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Introduction:

- Bilateral bargaining between upstream and downstream firms pervasive in many economic environments
  - In 2013, private insurers in the U.S. paid $348 billion to hospitals and $267 billion to physicians and clinics
  - These payments are determined neither by perfect competition, nor by take-it-or-leave-it offers (à la Bertrand)
  - Predominantly determined through bilateral negotiations
- This paper considers bilateral oligopoly
  - Sectors with multiple *upstream* and *downstream* firms
- Substantial theoretical literature on bilateral oligopoly, examining, e.g.:
  - Network environments and efficiency, Kranton and Minehart, 2001
  - Network centrality and pricing, Manea, 2011
- Concurrently, an applied literature has examined:
  - Horizontal mergers, Chipty and Synder, 1999
  - Vertical integration, Crawford et al., 2015
  - Hospital competition, Gowrisankaran, Nevo, and Town, 2015
  - The literature is influencing antitrust and regulatory policy
    - Cable mergers, hospital mergers, …
The two network literatures have proceeded in different directions

- Theoretical literature has focused on equilibrium properties
  - Often assumed simple underlying structure to payoffs
  - E.g., value of an agreement between two firms is independent of agreements formed by other parties
  - Developed analytically tractable models for complex network environments

- Applied literature allowed more general externalities across links
  - These are often fundamental in an applied setting
    - Why do hospitals merge?
    - Removing a second hospital from insurer network is worse than removing the first one
    - This bargaining leverage generates higher prices if the hospitals merge
    - It also creates interdependencies across links
  - As a tradeoff, literature focuses on simple solution concepts
    - Most commonly used: the “Nash-in-Nash” bargaining solution
The Nash-in-Nash bargaining solution

- Developed by Horn and Wolinsky (1988)
- Specifies a transfer price for each upstream-downstream pair
  - Each transfer price is the Nash bargaining solution fixing all other links
- Solution nests bilateral Nash bargains within a Nash equilibrium
  - Hence, we have named it “Nash-in-Nash”
- Criticism: ad hoc solution that nests a cooperative game theory concept within a non-cooperative equilibrium
- Previous non-cooperative rationalizations exist, but unsatisfying:
  
  *Each distributor and each conglomerate sends separate representatives to each meeting. Once negotiations start, representatives of the same firm do not coordinate with each other. We view this absence of informational asymmetries as a weakness of the bargaining model.*

  – Crawford and Yurukoglu (2012)
This paper

- Provides support for Nash-in-Nash as a viable surplus division rule
  - Develops a simple extensive form bargaining game
  - Game extends Rubinstein (1982) to bilateral oligopoly

- We prove two main results
  1. Existence: gives necessary and sufficient conditions for existence of equilibrium with “Rubinstein” prices
  2. Uniqueness: stronger conditions for these equilibrium prices to be unique

- What conditions do we need?
  - We focus on case where each link adds value at the margin
  - For existence, we need *declining contribution* type of assumption:
    - Adding a link at the margin adds weakly less value than adding the same link at an inframarginal point
  - For uniqueness, also need to limit externalities in some way
    - E.g., negative externalities can lead rational firms to not make offers if other offers are not happening
    - This then leads to multiple equilibria, some with offers, others without
  - We impose no stationarity conditions, such as Markov-Perfect Equilibrium
    - Both lack of knowledge about contracts to other firms and belief restrictions on off-equilibrium-path nodes play a role here
Remainder of talk

Background

Model

Existence

Uniqueness

Conclusions
Price formation in bilateral oligopolies with externalities

- Consider an industry where there are three upstream content providers and one downstream firm
- The marginal cost of content provision is 0
- There are declining (but positive) returns to adding more content
- We’ll consider price determination before and after mergers
- “Content” could be hospitals or TV channels
  - We’ll focus on TV channels for now
**Cable TV example**

\[ \pi_{TW}(D) = 0.5 \quad \pi_{TW}(HBO) = 0.5 \quad \pi_{TW}(S) = 0.5 \]

\[ \pi_{TW}(D, S) = 0.9 \quad \pi_{TW}(D, HBO) = 0.8 \quad \pi_{TW}(HBO, S) = 0.8 \]

\[ \pi_{TW}(D, HBO, S) = 1 \]
Take-it-or-leave-it offers

Consider a merger between all three upstream firms

- Suppose that downstream firm (Time Warner Cable) makes take-it or leave-it offers
  - Prices before merger
    \[ p_D = 0, \quad p_{HBO} = 0, \quad p_S = 0 \]
  - Prices after merger
    \[ p_D = 0, \quad p_{HBO} = 0, \quad p_S = 0 \]

