ONLINE APPENDIX

"Nash-in-Nash" Bargaining: A Microfoundation for Applied Work*

Allan Collard-Wexler, Duke & NBER

Gautam Gowrisankaran, Arizona, HEC Montreal & NBER

Robin S. Lee, Harvard & NBER

September 2017

Contents

A Formal Results for the Delegated Agent Model

In this section, we present formal results for the *delegated agent model*, discussed in Section 2.3. The extensive form for this representation involves separate bilateral negotiations between delegated agents, or representatives, of each firm.[1] We develop this model and show that this representation also admits the Nash-in-Nash bargaining solution as an equilibrium outcome if A.GFT holds.

Our model is as follows. For every negotiation $ij \in \mathcal{G}$, U_i and D_j send individual representatives, denoted as $\mathcal{U}^{ij}$ and $\mathcal{D}^{ij}$, who engage in the alternating-offers bargaining protocol of Binmore, Rubinstein, and Wolinsky (1986), where negotiation breakdowns are independent across negotiations and profits are realized once all negotiations have concluded or broken down. Each representative seeks to maximize her firm's total expected profits across all bargains. However, she does not know the outcome of any other bilateral bargain until her own bargain has concluded or broken down. One interpretation is that each pair

*Contact details: Collard-Wexler, collardwexler@gmail.com; Gowrisankaran, gautamg2@gmail.com; and Lee, robin-lee@fas.harvard.edu. The usual disclaimer applies.

[1] Chipty and Snyder (1999) (see footnote 10) provides a sketch of this argument in the context of a single supplier negotiating with multiple buyers; see also Björnerstedt and Stennek (2007).

of agents for a negotiation are sequestered in separate bargaining rooms, and no one outside the room knows the status of the bargain until it has concluded or broken down.

Under these conditions, we show that a Nash-in-Nash limit equilibrium exists:

Theorem A.1 *Assume A.GFT and that every negotiation $ij \in \mathcal{G}$ is conducted by delegated agents from $\mathcal{U}_i$ and $\mathcal{D}_j$. Then there exists an equilibrium of the* delegated agent model *where all agreements $ij \in \mathcal{G}$ are immediately formed at prices $\hat{p}_{ij} = p^R_{ij,D}$ ($p^R_{ij,U}$) if t_0 is odd (even).*

Proof. Let the delegated agents employ the following candidate set of strategies: $\mathcal{U}^{ij}$ offers $p^R_{ij,U}$ in even periods and only accepts offers equal to or above $p^R_{ij,D}$ in odd periods; $\mathcal{D}^{ij}$ offers $p^R_{ij,D}$ in odd periods, and accepts offers equal to or below $p^R_{ij,U}$ in even periods.

Given passive beliefs, when an agent sees an off-equilibrium action by one party, it still perceives that the other parties are following their equilibrium actions.[2] Thus, if a delegated agent—e.g., $\mathcal{D}^{ij}$—sees a deviation from the above strategies by its rival—$\mathcal{U}^{ij}$ in this case—then it perceives that all other negotiations (which are all run by separate delegated agents) follow equilibrium strategies. Hence, both $\mathcal{D}^{ij}$ and $\mathcal{U}^{ij}$ assume that all other agreements immediately form regardless of what occurs in their respective bargain. By Binmore, Rubinstein, and Wolinsky (1986), the above strategies then comprise the unique equilibrium for each ij negotiation. □

Note that this does not necessarily imply that there is a unique equilibrium for the delegated agent model. Indeed, consider our three supplier counterexample from Section 3.1. In this setting, if every negotiation is conducted by delegated agents by the downstream manufacturer, and each agent believes that all other agreements will form at Rubinstein prices (i.e., at prices $\delta/(1+\delta)$ it t_0 odd), then their agreement will also form at this price. However, there exists another equilibrium in which no agreements are ever formed: e.g., if each delegated agent for the downstream manufacturer believes that no other agreements will form, then upstream suppliers always demanding $p_{i1,U} > 0$ in each even period (and the manufacturer rejecting any offer greater than 0), and the manufacturer offering a price of $p_{i1,D} < 0$ in each odd period (and suppliers rejecting any offer less than 0) comprise equilibrium strategies.

While we do not prove that there exists a unique equilibrium of the delegated agent model, the proof of Theorem A.1 nevertheless implies that conditional on all agreements in $\mathcal{G}$ forming, the equilibrium outcome of agreements all forming at Rubinstein prices is unique. This result is the analog of Theorem 4.2.

B Proofs of Lemmas from Main Text

Proof of Lemma 2.1 Using l'Hospital's rule:

$$\lim_{\Lambda\to 0} \frac{\delta_{i,U}(1-\delta_{j,D})}{1-\delta_{i,U}\delta_{j,D}} = \lim_{\Lambda\to 0} \frac{e^{-r_{i,U}\Lambda}(1-e^{-r_{j,D}\Lambda})}{1-e^{-(r_{i,U}+r_{j,D})\Lambda}} = \frac{r_{j,D}}{r_{i,U}+r_{j,D}} \ ,$$

and

$$\lim_{\Lambda\to 0} \frac{1-\delta_{j,D}}{1-\delta_{i,U}\delta_{j,D}} = \lim_{\Lambda\to 0} \frac{1-e^{-r_{j,D}\Lambda}}{1-e^{-(r_{i,U}+r_{j,D})\Lambda}} = \frac{r_{j,D}}{r_{i,U}+r_{j,D}} \ .$$

