strategies

4. Open Q:
   - With complete agreement & delay, or with weaker assumptions on payoffs, do all equilibria have prices arbitrarily close to Nash-in-Nash?