- Suppose that upstream firms make take-it or leave-it offers
  - Bertrand equilibrium prices before merger
    \[ p_D = 0.2, \quad p_{HBO} = 0.1, \quad p_S = 0.2 \]
    (Price for bundle: 0.5)
  - Bertrand equilibrium bundle price after merger
    \[ p_D + p_{HBO} + p_S = 1 \]

Conclusion: both cases seem to provide lopsided results
- Merged upstream firm either gets no surplus or all surplus!
Nash-in-Nash prices for cable TV example

▶ Vector of prices that maximizes Nash bargaining product given all other agreements form

▶ Consider determination of \( p^{Nash}_{HBO} \)

\[
p^{Nash}_{HBO} = \arg \max_p \left\{ \left[ \pi_{TW}(D, HBO, S) - \pi_{TW}(D, S) - p \right]^{b_{TW}} \times [p]^{b_{HBO}} \right\}
\]

▶ If discount factors satisfy \( b_{TW} = b_{HBO} \),

\[
p = \frac{1}{2} \left[ \pi_{TW}(D, HBO, S) - \pi_{TW}(D, S) \right]
\]

▶ The marginal contribution is split evenly between the two parties
Effects of cable merger with Nash-in-Nash prices

\[ \pi_{TW}(D) = 0.5 \quad \pi_{TW}(HBO) = 0.5 \quad \pi_{TW}(S) = 0.5 \]
\[ \pi_{TW}(D, S) = 0.9 \quad \pi_{TW}(D, HBO) = 0.8 \quad \pi_{TW}(HBO, S) = 0.8 \]
\[ \pi_{TW}(D, HBO, S) = 1 \]

- We focus on case where bargaining weights are equal
- What are pre-merger prices?
  \[ p_{D}^{Nash} = 0.1, \quad p_{HBO}^{Nash} = 0.05, \quad p_{S}^{Nash} = 0.1 \]
  (Price for bundle: 0.25)
- Say HBO and Disney merge. What prices do they receive?
  \[ p_{D+HBO}^{Nash} = \frac{1}{2} [\pi_{TW}(D, HBO, S) - \pi_{TW}(S)] = 0.25 > 0.15 \]
- What about merger to monopoly?
  \[ p_{D+HBO+S}^{Nash} = 0.5 \]
- Overall, predictions of Nash-in-Nash seem pretty intuitive
Nash-in-Nash prices not always reasonable

What are Nash-in-Nash prices? What are surpluses?

- Underlying problem is increasing marginal contributions!
Firms and payoffs

- $N$ Upstream firms $U_1, \ldots, U_N$
- $M$ Downstream firms $D_1, \ldots, D_M$
- $\mathcal{G}$ is set of all agreements
- Primitives:
  - Profits $\pi^U_i(\mathcal{A})$ and $\pi^D_j(\mathcal{A})$, $\forall \mathcal{A} \subseteq \mathcal{G}$
  - Discount rates $r_{i,U}$, $r_{j,D}$
  - Period length $\Lambda$
    - Leads to discount factor between periods of $\exp(-r_{i,U}\Lambda)$, $\exp(-r_{j,D}\Lambda)$
- $U_i$ and $D_j$ bargain over $p_{ij}$
  - WLOG, price is paid from downstream to upstream firm
  - Can be negative (and will be if profits accrue mostly to upstream firms)
Nash-in-Nash prices for our model

Definition:
\[
\Delta \pi_j^D(A, B) \equiv \pi_j^D(A) - \pi_j^D(A \setminus B), \quad \text{for} \quad B \subseteq A \subseteq G
\]

Measures change in profits from dropping agreements in \(B\)

