"Nash-in-Nash" Bargaining: A Microfoundation for Applied Work

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October 29, 2016
Columbia/Duke/MIT/Northwestern IO Theory Conference

Introduction

Motivation

- Bilateral bargaining in industry is pervasive
  - Health care (over $600B in negotiated US private health care payments)
  - Content distribution deals (e.g., AT&T - Time Warner)
  - Retail - Manufacturer relationships

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- Important to understand the determinants of surplus division
  - Provides incentives for investment, entry, exit, mergers, integration...
  - Enables the evaluation of welfare and competitive effects of: market structure changes (mergers, integration, entry/exit), regulatory interventions (limiting price discrimination, bundling, vertical restraints), ...

Literature on Bilateral Bargaining

Theoretical Approaches: *Focus on analytically tractable models, typically w/ single agent on one side and/or limited externalities. E.g.,*

- Wage Bargaining
  [Davidson 88, Jun 89, Stole Zwiebel 96, Westermark 03, ...]
- Bargaining on Networks
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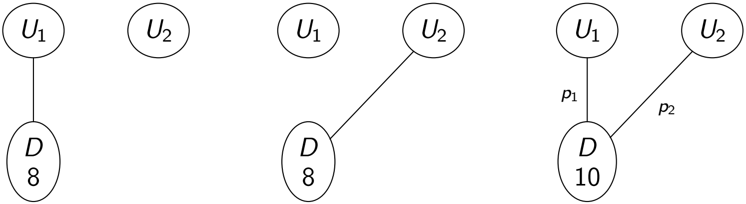
Empirical: *Emphasis on "many-to-many" environments & general externalities*

- Motivated by institutional details of various industries, e.g.:
  - Retailer-Manufacturer [Draganska et al. 10,...]
  - Content Distribution [Crawford Yurukoglu 12, Crawford et al. 15,...]
  - Health Care [Grennan 13, Gowrisankaran et al. 15, Ho Lee 16]
- Empirical literature often restricts attention to surplus division for a fixed "network" and either linear or lump-sum contracts; has used "simple" solution concepts, which includes the "Nash-in-Nash" bargaining solution
- Related to theory literature on vertical contracting
  [Hart Tirole 90, O'Brien Schaffer 92, McAfee Schwarz 94, Segal 99, Rey Vergé 04, ...]

"Nash-in-Nash" Bargaining Solution: Example



- D needs to contract w/ upstream firms to generate surplus
  - E.g., say D represents DirecTV and U_1 and U_2 are channels (CNN, Fox News)
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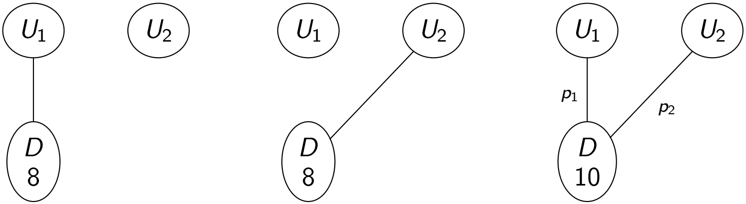

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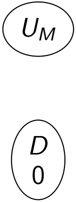
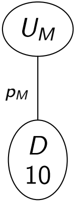
- How to determine p_1 and p_2?
- Nash-in-Nash solution:

$$p_1 = p_2 = \arg\max_p [(10 - 8) - p] \times [p] = 1$$

Total payments are $p_1 + p_2 = 2$

“Nash-in-Nash” Bargaining Solution: Example





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- Nash-in-Nash:

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- Can nest take-it-or-leave-it (TIOLI) predictions w/ asymmetric weights:
  - If D makes TIOLI offers, payments are 0 both pre- and post-merger
  - If upstream firms make TIOLI offers, payments increase from 4 to 10

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- Simplicity and tractability has been appealing for applied work