Similarly,

$$\lim_{\Lambda\to 0} \frac{\delta_{j,D}(1-\delta_{i,U})}{1-\delta_{i,U}\delta_{j,D}} = \lim_{\Lambda\to 0} \frac{(1-\delta_{i,U})}{1-\delta_{i,U}\delta_{j,D}} = \frac{r_{i,U}}{r_{i,U}+r_{j,D}} \ ,$$

which proves the lemma. □

Proof of Lemma 2.2 Assume A.GFT. For any $ij \in \mathcal{G}$, since $0 < \delta_{i,U} < 1$ and $0 < \delta_{j,D} < 1$, note:

$$(\Delta\pi_{j,D}(\mathcal{G},\{ij\}) - p^R_{ij,D}) = \underbrace{\frac{(1-\delta_{i,U})}{(1-\delta_{i,U}\delta_{j,D})}}_{>0} \underbrace{(\Delta\pi_{j,D}(\mathcal{G},\{ij\}) + \Delta\pi_{i,U}(\mathcal{G},\{ij\}))}_{>0 \text{ by A.GFT}} \ .$$

[2]This property holds for sequential equilibria (without public signals), and not just for weak perfect Bayesian equilibria with passive beliefs.

Thus $\Delta\pi_{j,D}(\mathcal{G},\{ij\}) > p^R_{ij,D}$. Also, note:

$$(\Delta\pi_{i,U}(\mathcal{G},\{ij\}) + p^R_{ij,U}) = \underbrace{\frac{(1-\delta_{j,D})}{(1-\delta_{i,U}\delta_{j,D})}}_{>0} \underbrace{(\Delta\pi_{j,D}(\mathcal{G},\{ij\}) + \Delta\pi_{i,U}(\mathcal{G},\{ij\}))}_{>0 \text{ by A.GFT}} .$$

Thus $\Delta\pi_{i,U}(\mathcal{G},\{ij\}) > -p^R_{ij,U} >$. Adding the previous two inequalities and rearranging, we obtain:

$$\begin{aligned} p^R_{ij,U} - p^R_{ij,D} &= \Big(\Delta\pi_{j,D}(\mathcal{G},\{ij\}) + \Delta\pi_{i,U}(\mathcal{G},\{ij\})\Big)\Big(\frac{(1-\delta_{i,U})}{(1-\delta_{i,U}\delta_{j,D})} + \frac{(1-\delta_{j,D})}{(1-\delta_{i,U}\delta_{j,D})} - 1\Big) \\ &= \frac{1}{1-\delta_{i,U}\delta_{j,D}}\Big(\Delta\pi_{j,D}(\mathcal{G},\{ij\}) + \Delta\pi_{i,U}(\mathcal{G},\{ij\})\Big)\Big(1 - \delta_{i,U} - \delta_{j,D} + \delta_{j,D}\delta_{i,U}\Big). \end{aligned}$$

Again, all three terms on the second line are positive; thus $p^R_{ij,U} > p^R_{ij,D}$. Finally, substituting in the definition of $\delta_{i,U}$ and $\delta_{j,D}$ into the definition of $p^R_{ij,D}$, it is straightforward to show $\partial p^R_{ij,D}(\Lambda)/\partial\Lambda < 0 \; \forall \; \Lambda > 0$; thus, as $lim_{\Lambda\to 0} p^R_{ij,D} = p^{Nash}_{ij}$ by Lemma 2.1, it follows that $p^{Nash}_{ij} > p^R_{ij,D}$. A similar approach can be used to show that $p^{Nash}_{ij} < p^R_{ij,U}$. □

Proof of Lemma 3.3 Assume A.GFT. We prove the lemma using the following four claims:

1. A.SCDMC ⇒ A.WCDMC

   We prove A.SCDMC(b) (for downstream firms) implies A.WCDMC holds for downstream firms; the proof that A.SCDMC(a) (for upstream firms) implies A.WCDMC holds for upstream firms is symmetric and omitted.

   A.SCDMC(b) states: $\pi_{j,D}(\mathcal{A}\cup\mathcal{B}\cup\{ij\}) - \pi_{j,D}(\mathcal{A}'\cup\mathcal{B}) \geq \Delta\pi_{j,D}(\mathcal{G},\{ij\})$ for all $ij \in \mathcal{G}$, $\mathcal{B} \subseteq \mathcal{G}_{-i,U}$, and $\mathcal{A},\mathcal{A}' \subseteq \mathcal{G}_{i,U}\setminus\{ij\}$. For the case where $\mathcal{A} = \mathcal{A}'$, A.SCDMC implies:

   $$\Delta\pi_{j,D}(\mathcal{A}\cup\mathcal{B},\{ij\}) \geq \Delta\pi_{j,D}(\mathcal{G},\{ij\}) \qquad \forall\; ij\in\mathcal{G}, \mathcal{B}\subseteq\mathcal{G}_{-i,U}, \mathcal{A}\subseteq\mathcal{G}_{i,U}\setminus\{ij\} . \tag{B.1}$$

   Index agreements in $\mathcal{A}$ from $k = 1,\cdots,|\mathcal{A}|$, and let a_k represent the kth agreement in $\mathcal{A}$. This allows us to create a sequence of sets of agreements, starting at $\mathcal{B}\equiv\mathcal{G}\setminus\mathcal{A}$, in which we add in each agreement one at a time, given by $\mathcal{D}_0 \equiv \mathcal{B}$, and $\mathcal{D}_k = \mathcal{D}_{k-1}\cup\{a_k\}$ for $k = 1,\cdots,|\mathcal{A}|$. Then, note that for any $\mathcal{A}\subseteq\mathcal{G}_{j,D}$:

   $$\begin{aligned} \Delta\pi_{j,D}(\mathcal{G},\mathcal{A}) = \Delta\pi_{j,D}(\mathcal{A}\cup\mathcal{B},\mathcal{A}) &= \sum_{k=1}^{|\mathcal{A}|}\Delta\pi_{j,D}(\mathcal{D}_k,\{a_k\}) \\ &\geq \sum_{k=1}^{|\mathcal{A}|}\Delta\pi_{j,D}(\mathcal{G},\{a_k\}) = \sum_{kj\in\mathcal{A}}\Delta\pi_{j,D}(\mathcal{G},\{kj\}), \end{aligned}$$

   where the last equality on the first line follows from the index for agreements for $\mathcal{A}$, and the inequality on the second line follows from (B.1). This coincides with the statement of A.WCDMC for downstream firms.