Define \(\Delta \pi_j^D(A, B)\) analogously

Nash-in-Nash prices, for \(i = 1, \ldots, N, j = 1, \ldots, M\)

\[
p_{ij}^{Nash} = \arg \max_p \left[ \pi_j^D(G) - \pi_j^D(G \setminus \{ij\}) - p \right]^{b_{j,D}} \times \left[ \pi_i^U(G) - \pi_i^U(G \setminus \{ij\}) + p \right]^{b_{i,U}}
\]

\[
= \frac{b_{i,U} \Delta \pi_j^D(G, \{ij\}) - b_{j,D} \Delta \pi_i^U(G, \{ij\})}{b_{i,U} + b_{j,D}}
\]

Generalization of Horn Wolinsky (1988) to \(N \times M\) case
Timing of extensive-form game

Start at period $t_0$ with some set of open agreements $\mathcal{A}_t^0 \subseteq \mathcal{G}$

1. Each odd (even) period, each downstream (upstream) firm simultaneously makes private offers $p_{ij}$ for its open agreements
2. Each receiving firm sees its offers, and simultaneously accepts/rejects any subset
3. Each $U_i$ receives $p_{ij}$ from $D_j$ for its accepted offers
4. Each $D_j$ receives flow payoff $(1 - \delta_{j,D})\pi_j^D(\mathcal{A}_t)$ (and each $U_i$ receives $(1 - \delta_{i,U})\pi_i^U(\mathcal{A}_t)$), where $\mathcal{A}_t$ are agreements formed by period $t$
   - Equivalent to assume that game terminates when all agreements are formed, with payoffs $\pi_j^D(\mathcal{G})$ and $\pi_i^D(\mathcal{G})$ at this point
5. All offers and acceptances are revealed
   - All results would hold if we assume that only acceptances are revealed
Timing

D → P₁ → U₁

P₂ → U₂

P₃ → U₃

t=1

D → P₁ → U₁

P₂ → U₂

P₃ →Accepted

U₃

t=2

Π(3) (1-δ)

D → Accepted

U₁

P₁ → U₁

P₂ → U₂

t=3

Π({1,2,3})

D → U₁

U₂

U₃

t=4
Assumptions for existence

Equilibrium concept

Perfect Bayesian Equilibrium (PBE) with passive beliefs

Assumption A.GFT: Gains From Trade

$$\Delta \pi^D_j(G, \{ij\}) + \Delta \pi^U_i(G, \{ij\}) > 0 \quad \forall i, j$$

Assumption A.WCDMC: Weak Conditional Decreasing Marginal Contribution

$$\Delta \pi^D_j(G, \mathcal{A}) \geq \sum_{hj \in \mathcal{A}} \Delta \pi^D_j(G, \{hj\}),$$

and the same holds for upstream firms
Declining marginal contribution

- **Time-Warner example** Consider Disney’s contribution:
  - Full network marginal contribution: \( \Delta \pi_{TW}(\{D, HBO, S\}, \{D\}) = \pi_{TW}(D, HBO, S) - \pi_{TW}(HBO, S) = 1 - 0.8 = 0.2 \)
  - Subnetwork marginal contributions:
    \[
    \begin{align*}
    \Delta \pi_{TW}(\{D, S\}, \{D\}) &= 0.9 - 0.5 = 0.4 \\
    \Delta \pi_{TW}(\{D, HBO\}, \{D\}) &= 0.8 - 0.5 = 0.3 \\
    \Delta \pi_{TW}(\{D\}, \{D\}) &= 0.5 - 0 = 0.5
    \end{align*}
    \]
- **Note**: the hospital merger model of Capps, Dranove, Satterthwaite (2003) satisfies A.WCMDC. Value of hospital network \( \mathcal{H} \) is the inclusive value for all patients \( i \):
  \[
  \pi_j^D(\mathcal{H}) = \sum_i \log \left( \sum_{k \in \mathcal{H}} \exp(X_k \beta_i) \right)
  \]
Decreasing marginal contribution: violations

Auto supplier example:
- Subnetwork marginal contributions:
  - $\Delta \pi_{Ford}(\{B, E\}, \{B\}) = \Delta \pi_{Ford}(\{B, C\}, \{B\}) = \Delta \pi_{Ford}(\{B\}, \{B\}) = 0$
- Full network marginal contribution:
  - $\Delta \pi_{Ford}(\{B, E, C\}, \{B\}) = 1$
- This model exhibits increasing marginal contributions
Candidate prices for equilibrium