“Nash-in-Nash” Bargaining Solution: Previous Motivations

- “Nash equilibrium in Nash bargains”
  - Each bilateral bargain represented by an agent, and each agent simultaneously maximizes its respective bargain’s Nash product
  - Form of *contract equilibrium* (Cremer Riordan 87)

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- A Non-Cooperative Interpretation?
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- Delegated Agent Representations
  [Chipty Snyder 99, Björnerstedt Stennek 07, Inderst Montez 14]
  - E.g., Crawford Yurukoglu 12 motivate approach as:

    "Each distributor and each conglomerate sends separate representatives to each meeting. Once negotiations start, representatives of the same firm do not coordinate with each other. We view this absence of informational asymmetries as a weakness of the bargaining model."

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    - Necessary & Sufficient conditions for "Rubinstein" prices
    - Sufficient conditions for prices that still "converge to" Nash-in-Nash
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- Provides guidance for when Nash-in-Nash may be appropriate
  - Contributes to *Nash program* of cooperative/non-cooperative anaylses of bargaining games
  - *Simple, tractable solution concept useful for applied work*
    [E.g., Möllers Normann Snyder (2016)]

Organization of talk

1. I. Model
2. II. Existence
3. III. Uniqueness
4. IV. Discussion and Conclusion

I. Model

I. Model: Primitives and Notation

Primitives:

- Firms: Upstream $U_1, \ldots, U_N$, Downstream $D_1, \ldots, D_M$
- Feasible Agreements/Network $\mathcal{G} \subseteq \{0,1\}^{N \times M}$ [Exogenous]
- Profits: $\{\pi_{i,U}(\mathcal{A})\}_{i=1,\ldots,N;\mathcal{A}\subseteq\mathcal{G}}$ and $\{\pi_{j,D}(\mathcal{A})\}_{j=1,\ldots,M;\mathcal{A}\subseteq\mathcal{G}}$
- Discounting: $\delta \equiv \exp(-r\Lambda)$ and Λ is period length
  [Paper has firm-specific discount factors & asymmetric "bargaining weights"]

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Notation:

- $\Delta\pi_{i,U}(\mathcal{A},\mathcal{B}) \equiv \pi_{i,U}(\mathcal{A}) - \pi_{i,U}(\mathcal{A} \setminus \mathcal{B})$ for $\mathcal{B} \subseteq \mathcal{A}$
  ["Marginal contribution to U_i of $\mathcal{B}$ at $\mathcal{A}$"]
- $\mathcal{A}_{i,U}$ are agreements in $\mathcal{A}$ involving U_i (Similarly for $\mathcal{A}_{j,D}$)

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In each period t:

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  - Each downstream firm D_j simultaneously makes private offers p_{ij} (lump-sum) to all upstream firms U_i s.t. $ij \notin \mathcal{A}^{t-1}$.
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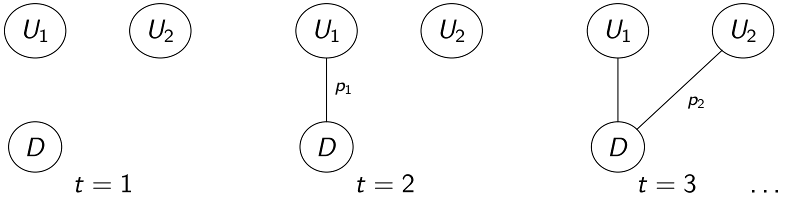
- If t is even:
  - Upstream firms make offers and downstream firms accept/reject.
- $\mathcal{A}^t \equiv \mathcal{A}^{t-1} \cup a^t$, where a^t are the set of agreements accepted at t.
- For each agreement $ij \in a^t$, D_j pays U_i the amount p_{ij}
- Each D_j receives $(1-\delta)\pi_{j,D}(\mathcal{A}^t)$ and each U_i receives $(1-\delta)\pi_{i,U}(\mathcal{A}^t)$.
- All offers and acceptances are revealed.