2. A.WCDMC ⇒ A.FEAS

   For all $ij\in\mathcal{G}$ and $\mathcal{A}\subseteq\mathcal{G}_{j,D}$:

   $$\begin{aligned} \Delta\pi_{j,D}(\mathcal{G},\mathcal{A}) - \sum_{ij\in\mathcal{A}} p^{Nash}_{ij} &\geq \sum_{kj\in\mathcal{A}}\Big(\Delta\pi_{j,D}(\mathcal{G},\{ij\}) - \frac{b_{i,U}\Delta\pi_{j,D}(\mathcal{G},\{ij\}) - b_{j,D}\Delta\pi_{i,U}(\mathcal{G},\{ij\})}{b_{i,U}+b_{j,D}}\Big) \\ &= \sum_{kj\in\mathcal{A}}\left(\Big(\Delta\pi_{j,D}(\mathcal{G},\{ij\}) + \Delta\pi_{i,U}(\mathcal{G},\{ij\})\Big)\frac{b_{j,D}}{b_{i,U}+b_{j,D}}\right) > 0 , \end{aligned}$$

where the first inequality follows from A.WCDMC (for downstream firms) and the definition of p_{ij}^{Nash}, and the second line is positive by A.GFT. Thus $\Delta\pi_{j,D}(\mathcal{G},\mathcal{A}) > \sum_{ij\in\mathcal{A}} p_{ij}^{Nash}$, and A.FEAS holds. The proof for upstream firms is symmetric and omitted.

3. A.WCDMC $\not\Rightarrow$ A.SCDMC

   Consider a single downstream firm and three upstream suppliers. Suppose that the downstream firm profits are: 0 without any supplier, 0.25 with one supplier, 0.7 with two suppliers, and 1 with all three suppliers; assume supplier profits are always 0. This example violates A.SCDMC(b) given by (B.1) because the surplus to the downstream firm from having one supplier (0.25) is less than the surplus from adding the third supplier (0.3). But, it does not violate A.WCDMC, because removing two or three suppliers both result in a greater loss than the sum of the marginal values (0.75 versus 0.6 when removing two suppliers, and 1 versus 0.9 when removing all three).

4. A.FEAS $\not\Rightarrow$ A.WCDMC

   The two automobile supplier example in the paper discussed in Section 3.1 satisfies A.FEAS but not A.WCDMC when $0 \leq a < 0.5$.

□

C Proof of Theorems on Existence

C.1 Proof of Theorem 3.2

We proceed by contradiction: assume that a Nash-in-Nash limit equilibrium exists where all agreements form immediately, but A.FEAS does not hold, so that for some D_j, there exists $\mathcal{A} \subseteq \mathcal{G}_{j,D}$ such that $\Delta\pi_{j,D}(\mathcal{G},\mathcal{A}) < \sum_{ij\in\mathcal{A}} p_{ij}^{Nash}$; the proof if A.FEAS is violated for some U_i is symmetric and omitted. Let $\varepsilon = \frac{1}{|\mathcal{A}|}\Big((\sum_{ij\in\mathcal{A}} p_{ij}^{Nash}) - \Delta\pi_{j,D}(\mathcal{G},\mathcal{A})\Big)$, which is positive by assumption. By the contradictory assumption, for any t_0, there exists $\bar{\Lambda} > 0$ such that for all $\Lambda \in (0,\bar{\Lambda}]$, there is an equilibrium where all agreements in $\mathcal{G}$ form at t_0 at prices $\{p_{ij}^*\}_{ij\in\mathcal{G}}$, where $|p_{ij}^* - p_{ij}^{Nash}| < \varepsilon$ for all $ij \in \mathcal{G}$. Assume that t_0 is even, and fix $\Lambda \in (0,\bar{\Lambda}]$. Consider the following multi-period deviation: D_j rejects offers $ij \in \mathcal{A}$ at t_0 and every subsequent even period and proposes offers that are sufficiently low that they will be rejected in odd periods. By assumption, in this equilibrium, all agreements in $\mathcal{G}\setminus\mathcal{A}$ will still form at t_0. Thus, this deviation (where agreements in $\mathcal{A}$ never form) will increase D_j's payoffs by:

$$\begin{aligned}\Big(\sum_{ij\in\mathcal{A}} p_{ij}^*\Big) - \Delta\pi_{j,D}(\mathcal{G},\mathcal{A}) &> \Big(\sum_{ij\in\mathcal{A}} (p_{ij}^{Nash} - \varepsilon)\Big) - \Delta\pi_{j,D}(\mathcal{G},\mathcal{A}) \\ &= \Big((\sum_{ij\in\mathcal{A}} p_{ij}^{Nash}) - \Delta\pi_{j,D}(\mathcal{G},\mathcal{A})\Big) - |\mathcal{A}| \times \varepsilon \\ &= \Big((\sum_{ij\in\mathcal{A}} p_{ij}^{Nash}) - \Delta\pi_{j,D}(\mathcal{G},\mathcal{A})\Big) - \Big((\sum_{ij\in\mathcal{A}} p_{ij}^{Nash}) - \Delta\pi_{j,D}(\mathcal{G},\mathcal{A})\Big) = 0\end{aligned}$$

where the inequality on the first line follows from the definition of p_{ij}^*, the second line rearranges terms, and the third line follows from substituting in the definition of ε. Thus, this deviation is profitable, yielding a contradiction. □

C.2 Proof of Theorem 3.1 (Sufficiency Only) and Theorem 3.4

Assume A.GFT and *either* (i) A.WCDMC *or* (ii) A.FEAS and either A.SCDMC(a) or A.SCDMC(b). For condition set (ii), we detail the proof where A.SCDMC(a) (for upstream firms) holds; the proof when A.SCDMC(b) holds is symmetric and omitted.