Definition of “Rubinstein” prices

Odd period:

\[
p_{ij,D}^R = \frac{\delta_{i,U}(1 - \delta_{j,D})\Delta \pi_j^D(G, \{ij\}) - (1 - \delta_{i,U})\Delta \pi_i,U(G, \{ij\})}{1 - \delta_{i,U}\delta_{j,D}}
\]

Even period:

\[
p_{ij,U}^R = \frac{(1 - \delta_{j,D})\Delta \pi_j^D(G, \{ij\}) - \delta_{j,D}(1 - \delta_{i,U})\Delta \pi_i,U(G, \{ij\})}{1 - \delta_{i,U}\delta_{j,D}}
\]

- These are prices from Rubinstein (1982) if \(D_j\) and \(U_i\) are the only firms without agreement.
- Lemma 2.1: Rubinstein prices converge to Nash-in-Nash prices as \(\Lambda \to 0\). (See also Binmore, Rubinstein, and Wolinsky (1986).)
Existence of equilibrium: main result

Recall Nash-in-Nash prices:

\[ p_{ij}^{\text{Nash}} = \frac{b_{i,U} \Delta \pi_j^D(G, ij) - b_{j,D} \Delta \pi_i^U(G, ij)}{b_{i,U} + b_{j,D}}, \forall i = 1, \ldots, N, j = 1, \ldots, M \]

Existence theorem:

Assume A.GFT. Then there exists an equilibrium of the extensive form game for which:

(a) there is immediate agreement between all agents at every node;

(b) for all \( i \) and \( j \), equilibrium prices are \( p_{ij}^* = p_{ij,D}^R \) at each odd period node and \( p_{ij}^* = p_{ij,U}^R \) at each even period node; and

(c) \( p_{ij}^* \rightarrow p_{ij}^{\text{Nash}} \) \( \forall i, j \) as \( \Lambda \rightarrow 0 \) regardless of whether \( t_0 \) is odd or even, where \( b_{i,U} = r_{j,D}/(r_{i,U} + r_{j,D}) \) and \( b_{j,D} = r_{i,U}/(r_{i,U} + r_{j,D}) \);

if and only if A.WCDMC holds.
Intuition of existence proof

Logic for Rubinstein prices (odd period):
Downstream firm’s offer makes the upstream firm just indifferent between accepting all or rejecting one offer:

\[
(1 - \delta_{i,U}) \Delta \pi_i^U (G, \{ij\}) = \delta_{i,U} p_{ij,U}^R - p_{ij,D}^R
\]

(1)

Loss in profit from waiting

Decrease in transfer payment from waiting

Now suppose \( U_i \) is deciding whether to accept some subset \( A \) of offers. Value from accepting them (instead of waiting until next period) is:

\[
(1 - \delta_{i,U}) \Delta \pi_i^U (G, A) + \sum_{ik \in A} [p_{ik,D}^R - \delta_{i,U} p_{ik,U}^R]
\]

\[
\geq \sum_{ik \in A} [(1 - \delta_{i,U}) \Delta \pi_i^U (G, \{ik\}) + p_{ik,D}^R - \delta_{i,U} p_{ik,U}^R] = 0.
\]

(2)

The inequality follows from A.WCDMC and the inequality from the above.
Intuition of existence proof (continued)

- Odd periods
  - $D_j$ makes offers $p_{ij,D}^R$
  - Each $U_i$ accepts all offers if all offers equal to Rubinstein
  - If one offer is not Rubinstein, $U_i$ accepts all others and rejects just that offer $\iff$ it’s less than Rubinstein
  - If multiple offers are not Rubinstein, plays arbitrary best response

- Even periods
  - Symmetric to odd periods

- Why is this an equilibrium?
  - For receiving firm:
    - With all Rubinstein offers, full acceptance is a best response (per last slide)
    - Other strategies designed to best respond in off-equilibrium-path nodes
  - For proposing firm:
    - Strictly worse off from waiting a period (being a proposer is better)

- Why is this not an equilibrium when A.WCDMC is violated?
  - Find the node at which it’s violated
  - Receiving firm wants to drop the set of offers that violate A.WCDMC

- Convergence to Nash-in-Nash prices follows from Lemma 2.1
Simpler set of assumptions for uniqueness