I. Model: Timing & Payoffs Example



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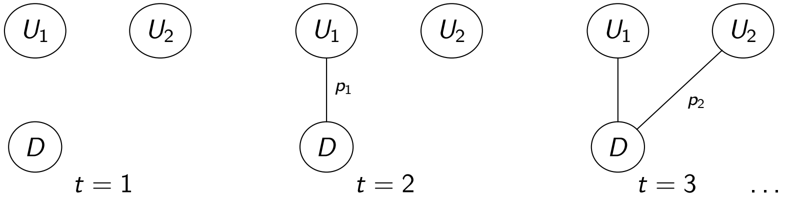
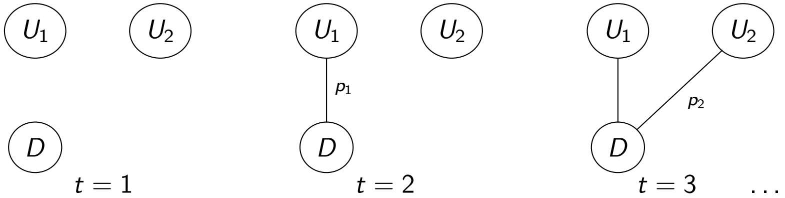


Total payoffs to D (in period $t = 1$ units):

$$(1-\delta)\pi_D(\emptyset) + \delta\Big[(1-\delta)\pi_D(\{1\}) - p_1\Big] + \delta^2\Big[(1-\delta)\pi_D(\{1,2\}) - p_2\Big]$$
$$+ \delta^3\Big[(1-\delta)\pi_D(\{1,2\})\Big] + \ldots$$



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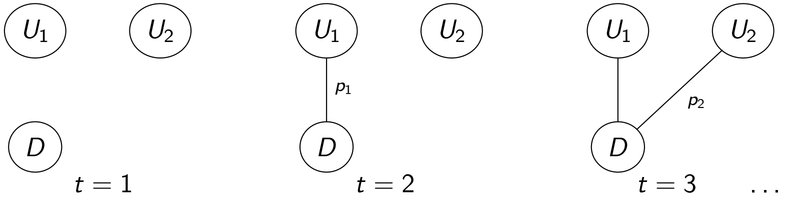

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Total payoffs to U_1 (in period $t = 1$ units):

$$(1-\delta)\pi_{1,U}(\emptyset) + \delta\Big[(1-\delta)\pi_{1,U}(\{1\}) + p_1\Big] + \delta^2\Big[\pi_{1,U}(\{1,2\})\Big]$$

I. Model: Equilibrium and Maintained Assumptions

- Strategies: offers (when proposing); acceptances (when receiving)
  - No stationarity restriction on strategies: can condition on full history h^t

- Equilibrium Concept: PBE w/ passive beliefs

  When a firm receives an out-of-equilibrium price offer following any history of play h^t, the firm does not update its beliefs over any unobserved actions in the current period.

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- Maintained assumptions:
  - Payments are non-contingent lump-sum transfers
  - **Assumption Gains From Trade (A.GFT)**:

$$\Delta\pi_{j,D}(\mathcal{G},\{ij\}) + \Delta\pi_{i,U}(\mathcal{G},\{ij\}) > 0 \qquad \forall ij \in \mathcal{G}$$

    Often agreements in $\mathcal{G}$ have been observed to or are expected to form (e.g., announced in a prior network formation stage as in Lee Fong 13)

I. Model: Nash-in-Nash Prices

Nash-in-Nash prices $\boldsymbol{p}^{Nash} \equiv \{p_{ij}\}_{i=1,\ldots,N;j=1,\ldots,M}$

$$p_{ij}^{Nash} = \arg\max_{p} [\Delta\pi_{j,D}(\mathcal{G},\{ij\}) - p]^{b_{j,D}} \times [\Delta\pi_{i,U}(\mathcal{G},\{ij\}) + p]^{b_{i,U}}$$

$$= \frac{\Delta\pi_{j,D}(\mathcal{G},\{ij\}) - \Delta\pi_{i,U}(\mathcal{G},\{ij\})}{2} \qquad \text{when } b_{i,U} = b_{j,D}$$