We first detail our candidate equilibrium strategies:

- In every odd period, each D_j makes offers $p^R_{ij,D}$ to all firms U_i with which it has not already formed an agreement. If all price offers that it receives are equal to $p^R_{ij,D}$, U_i accepts all offers. If U_i receives exactly one non-equilibrium offer from some D_j, it accepts all other offers and rejects D_j's offer if and only if the offer is lower than $p^R_{ij,D}$. Finally, if U_i receives multiple non-equilibrium offers, it plays an arbitrary best response in its acceptance decision, respecting passive beliefs.

- In every even period with open agreements given by $\mathcal{C}$, each U_i makes offers $p_{ij,U}(\mathcal{C})$ (defined below) to all firms $D_j \in \mathcal{C}_{i,U}$. If all price offers that it receives are equal to $p_{ij,U}(\mathcal{C})$, D_j accepts all offers. If D_j receives exactly one non-equilibrium offer from some U_i and that offer is lower than $p_{ij,U}(\mathcal{C})$, then D_j still accepts all offers. If D_j receives exactly one non-equilibrium offer from some U_i and that offer is higher than $p_{ij,U}(\mathcal{C})$, then: (i) under A.WCDMC, D_j rejects U_i's offer and accepts all other offers; (ii) under A.FEAS and A.SCDMC, D_j rejects U_i's offer and plays an arbitrary best response in its acceptance decision with other offers (respecting passive beliefs). If D_j receives multiple non-equilibrium offers, it plays an arbitrary best response in its acceptance decision, respecting passive beliefs.

The prescribed strategy profile dictates that every firm makes proposals that are Rubinstein prices in odd periods, and may differ from Rubinstein prices in even periods. On the equilibrium path, all offers are accepted regardless of whether the period is odd or even.

Note that our candidate equilibrium strategies do not completely specify a receiving firm's best response upon receiving multiple non-equilibrium offers; indeed, at certain nodes, there may be multiple actions that satisfy our equilibrium construction. Generally, determining a receiving firm's best response upon receiving multiple non-equilibrium offers may depend on actions taken in a subsequent subgame when another firm receives multiple non-equilibrium offers; in such circumstances, best responses may not be straightforward to determine or even well defined. However, this is not an issue in our setting: given our candidate equilibrium strategies, a receiving firm's value from accepting any set of offers (regardless of whether any offers are non-equilibrium offers) in any period t does not depend on future actions taken at nodes with multiple non-equilibrium offers. The reason for this is that a receiving firm anticipates that all agreements that are not formed in period t (including any offers that it rejects) will be formed at candidate equilibrium prices in the next period $t+1$. Hence, regardless of the receiving firm's actions in a given period, it will never expect to reach another subgame where any firm receives multiple non-equilibrium offers. Consequently, our candidate equilibrium strategies are well defined; furthermore, in our proof below, we explicitly characterize a firm's best response to receiving multiple non-equilibrium offers at any history of play.

Construction of Even Period ($p_{ij,U}(\mathcal{C})$) Prices. We now define candidate even-period equilibrium pricing strategies $p_{ij,U}(\mathcal{C})$ iteratively as follows. For each set of open agreements $\mathcal{C} \subseteq \mathcal{G}$, consider the constraints:

$$\underbrace{\sum_{ij \in \mathcal{B}} p_{ij,U}(\mathcal{C})}_{LS} \leq \underbrace{(1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\mathcal{B}) + \delta_{j,D} \sum_{ij \in \mathcal{B}} p^R_{ij,D}}_{RS} \qquad \forall j \text{ s.t. } \mathcal{C}_{j,D} \neq \emptyset, \forall \mathcal{B} \subseteq \mathcal{C}_{j,D}. \tag{C.2}$$

where the constraint ensures that each downstream firm D_j with open agreements in $\mathcal{C}$ wishes to accept prices $p_{ij,U}(\mathcal{C})$ for any subset of agreements $\mathcal{B} \subseteq \mathcal{C}_{j,D}$ at an even period as opposed to forming those agreements in the next period at Rubinstein prices $p^R_{ij,D}$.

Step 1. Initialize $p_{ij,U}(\mathcal{C}) = p^R_{ij,D}, \forall ij$. At these values, the constraints specified by (C.2) are strictly satisfied:

$$\underbrace{(1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\mathcal{B}) + \delta_{j,D}\sum_{ij\in\mathcal{B}} p^R_{ij,D}}_{RS} \geq (1-\delta_{j,D})\sum_{ij\in\mathcal{B}} p^{Nash}_{ij} + \delta_{j,D}\sum_{ij\in\mathcal{B}} p^R_{ij,D}$$
$$> \underbrace{(1-\delta_{j,D})\sum_{ij\in\mathcal{B}} p^R_{ij,D} + \delta_{j,D}\sum_{ij\in\mathcal{B}} p^R_{ij,D}}_{=LS},$$

where the inequality on the first line follows from A.FEAS, and the second line from Lemma 2.2.