**Assumption A.CDMC: Conditional Decreasing Marginal Contribution**

\[
\Delta \pi^D_j(A, \{ij\}) \geq \Delta \pi^D_j(G, \{ij\}) \quad \forall ij \in A, \forall A \subseteq G,
\]

and the same holds for upstream firms

**Assumption A.LEXT: Limited Externalities**

\[
\pi^U_i(A \cup B) = \pi^U_i(A \cup B') \quad \forall i; \forall A \subseteq G^U_i; \forall B, B' \subseteq G^U_{-i},
\]

\[
\pi^D_j(A \cup B) = \pi^D_j(A \cup B') \quad \forall j; \forall A \subseteq G^D_j; \forall B, B' \subseteq G^D_{-j}.
\]

where \(G^U_i\) are agreements that involve \(U_i\) and \(G^U_{-i}\) are agreements that don’t
Discussion of A.CDMC

- A.CDMC is stronger than A.WCDMC
  - A.WCDMC: the inframarginal value of a set of agreements is less than the sum of the values of those agreements at the margin
  - A.CDMC: the inframarginal value of any agreement is less than its marginal value

- Consider the following example:
  - 3 upstream firms $U_1, U_2, U_3$; 1 downstream firm $D$; profits only to $D$
  - Upstream firms are symmetric
  - $\pi^D(\{U_1\}) = 0.25, \pi^D(\{U_1, U_2\}) = 0.7, \pi^D(\{U_1, U_2, U_3\}) = 1$
  - The value of adding the first agreement (0.25) is less than the value a firm at the margin (0.3)
    - A.CDMC is violated
  - But, adding 2 agreements gives 0.75 (versus 0.6 at the margin) and adding 3 agreements gives 1 (versus 0.9 at the margin)
    - A.WCDMC holds
Discussion of A.LEXT

- Cable TV is an example of a market that satisfies A.LEXT
  - Content providers have zero marginal costs
  - They receive payoffs only from transfers from cable operators
  - Downstream firms do not compete with each other

- Hospitals are also an example if:
  - MCOs reimburse for marginal costs plus a fixed negotiated fee
  - Hospital disagreement with an MCO does not lead to “spill” back to the hospital from enrollees switching MCOs

- More generally, when profits flow just to downstream firms, A.LEXT
  - allows for competition between upstream firms
  - . . . but not between downstream firms
Sharper set of assumptions for uniqueness

**Assumption A.ASR: Acceptance Strategies Refinement**
Firms accept offers when they are indifferent between accept and reject

**Assumption A.SCDMC: Strong Conditional Decreasing Marginal Contribution**

\[
\pi^D_j (A \cup B \cup \{ij\}) - \pi^D_j (A' \cup B) \geq \Delta \pi^D_j (G, \{ij\})
\]

for all \( ij \in G, \ B \subseteq G^U_{-i}, \) and \( A, A' \subseteq G^U_i \setminus \{ij\}, \)

and the same holds for upstream firms

Interpretation: if \( U_i \) rearranges its acceptances, \( D_j \) still has a higher inframarginal value from accepting \( ij \)
Assumption A.LNEXT: Limited Negative Externalities

∀C ⊆ G, ∃ij ∈ C such that:

\[ \Delta \pi_i^U(G, C) \geq \sum_{ik \in C_i^U} \Delta \pi_i^U(G, \{ik\}) \] and

\[ \Delta \pi_j^D(G, C) \geq \sum_{hj \in C_j^D} \Delta \pi_j^D(G, \{hj\}) \]

where \( C_i^U \) are the agreements in \( C \) that involve \( U_i \) (and analogously for \( C_j^D \))

Interpretation: there is a pair of firms \( U_i \) and \( D_j \) for which the values of all open agreements forming (including those that do not include them) is higher than the sum of the marginal values of their open agreements.
Discussion of A.SCDMC and A.LNEXT

- A.SCDMC is stronger than A.CDMC
  - The value at the margin must be greater than the inframarginal even if \( U_i \) were to rearrange its acceptances; A.CDMC holds other acceptances fixed