- Adaptation of Horn Wolinsky (1988) to $N \times M$ case w/ lump-sum transfers
- Payment based on "marginal contributions of ij at $\mathcal{G}$":
  i.e., firms split their "gains-from-trade" given all other agreements form

I. Model: Rubinstein Prices

"Rubinstein" prices $\boldsymbol{p}^R$ (with common discount factor)

Odd period:

$$p^R_{ij,D} = \frac{\delta \Delta\pi^D_j(\mathcal{G}, \{ij\}) - \Delta\pi_{i,U}(\mathcal{G}, \{ij\})}{1+\delta}$$

Even period:

$$p^R_{ij,U} = \frac{\Delta\pi^D_j(\mathcal{G}, \{ij\}) - \delta\Delta\pi_{i,U}(\mathcal{G}, \{ij\})}{1+\delta}$$

- Corresponds to Rubinstein (1982) SPE prices for $N = M = 1$
- $p^R_{ij,D} < p^{Nash}_{ij} < p^R_{ij,U}$
- Lemma 2.2: Prices converge to Nash-in-Nash prices as $\Lambda \to 0$.
  [Binmore, Rubinstein, and Wolinsky (1986)]

II. Existence

II. Existence Results: Overview

First set of results concerns the existence of a *Nash-in-Nash limit equilibrium*:

> For any $\varepsilon > 0$, there exists $\bar{\Lambda} > 0$ s.t. $\forall \Lambda \in (0, \bar{\Lambda}]$, there is an equilibrium with complete agreements at prices within ε of Nash-in-Nash prices.

We provide:

1. A necessary and sufficient condition on *decreasing marginal contributions of agreements* for there to exist an equilibrium where all open agreements immediately form at Rubinstein prices

2. Sufficient conditions for there to exist a Nash-in-Nash limit equilibrium
   - Admits complementarities on one "side" of the market

II. Existence (1): Necessary and Sufficient Condition

A.WCDMC: Weak Conditional Decreasing Marginal Contribution

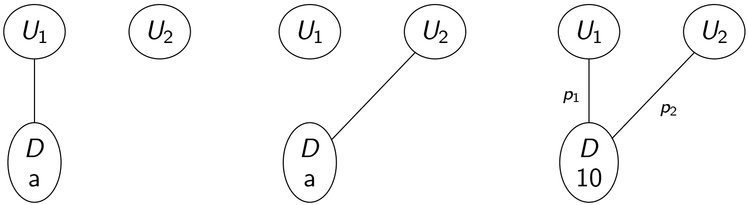
$$\Delta\pi_{j,D}(\mathcal{G}, \mathcal{A}) \geq \sum_{hj \in \mathcal{A}} \Delta\pi_{j,D}(\mathcal{G}, \{hj\}) \qquad \forall j = 1, \ldots, M; \forall \mathcal{A} \subseteq \mathcal{G}_{j,D}$$

(and the same holds for upstream firms)

[Similar to conditions in Stole Zwiebel 96, Westermark 03, Bloch Jackson 07, Hellman 13]

- Marginal contribution of any set of agreements is weakly greater than the sum of the marginal contributions of individual agreements within the set when $\mathcal{G}$ is formed
- Generally satisfied if firms on same side of market are substitutes
  - A.WCDMC is also implied by assuming that the value of an agreement to a firm is lower as additional agreements are signed

II. Existence (1): A.WCDMC Example



- A.WCDMC is satisfied here iff $a \geq 5$:

$$\pi_{1,D}(\{1,2\}) - \pi_{1,D}(\emptyset) \geq \Delta\pi_{1,D}(\{1,2\},\{1\}) + \Delta\pi_{1,D}(\{1,2\},\{2\})$$
$$10 \geq (10 - a) + (10 - a)$$

II. Existence (1): Result and Intuition

Theorem 3.2: A.WCDMC is necessary and sufficient for there to exist an equilibrium where at every period t and history h^t, all open agreements are immediately formed at Rubinstein prices.