Step 2. Now, for each set of open agreements $\mathcal{C}$, fix an arbitrary ordering over agreements within that set. Start with the first open agreement $ij \in \mathcal{C}$, and increase $p_{ij,U}(\mathcal{C})$ until (at least) one of the constraints given by (C.2) binds. Move on to the second open agreement, and do the same. Continue through all the open agreements in $\mathcal{C}$. Define the candidate set of equilibrium offers $p_{ij,U}(\mathcal{C})$ to be the offers resulting from this process. For these prices, all constraints (C.2) still hold. Moreover, by construction, at these prices each open agreement $ij \in \mathcal{C}$ has at least one constraint (C.2) that binds.

Next, we prove the following supporting Lemma.

Lemma C.1 *Candidate equilibrium prices satisfy the following properties:*

1. $p_{ij,U}(\mathcal{C}) \geq p^R_{ij,D}, \forall ij \in \mathcal{C}, \mathcal{C} \subseteq \mathcal{G}$.
2. $p_{ij,U}(\mathcal{C}) \leq p^R_{ij,U}, \forall ij \in \mathcal{C}, \mathcal{C} \subseteq \mathcal{G}$.
3. $p_{ij,U}(\{ij\}) = p^R_{ij,U}, \forall ij \in \mathcal{G}$.
4. *Assume A.WCDMC. Then* $p_{ij,U}(\mathcal{C}) = p^R_{ij,U}, \forall ij \in \mathcal{C}, \forall \mathcal{C} \subseteq \mathcal{G}$.
5. *All candidate equilibrium prices converge to Nash-in-Nash prices as* $\Lambda \to 0$.

Proof. We prove that each property holds in turn.

1. This follows directly from the iterative procedure: we start with $p^R_{ij,D}$ and then weakly increase prices to arrive at $p_{ij,U}(\mathcal{C})$.
2. By (C.2), $p_{ij,U}(\mathcal{C}) \leq (1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G}, ij) + \delta_{j,D} p^R_{ij,D} = p^R_{ij,U}$.
3. By construction, $p_{ij,U}(\{ij\}) = (1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\{ij\}) + \delta_{j,D} p^R_{ij,D} = p^R_{ij,U}$, with the equality following again from (C.2).
4. Suppose, by contradiction, that A.WCDMC holds but $\exists \mathcal{C} \subseteq \mathcal{G}$ and $lm \in \mathcal{C}$ such that $p_{lm,U}(\mathcal{C}) < p^R_{lm,U}$ (Claim 2 rules out the inequality in the other direction). Then, by the construction of $p_{lm,U}(\mathcal{C})$ there must exist $\mathcal{B} \subseteq \mathcal{C}, lm \in \mathcal{B}$ for which the constraint in (C.2) binds. Using this $\mathcal{B}$, we arrive at a contradiction:

$$\sum_{ij\in\mathcal{B}} p^R_{ij,U} > \sum_{ij\in\mathcal{B}} p_{ij,U}(\mathcal{C})$$
$$= (1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\mathcal{B}) + \delta_{j,D}\sum_{ij\in\mathcal{B}} p^R_{ij,D}$$
$$\geq (1-\delta_{j,D})\sum_{ij\in\mathcal{B}} \Delta\pi_{j,D}(\mathcal{G}, ij) + \delta_{j,D}\sum_{ij\in\mathcal{B}} p^R_{ij,D} = \sum_{ij\in\mathcal{B}} p^R_{ij,U},$$

where the first line follows from Claim 2 and the contradictory assumption, the second line from our choice of $\mathcal{B}$, the third line inequality from A.WCDMC, and the final equality from (1).

5. In odd periods, offers are $p^R_{ij,D}$ which converge to Nash-in-Nash prices as $\Lambda \to 0$ by Lemma 2.1. In even periods, offers satisfy $p^R_{ij,D} \leq p_{ij,U}(\mathcal{C}) \leq p^R_{ij,U}$. Since the offers lie between two sets of offers that converge to Nash-in-Nash prices, by the sandwich theorem, they also converge to Nash-in-Nash prices. $\square$

We now prove the main result.

Lemma C.2 *The candidate strategies described above comprise an equilibrium.*

Proof. We now prove that no unilateral deviation is profitable on the part of any firm. Consider any period t where there are $\mathcal{C} \subseteq \mathcal{G}$ open agreements.

Upstream firm, t odd. Consider first an upstream firm U_i's decision of which offers to accept at an odd period t given it receives offers $\{\tilde{p}_{ij}\}_{ij \in \mathcal{C}_{i,U}}$. If U_i engages in a one-shot deviation by rejecting a subset $\mathcal{K} \subseteq \mathcal{C}_{i,U}$ of its open agreements, U_i expects that: (i) all other upstream firms will accept all of their open agreements at period t; and (ii) all non-accepted agreements $\mathcal{K}$ will form at period $t+1$ at prices $p_{ij,U}(\mathcal{K})$ (as we only consider one-shot deviations, and play is expected to follow the prescribed equilibrium strategies from $t+1$ onwards). We define the increase in U_i's payoffs from rejecting agreements $\mathcal{K} \subseteq \mathcal{C}_{i,U}$ (and following equilibrium strategies thereafter) as:

$$F(\mathcal{K}) \equiv -\left[(1-\delta_{i,U})\Delta\pi_{i,U}(\mathcal{G},\mathcal{K}) + \sum_{ij \in \mathcal{K}} [\tilde{p}_{ij} - \delta_{i,U} p_{ij,U}(\mathcal{K})]\right], \tag{C.3}$$

where we omit the fact that F is implicitly a function of the firm U_i, the set of open agreements $\mathcal{C}$, the set of candidate equilibrium strategies employed by other firms, and the set of prices that U_i receives. U_i chooses to reject agreements $\hat{\mathcal{K}} = \arg\max_{\mathcal{K} \subseteq \mathcal{C}_{i,U}} F(\mathcal{K})$, as this set of rejections maximizes its payoffs.