- Consider the following “vertical relations” example:
  - One upstream firm \( U \); 2 downstream firms \( D_1 \) and \( D_2 \); profits only to \( D \)
  - Upstream firms are again symmetric
  - \( \pi^D_1(\{D_1\}) = 0.6, \pi^D_1(\{D_2\}) = -100.5, \pi^D_1(\{D_1, D_2\}) = -100 \)
  - A.CDMC holds here
    - The inframarginal value to \( D_1 \) of its link is 0.6; the marginal value is 0.5

- But, A.SCDMC does not hold
  - The value to \( D_1 \) of adding its link – if \( U \) is allowed to change its links – is potentially 101.1 (\( = 0.6 - (-100.5) \)) which is greater than 0.5

- A.LNEXT also does not hold
  - The marginal value to \( D_1 \) of \textit{its own} agreement is 0.5
    - But the value of all agreements is \(-100\) (which is lower than 0.5)
  - For \( 2 \times 2 \) case, examples where A.LNEXT is violated but A.SCDMC holds
## Uniqueness of equilibrium: main result

### Uniqueness theorem:

Assume A.GFT. In addition, assume either (i) A.CDMC and A.LEXT, or (ii) A.SCMDC, A.LNEXT, and A.ASR. Then every equilibrium of the bargaining game satisfies:

(a) immediate agreement between all agents at every node;

(b) for all $i$ and $j$, equilibrium prices are $p_{ij}^* = p_{ij,D}^R$ at each odd period node and $p_{ij}^* = p_{ij,U}^R$ at each even period node; and

(c) $p_{ij}^* \rightarrow p_{ij,Nash}^{Nash} \ \forall i,j$ as $\Lambda \rightarrow 0$
Role of assumptions

Properties of assumptions:

1. A.SCDMC \Rightarrow A.CDMC \Rightarrow A.WCDMC
2. A.CDMC + A.LEXT \Rightarrow A.SCDMC + A.LNEXT

Why do we need A.CDMC (instead of A.WCDMC in existence proof)?
- We need to show that Rubinstein offers are always desirable
- Unlike with existence proof, can’t assume all other offers happen

Why do we need to limit externalities?
- Example with A.SCDMC not holding is illustrative here
  - Best outcome is no agreements
  - But \( D_1 \) prefers an agreement if \( D_2 \) has one
  - Multiple equilibria – similar to repeated prisoner’s dilemma

Why do we need A.ASR if we don’t assume A.LEXT?
- Upon receiving an offer from \( D_1 \), \( U \) might renege on a different offer over which it is indifferent but which is crucial to \( D_1 \)
  - A.ASR rules out the possibility, and A.LEXT rules out \( D_1 \) caring about it
Proof structure

We prove the theorem by induction:

- Consider a set of open agreements $\mathcal{C}$
- Assume theorem holds for all proper subsets $\mathcal{B} \subset \mathcal{C}$
- WTS that theorem holds for $\mathcal{C}$

We have three cases to verify:

1. Base case: $1 \times 1$
   - This is the Rubinstein (1982) case
   - A.ASR is all that is needed here
   - We don’t discuss this further

2. Inductive case with multiple firms on one side and 1 firms on other side
   - Hardest case so we discuss it last
   - WLOG, focus on $N$ upstream firms and 1 downstream firm

3. Inductive case with multiple firms on both sides of market
Flowchart of $M \times N$ proof

- **$N$ upstream $\times M$ downstream**
  - **First agreement at odd period (Lemma D.11)**
    - Claim A – simultaneity:
      - Uses “pulled up” agreements.
      - Requires A.GFT.
    - Claim B – $\hat{p}_{ij} = p_{ij,D}^R$:
      - Uses threat of postponing agreement.
      - Requires A.GFT.
- **First agreement at even period (Lemma D.12)**
  - Symmetric to odd period.
  - Claim A – simultaneity:
    - Uses “pulled up” agreements.
    - Requires A.GFT.
- **Immediacy of agreement (Lemma D.13)**
  - Uses “pulled up” agreements.
  - Requires A.GFT.