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Intuition: eq. strategies are always offer & accept Rubinstein prices, reject worse than Rubinstein prices; these ensure that agents do not wish to delay agreement.

- Consider D_j's gains from accepting $\mathcal{A}$ agreements vs. rejecting them (and forming them next period given candidate strategies) in an even period:

$$\underbrace{(1-\delta)\Delta\pi_{j,D}(\mathcal{G},\mathcal{A}) + \sum_{hj\in\mathcal{A}}[-p^R_{hj,U} + \delta p^R_{hj,D}]}_{\text{change in } D_j\text{' s profits from accepting vs. rejecting offers in } \mathcal{A}}$$

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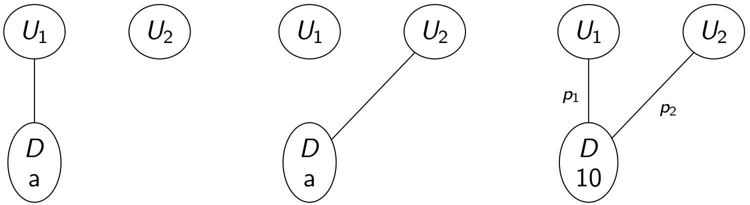
$$\geq \sum_{hj\in\mathcal{A}}\left[(1-\delta)\Delta\pi_{j,D}(\mathcal{G},\{hj\}) - p^R_{hj,U} + \delta p^R_{hj,D}\right] = 0$$

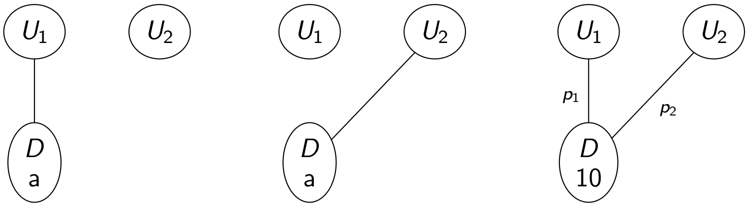
where inequality follows by A.WCDMC, and equality by Rubinstein prices.

II. Existence (1): A.WCDMC Example (Cont'd)



- A.WCDMC is violated if $a < 5$:
  - If $a < 5$, complementarities imply that in an even period, downstream firm will prefer rejecting *both* offers if Rubinstein and coming to agreement next period.
    - E.g., if $a = 0$, $p^R_{i,U} > 5$, and D will earn negative profits at these prices

II. Existence (1): A.WCDMC Example (Cont'd)



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    - E.g., if $a = 0$, $p_{i,U}^{R} > 5$, and D will earn negative profits at these prices
  - However, there will exist an equilibrium at prices that need not be Rubinstein in each period, but still converge to Nash-in-Nash prices as $\Lambda \to 0$. [Example related to setting in Jun 89]

II. Existence (2): Existence w/ Complementarities

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A.FEAS: Feasibility [Related to condition in Stole Zwiebel 96]

$$\Delta\pi_{j,D}(\mathcal{G},\mathcal{A}) \geq \sum_{ij\in\mathcal{A}} p_{ij}^{Nash} \qquad \forall j = 1,\ldots,M; \forall\mathcal{A}\subseteq\mathcal{G}_{j,D}$$

(and the same holds for upstream firms)

- Ensures no firm would wish to drop any subset of agreements at Nash-in-Nash prices