Consider first the case where U_i receives candidate equilibrium offers $\{p^R_{ij,D}\}_{ij \in \mathcal{C}_{i,U}}$. Substituting these prices into (C.3), we obtain:

$$\begin{aligned} F(\mathcal{K}) &= -\left[(1-\delta_{i,U})\Delta\pi_{i,U}(\mathcal{G},\mathcal{K}) + \sum_{ij \in \mathcal{K}} \left[p^R_{ij,D} - \delta_{i,U} p_{ij,U}(\mathcal{K})\right]\right] \\ &\leq -\left[(1-\delta_{i,U})\Delta\pi_{i,U}(\mathcal{G},\mathcal{K}) + \sum_{ij \in \mathcal{K}} \left[p^R_{ij,D} - \delta_{i,U} p^R_{ij,U}\right]\right] \\ &\leq -\left[\sum_{ij \in \mathcal{K}} \left[(1-\delta_{i,U})\Delta\pi_{i,U}(\mathcal{G},\{ij\}) + p^R_{ij,D} - \delta_{i,U} p^R_{ij,U}\right]\right] = 0\,, \end{aligned} \tag{C.4}$$

where the second line uses Lemma C.1(2) ($p_{ij,U}(\mathcal{K}) \leq p^R_{ij,U}$), the third line inequality follows from A.WCDMC or A.SCDMC (a), and the third line equality uses (2). Since $F(\emptyset) = 0$, $F(\mathcal{K})$ is maximized for $\mathcal{K} = \emptyset$. This implies that at equilibrium prices, U_i maximizes surplus by rejecting no offer, or equivalently, accepting all offers. Thus, in this case, U_i cannot gain by deviating from its candidate equilibrium strategy.

Consider next the case where U_i receives exactly one non-equilibrium offer, $\tilde{p}_{ij} \neq p^R_{ij,D}$. In this case, the increase in U_i's payoffs from rejecting agreements $\mathcal{K} \subseteq \mathcal{C}_{i,U}$ can be expressed as:

$$\begin{aligned} F(\mathcal{K}) &= \underbrace{-\left[(1-\delta_{i,U})\Delta\pi_{i,U}(\mathcal{G},\mathcal{K}) + \sum_{kj \in \mathcal{K}} \left[p^R_{kj,D} - \delta_{k,U} p_{kj,U}(\mathcal{K})\right]\right]}_{\leq 0 \text{ by (C.4)}} + 1_{\mathcal{K}}(\{ij\})(p^R_{ij,D} - \tilde{p}_{ij}) \\ &\leq 1_{\mathcal{K}}(\{ij\})(p^R_{ij,D} - \tilde{p}_{ij})\,, \end{aligned}$$

where $1_{\mathcal{K}}(\{ij\})$ is an indicator function for $ij \in \mathcal{K}$. Note that the increase in U_i's profits from rejecting only agreement ij can be expressed as:

$$\begin{aligned}
F(\{ij\}) &= -\Big[(1-\delta_{i,U})\Delta\pi_{i,U}(\mathcal{G},\{ij\}) + \tilde{p}_{ij} - \delta_{i,U} p_{ij,U}(\{ij\})\Big] \\
&= \underbrace{-\Big[(1-\delta_{i,U})\Delta\pi_{i,U}(\mathcal{G},\{ij\}) + p^R_{ij,D} - \delta_{i,U} p^R_{ij,U}\Big]}_{=0 \text{ by } (2)} + (p^R_{ij,D} - \tilde{p}_{ij}) \\
&= (p^R_{ij,D} - \tilde{p}_{ij}) \ ,
\end{aligned}$$

where the second line follows from an application of Lemma C.1(3) ($p_{ij,U}(\{ij\}) = p^R_{ij,U}$) and a re-arranging of terms. Thus, if $\tilde{p}_{ij} \geq p^R_{ij,D}$, $F(\mathcal{K})$ is again maximized for $\mathcal{K} = \emptyset$ (accept all offers); if $\tilde{p}_{ij} < p^R_{ij,D}$, $F(\mathcal{K})$ is maximized for $\mathcal{K} = \{ij\}$ (reject only D_j's offer). Hence, upon receiving one non-equilibrium offer, U_i does not have a profitable deviation from its prescribed equilibrium strategy.

Finally, in cases with multiple off-equilibrium offers, the strategy profile specified above states that U_i picks an arbitrary $\hat{\mathcal{K}}$ that maximizes $F(\mathcal{K})$. By definition then, this is a best response and no unilateral deviation is profitable. Note that receiving multiple off-equilibrium offers is unreachable from candidate equilibrium play by any unilateral deviation.

Downstream firm, t even. Next consider a downstream firm D_j's decision of which offers to accept at an even period t given it receives offers $\{\tilde{p}_{ij}\}_{ij \in \mathcal{C}_{j,D}}$. If D_j engages in a one-shot deviation by rejecting a subset $\mathcal{K} \subseteq \mathcal{C}_{j,D}$ of its open agreements, D_j expects that: (i) all other downstream firms will accept all of their open agreements at period t; and (ii) all non-accepted offers $\mathcal{K}$ will form agreement at period $t+1$ at Rubinstein prices $p^R_{ij,D}$. We define the increase in D_j's payoffs from rejecting agreements $\mathcal{K} \subseteq \mathcal{C}_{j,D}$ (and following equilibrium strategies thereafter) as:

$$F(\mathcal{K}) \equiv -\left[(1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\mathcal{K}) + \sum_{ij \in \mathcal{K}} \left[-\tilde{p}_{ij} + \delta_{j,D} p^R_{ij,D}\right]\right] , \tag{C.5}$$

where we omit the fact that F is implicitly a function of the firm D_j, the set of open agreements $\mathcal{C}$, the set of candidate equilibrium strategies employed by other firms, and the set of prices that D_j receives. D_j chooses to reject agreements $\hat{\mathcal{K}} = \arg\max_{\mathcal{K} \subseteq \mathcal{C}_{j,D}} F(\mathcal{K})$, as this set of rejections maximizes its payoffs.