Note: Downstream firms propose in odd periods and upstream firms propose in even periods. All parts use A.GFT.
More on $M \times N$ proof

- If first agreement at odd period $t$, all agreements occur simultaneously
  - Proof is by contradiction
  - Suppose some $U_i$ doesn’t accept some offer $ij$
  - By inductive hypothesis, fewer open agreements at $t + 1$
    - $U_i$ knows that all agreements will form at $t + 1$ for $p_{ij,u}^R$
    - Given A.SCDMC, $U_i$ will accept $p_{ij,D}^R + \varepsilon$

- All offers are for Rubinstein prices
  - Consider again proof by contradiction, using the inductive hypothesis
  - If an offer $ij$ is lower than $p_{ij,D}^R$, $U_i$ should reject, counterproposing at $t + 1$
  - If an offer is higher than $p_{ij,D}^R$, $D_j$ can lower the offer
    - $U_i$ will still accept offer, but is this good for $D_j$?
    - $U_i$ might reject some other offer over which it is indifferent
    - By A.LEXT, $D_j$ doesn’t care about this; by A.ASR, $U_i$ won’t do this

- Immediacy of agreements
  - If $D_j$ can sign up even one $U_i$, this will break equilibrium with delay
  - We know potential payoffs with delay from above results
  - Given A.LNEXT and A.SCDMC, $U_i$ will accept $p_{ij,D}^R + \varepsilon$
# Flowchart of Proof $1 \times N$

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$N$ upstream $\times$ 1 downstream</td>
</tr>
<tr>
<td>2.</td>
<td>First agreement at odd period (Lemma D.8)</td>
</tr>
<tr>
<td>4.</td>
<td>Claim B – $\hat{p}<em>{ij} = p</em>{ij,D}^R$: Uses threat of postponing agreement. Requires A.CDMC.</td>
</tr>
<tr>
<td>5.</td>
<td>First agreement at even period (Lemma D.9)</td>
</tr>
<tr>
<td>6.</td>
<td>Claim A: $\hat{p}<em>{ij} \geq p</em>{ij,D}^R$: Uses “explosion” argument. Requires A.LEXT or A.ASR.</td>
</tr>
<tr>
<td>8.</td>
<td>Claim C – $\hat{p}<em>{ij} = p</em>{ij,U}^R$: Requires A.SCDMC and A.ASR or A.LEXT.</td>
</tr>
<tr>
<td>9.</td>
<td>Immediacy of agreement (Lemma D.10)</td>
</tr>
<tr>
<td>10.</td>
<td>Uses proof from Lemma D.13</td>
</tr>
</tbody>
</table>

Note: Downstream firms propose in odd periods and upstream firms propose in even periods. All parts use A.GFT.

Figure 1: Diagram of Proof

Proof from Rubinstein (1982) (Proposition D.6) Requires A.GFT...
More on $1 \times N$ proof

We focus on second bubble (when first agreement is at even period)

- The first and third bubbles are special cases of the $M \times N$ proof
- The second bubble isn’t: can’t apply inductive hypothesis
  - (The unique) $D_j$ could reject all open offers

Three steps to proof here:

1. Show that prices have a lower bound of $p_{ij,D}^R$
   - With A.ASR or A.LEXT, credible rejection of an offer higher than $p_{ij,D}^R$ by $D_j$ requires a credible rejection of a higher offer in the future
   - Eventually leads to contradiction
   - Similar to Rubinstein’s original proof, except that $D_j$’s constraint solved by induction, so we only solve upstream side

2. Show simultaneity of offers
   - Lower bound on prices puts an upper bound on $D_j$’s profits from rejection
   - With A.SCDMC, enough to show result similar to $M \times N$ case

3. Show that offers are exactly $p_{ij,U}^R$
   - Proof very similar to $M \times N$ case
Conclusions

- We provide a non-cooperative foundation for “Nash-in-Nash” solution
  - Our model has quite flexible externalities between parties

- Three general takeaways from our work:
  1. Nash-in-Nash is a well-founded concept that can be generated from an extensive form that generalizes Rubinstein (1982)
     - Provides a microfoundation for the many bilateral oligopoly applied papers that use Nash-in-Nash
  2. Existence results show that Nash-in-Nash is only appropriate for environments with declining returns from additional links
  3. Uniqueness results show that Nash-in-Nash is relatively robust
     - We don’t need any stationarity assumptions
     - Leverage simultaneity and belief restrictions after off-equilibrium actions

- Limitations:
  - We have nothing to say about cases with non-transferable utility
    - Nor does the literature restricted to two-party Rubinstein games
  - We are silent on network formation too: exclusion in particular