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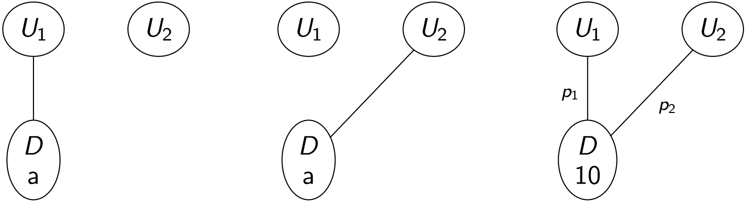
- Ensures no firm would wish to drop any subset of agreements at Nash-in-Nash prices
- Theorem 3.4: A.FEAS is necessary for there to exist a Nash-in-Nash limit equilibrium where all agreements in $\mathcal{G}$ immediately form.
- A.WCDMC implies A.FEAS

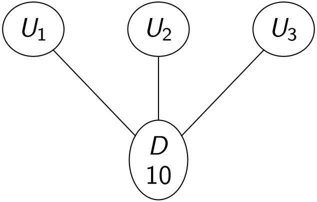
II. Existence (2): A.FEAS Example #1



- A.FEAS is satisfied here if $a \geq 0$, as will imply that $p_i^{Nash} \leq 5$ and:

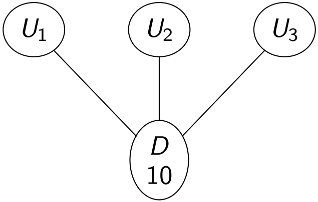
$$10 - p_1^{Nash} - p_2^{Nash} \geq 0$$

II. Existence (2): A.FEAS Example #2



- Now imagine that D needs all 3 upstream firms to generate any surplus.

II. Existence (2): A.FEAS Example #2



- Now imagine that D needs all 3 upstream firms to generate any surplus.
- D_A would prefer no agreement than paying Nash-in-Nash prices to all upstream firms as $p_{1A}^{Nash} = p_{2A}^{Nash} = p_{3A}^{Nash} = 5$.
- However, if D_A generated at least a surplus of $10/3$ with 2 upstream firms, then A.FEAS would be satisfied (as then $p_{iA}^{Nash} \leq 10/3$ for all i).

II. Existence (2): Existence w/ Complementarities

Moving away from Rubinstein prices expands cases where a Nash-in-Nash outcome can still emerge in equilibrium. Consider alternative set of assumptions:

A.FEAS: Feasibility

A.SCDMC(b): Strong Conditional Decreasing Marginal Contribution

$$\pi_{j,D}(\mathcal{A} \cup \mathcal{B} \cup \{ij\}) - \pi_{j,D}(\mathcal{A}' \cup \mathcal{B}) \geq \Delta\pi_{j,D}(\mathcal{G}, \{ij\})$$

$$\forall ij \in \mathcal{G}; \mathcal{B} \subseteq \mathcal{G}_{-i,U}; \mathcal{A}, \mathcal{A}' \subseteq \mathcal{G}_{i,U} \setminus \{ij\}$$

[A.SCDMC(a) is same assumption for upstream firms]

- Requires that inframarginal contribution to a firm of a link is greater than marginal contribution at full network, even if counterparty adjusts
- Stronger than A.WCDMC (on one side)

II. Existence (2): Existence w/ Complementarities

Theorem 3.7: A.FEAS and either A.SCDMC(a) or A.SCDMC(b) is sufficient for there to exist a Nash-in-Nash limit equilibrium.

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- A.SCDMC(a)/(b) is trivially satisfied when profits accrue only to one-side
- A.FEAS admits some form of "complementarities"
- Intuition: Assume that upstream firms' profits satisfy A.SCDMC and downstream firms' profits satisfy A.FEAS.
  - Odd periods (downstream proposing): Equilibrium offers are Rubinstein
  - Even periods (upstream proposing): Equilibrium offers are lower than Rubinstein, as upstream firms need to insure that downstream firms do not wish to reject multiple offers.

III. Uniqueness

III. Uniqueness Results: Overview

Second set of results concerns the uniqueness of *Nash-in-Nash* outcomes.