Consider first the case where D_j receives candidate equilibrium offers $\{p_{ij,U}(\mathcal{C})\}_{ij \in \mathcal{C}_{j,D}}$. Substituting these prices into (C.5) and then applying (C.2), we obtain:

$$F(\mathcal{K}) = -\left[(1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\mathcal{K}) + \sum_{ij \in \mathcal{K}} \left[-p_{ij,U}(\mathcal{C}) + \delta_{j,D} p^R_{ij,D}\right]\right] \leq 0. \tag{C.6}$$

Since $F(\emptyset) = 0$, $F(\mathcal{K})$ is maximized for $\mathcal{K} = \emptyset$. This implies that at equilibrium prices, D_j maximizes surplus by rejecting no offer, or equivalently, accepting all offers. Thus, in this case, D_j cannot gain by deviating from its candidate equilibrium strategy.

Consider next the case where D_j receives exactly one non-equilibrium offer, $\tilde{p}_{ij} \neq p_{ij,U}(\mathcal{C})$. In this case, the increase in D_j's payoffs from rejecting agreements $\mathcal{K} \subseteq \mathcal{C}_{j,D}$ can be expressed as:

$$\begin{aligned}
F(\mathcal{K}) &= \underbrace{-\left[(1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\mathcal{K}) + \sum_{ik \in \mathcal{K}} \left[-p_{ik,U}(\mathcal{C}) + \delta_{j,D} p^R_{ik,D}\right]\right]}_{\leq 0 \text{ by (C.6)}} + 1_{\mathcal{K}}(\{ij\})(-p_{ij,U}(\mathcal{C}) + \tilde{p}_{ij}) \\
&\leq 1_{\mathcal{K}}(\{ij\})(-p_{ij,U}(\mathcal{C}) + \tilde{p}_{ij}) \ .
\end{aligned} \tag{C.7}$$

From (C.7), if $\tilde{p}_{ij} < p_{ij,U}(\mathcal{C})$, then there is no profitable deviation from the candidate equilibrium strategy of accepting all offers (as $F(\mathcal{K})$ is maximized at $\mathcal{K} = \emptyset$). If $\tilde{p}_{ij} \geq p_{ij,U}(\mathcal{C})$, there are two cases to consider:

1. Under A.WCDMC, by Lemma C.1(4), $p_{ij,U}(\mathcal{C}) = p^R_{ij,U}$. By substituting this in to (C.7) and then

applying (2), the inequality can be shown to be an equality for $\mathcal{K} = \{ij\}$; thus, $F(\mathcal{K})$ is maximized for $\mathcal{K} = \{ij\}$ (reject only U_i's offer).

2. Otherwise, note first that if D_j rejects some set of offers $\mathcal{K}$ that does not include ij, then $F(\mathcal{K}) \leq 0$ from (C.7); if D_j rejects no offers, $F(\emptyset) = 0$. By rejecting a set of offers $\mathcal{K}$ that includes ij and all other offers which are included in some constraint from (C.2) that binds, $F(\mathcal{K}) = p_{ij,U}(\mathcal{C}) - \tilde{p}_{ij} > 0$. Thus, D_j's best response must include rejecting U_i's offer, and potentially includes rejecting other offers.

Again, there are no profitable deviations from prescribed strategies.

Finally, in cases with multiple off-equilibrium offers, the strategy profile specified above states that D_j picks an arbitrary $\hat{\mathcal{K}}$ that maximizes $F(\mathcal{K})$. By definition, this is a best response and no unilateral deviation is profitable.

Upstream firm, t even. Next, we consider the decision for an upstream firm U_i of what offers to propose at an even period t. Consider the possibility that U_i deviates from the candidate equilibrium strategies and offers prices $\tilde{p}_{ij} \neq p_{ij,U}(\mathcal{C})$ for all ij in some $\mathcal{K} \subseteq \mathcal{C}_{i,U}$. By passive beliefs, each firm D_j receiving $\tilde{p}_{ij}$ perceives that it is the only one to have received an off-equilibrium offer. Given the above discussion regarding D's strategies in this case, if $\tilde{p}_{ij} < p_{ij,U}(\mathcal{C})$, then it will be accepted, while if $\tilde{p}_{ij} > p_{ij,U}(\mathcal{C})$, then it will be rejected, potentially along with some other offers kj. Clearly, U_i will never choose to offer $\tilde{p}_{ij} < p_{ij,U}(\mathcal{C})$, since it can always offer $p_{ij,U}(\mathcal{C})$ instead, without affecting the set of acceptances.

Thus, the only possible profitable deviation left is for U_i to offer $\tilde{p}_{ij} > p_{ij,U}(\mathcal{C})$ for all ij in $\mathcal{K}$. Given the candidate equilibrium strategies, these offers will be rejected at period t, and then accepted at $t+1$ at prices $p^R_{ij,D}$. Let $a_1, \ldots, a_{|\mathcal{K}|}$ denote the downstream firms with offers in $\mathcal{K}$ and let C_k denote the set of offers rejected by D_{a_k} following its deviant offer from U_i. Given the downstream firms' strategies, it will be the case each downstream firm D_{a_k} will reject ia_k, and $ia_k \in C_k$. Note that under A.WCDMC, given the prescribed equilibrium strategy profiles each downstream firm D_{a_k} will *only* reject ia_k, and $C_k = \{ia_k\}$; under A.FEAS, a downstream firm may also reject additional agreements.