1. We prove that all equilibria where all agreements are formed without delay have equilibrium prices arbitrarily close to Nash-in-Nash prices as $\Lambda \to 0$
2. We provide sufficient conditions for any equilibrium to result in all agreements being formed immediately at Rubinstein prices

III. Uniqueness (1): No-Delay Result

Theorem 4.1: For any $\varepsilon > 0$, there exists $\bar{\Lambda} > 0$ s.t. $\forall \Lambda \in (0, \bar{\Lambda}]$, any *no-delay equilibrium*—i.e., an equilibrium in which all open agreements at any history immediately form—has prices at every period t and history h^t that are within ε of Nash-in-Nash prices.

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- Does not require *any* assumptions on underlying payoffs
- Similar to restrictions used in literature:
  E.g., Ray Vohra (2015): in complete information games, delays are "more artificial [than in incomplete information games] and stem from two possible sources...a typical folk theorem-like reason...[and in] protocols that are sensitive to the identity of previous rejectors."

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- Intuition for result:
  - Receiving firm cannot obtain a worse price: it is better off rejecting a single offer and coming to agreement next period.
  - Proposing firm cannot obtain a worse price: if time periods are sufficiently small, it is better off "withdrawing" (potentially induce multiple rejections) and coming to agreement next period.

III. Uniqueness (2): Stronger Conditions

With stronger conditions, all equilibria are no-delay (and yield Rubinstein prices).

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A.LNEXT: Limited Negative Externalities

For all non-empty $\mathcal{A} \subseteq \mathcal{G}$, there exists $ij \in \mathcal{A}$ s.t.

$$\Delta\pi_{i,U}(\mathcal{G}, \mathcal{A}) \geq \sum_{ik \in \mathcal{A}_{i,U}} \Delta\pi_{i,U}(\mathcal{G}, \{ik\}) \text{ and}$$

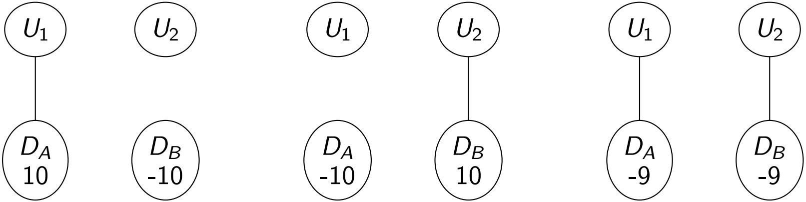
$$\Delta\pi_{j,D}(\mathcal{G}, \mathcal{A}) \geq \sum_{hj \in \mathcal{A}_{j,D}} \Delta\pi_{j,D}(\mathcal{G}, \{hj\})$$

- Requires for any subset of agreements $\mathcal{A}$ that may not yet have formed, there is *some* pair $ij \in \mathcal{A}$ such that for each firm in the pair, having all remaining agreements form is sufficiently beneficial.
- Paired with A.SCDMC, limits the degree of negative externalities imposed on at least one pair of firms by other agreements not involving them

[Implied by Bloch Jackson 07 nonnegative externalities condition]

III. Uniqueness (2): Violation of A.LNEXT

Assume that $\mathcal{G} = \{1A, 2B\}$ and $\pi_{1,D}(\emptyset) = \pi_{2,D}(\emptyset) = 0$



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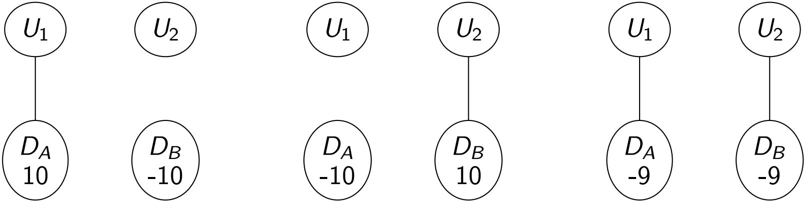

- Nash-in-Nash prices are $1/2$.
- A.LNEXT is violated for $\mathcal{A} = \mathcal{G}$; e.g.,

$$\pi_{A,D}(\{1A, 2B\}) - \pi_{A,D}(\emptyset) \not\geq \Delta\pi_{A,D}(\{1A, 2B\}, \{1A\})$$
$$-9 - 0 \not\geq 1$$

- Thus, there exists an equilibrium (starting at empty network) where no agreements are ever formed. [prisoners' dilemma]

III. Uniqueness (2): Result

Theorem 4.3: Assume A.SCDMC(a)+(b) and A.LNEXT. For any $\Lambda > 0$, every common tie-breaking equilibrium results in all open agreements at every period t and history h^t immediately forming at Rubinstein prices.

- Common tie-breaking equilibrium: at any history of play h^t, if any receiving firm has the same set of best responses for two sets of offers, it chooses the same best response. [Can be relaxed if A.LNEXT is strengthened]

III. Uniqueness (2): Intuition for Proof

Induction on the set of open agreements $\mathcal{C}$:
if any subgame that begins with fewer open agreements than $\mathcal{C}$ results in immediate agreement at Rubinstein prices, then any subgame with $\mathcal{C}$ open agreements also results in immediate agreement at Rubinstein prices.

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  - Rubinstein (82) / Shaked Sutton (84) type arguments used to show agreement must be at Rubinstein prices.
- Immediacy of Agreement:
  - A.LNEXT rules out delay: always one pair of firms that wish to receive / pay Nash-in-Nash prices for their remaining agreements even if other agreements not involving them form.

IV. Discussion and Conclusion

Discussion

Connections to Applied Literatures:

- Wage bargaining: typically has assumed a single firm bargaining w/ multiple workers, profits accruing only to firm, lump-sum payments, & no externalities
  - W/ substitutable workers, our uniqueness + existence results typically apply
  - Extends results to M x N settings (w/ differentiated firms)

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  - W/ substitutable workers, our uniqueness + existence results typically apply
  - Extends results to M x N settings (w/ differentiated firms)
- IO / Vertical Market Settings:
  - Declining Marginal Contributions often satisfied when upstream firms are substitutable (though also depends on downstream firm competition)
  - Limited Negative Externalities satisfied when markets are separable and served by monopolists (e.g., Chipty Snyder 99), upstream firms earn zero profits (as above), or upstream firms are reimbursed marginal costs alongside lump sum payments (as in Capps et. al 03)
  - Hospital-insurer and content distribution bargaining are recent prominent applications, but insights are also relevant to many B2B interactions

Discussion

Other Bargaining Protocols

- Allowing for renegotiation or contingent contracts implies that surplus division is no longer is a function of *marginal contributions* alone
  [Stole Zweibel 96, Raskovich 03, de Fontenay Gans 14,...]

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Richer Contract Spaces

- Different contract spaces (e.g., linear fees) can imply profits depend on contracts that have been signed
  - Contract equilibrium (Cremer Riordan 87)
  - Delegated agent representation in applied work
  - Open Q: do similar results extend to these other environments?

Conclusions & Takeaways

1. "Nash-in-Nash" surplus division can emerge as an equilibrium outcome from a generalization of Rubinstein (1982)'s alternating offers bargaining game
   - Predicts surplus division for a *fixed & given* network
   - Establish connections b/w vertical contracting and bargaining literatures (e.g., passive beliefs / alternating simultaneous offers)
2. Results highlight importance of restrictions on marginal contributions
3. Uniqueness results demonstrate robustness along certain dimensions
   - Does not require restriction to stationary strategies

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2. Results highlight importance of restrictions on marginal contributions
3. Uniqueness results demonstrate robustness along certain dimensions
   - Does not require restriction to stationary strategies
4. Open Q:
   - With complete agreement & delay, or with weaker assumptions on payoffs, do all equilibria have prices arbitrarily close to Nash-in-Nash?