Then, the decrease in U_i's surplus from raising the price on some set of offers $\mathcal{K} \subseteq \mathcal{C}_{i,U}$ is:

$$\sum_{ij \in \mathcal{K}} [p_{ij,U}(\mathcal{C}) - \delta_{i,U} p^R_{ij,D}] + (1 - \delta_{i,U}) \Delta\pi_{i,U}(\mathcal{G}, C_1 \cup \ldots \cup C_{|K|}) \tag{C.8}$$

$$= \sum_{ij \in \mathcal{K}} \left[p_{ij,U}(\mathcal{C}) - \delta_{i,U} p^R_{ij,D}\right] + (1 - \delta_{i,U}) \sum_{k=1}^{|K|} \Delta\pi_{i,U}(\mathcal{G} \setminus (C_{k+1} \cup \ldots \cup C_{|K|}), C_k)$$

$$\geq \sum_{ij \in \mathcal{K}} \left[p_{ij,U}(\mathcal{C}) - \delta_{i,U} p^R_{ij,D}\right] + (1 - \delta_{i,U}) \sum_{k=1}^{|K|} \Delta\pi_{i,U}(\mathcal{G}, \{ia_k\})$$

$$= \sum_{ij \in \mathcal{K}} \left[p_{ij,U}(\mathcal{C}) - \delta_{i,U} p^R_{ij,D} + (1 - \delta_{i,U}) \Delta\pi_{i,U}(\mathcal{G}, ij)\right]$$

$$\geq \sum_{ij \in \mathcal{K}} \left[p^R_{ij,D} - \delta_{i,U} p^R_{ij,U} + (1 - \delta_{i,U}) \Delta\pi_{i,U}(\mathcal{G}, ij)\right] = 0 \ ,$$

implying that such a deviation is not profitable. In (C.8), the second term of the second line splits the change in surplus from the postponed agreements by downstream firms. The third line then either follows directly from A.WCDMC (as $C_k = \{ia_k\}$), or applies A.SCDMC to each element of the second term, which can be done since each element can be expressed as the difference in U_i's profits between when D_{a_k} accepts all its offers and when it rejects both U_i's offer and the other offers in C_k, holding constant the fact that all other agreements are formed except for $C_{k+1} \cup \ldots C_{|K|}$.[3] The fourth line rearranges terms. The final line inequality follows from Lemma C.1(1) and Lemma 2.2, and the last equality from (2).

Downstream firm, t odd. Finally, we consider the decision for a downstream firm D_j of what offers to propose at an odd period t. Consider the possibility that D_j deviates from the candidate equilibrium

[3]Let $\mathcal{B} \equiv \mathcal{G} \setminus (C_{k+1} \cup \ldots \cup C_{|K|})$, $\mathcal{A} \equiv C_k \setminus \{ia_k\}$, and $\mathcal{A}' \equiv \emptyset$. As $\mathcal{A}$ and $\mathcal{A}'$ only differ in agreements formed by D_{a_k}, $\Delta\pi_{i,U}(\mathcal{G} \setminus (C_{k+1} \cup \ldots \cup C_{|K|}), C_k) = \pi_{i,U}(\mathcal{A} \cup \mathcal{B} \cup \{ia_k\}) - \pi_{i,U}(\mathcal{A}' \cup \mathcal{B}) \geq \Delta\pi_{i,U}(\mathcal{G}, \{ia_k\})$, where the last inequality follows from A.SCDMC.

strategies and offers prices different from $\tilde{p}_{ij} \neq p^R_{ij,D}$ for all ij in some $\mathcal{K} \subseteq \mathcal{C}_{j,D}$. By passive beliefs, each firm U_i receiving $\tilde{p}_{ij}$ perceives that it is the only one to have received an off-equilibrium offer. Given candidate equilibrium strategies, if $\tilde{p}_{ij} > p^R_{ij,D}$, then it will be accepted, while if $\tilde{p}_{ij} < p^R_{ij,D}$, then it will be rejected, with no other impact on the acceptance of offers not in $\mathcal{K}$. D_j will not deviate and offer $\tilde{p}_{ij} > p^R_{ij,D}$ to any U_i, as it can always offer $p^R_{ij,D}$ instead and do strictly better (as no other agreements are affected). The only possible profitable deviation for D_j is to offer $\tilde{p}_{ij} < p^R_{ij,D}$ for all ij in $\mathcal{K}$. Again, given the candidate equilibrium strategies, these offers will be rejected at period t, and then accepted at $t+1$ at prices $p_{ij,U}(\mathcal{K})$. The decrease in D_j's payoffs from engaging in such a deviation is:

$$\begin{aligned}(1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\mathcal{K}) - &\sum_{ij\in\mathcal{K}}\left[p^R_{ij,D} - \delta_{j,D}p_{ij,U}(\mathcal{K})\right] \\ &\geq \sum_{ij\in\mathcal{K}}\left[(1-\delta_{j,D})p^{Nash}_{ij} - p^R_{ij,D} + \delta_{j,D}p_{ij,U}(\mathcal{K})\right] \\ &> \sum_{ij\in\mathcal{K}}\left[(1-\delta_{j,D})p^R_{ij,D} - p^R_{ij,D} + \delta_{j,D}p_{ij,U}(\mathcal{K})\right] \\ &= \delta_{j,D}\sum_{ij\in\mathcal{K}}\left[p_{ij,U}(\mathcal{K}) - p^R_{ij,D}\right] \geq 0\ ,\end{aligned} \tag{C.9}$$

where the second line of (C.9) follows from A.FEAS (and implied by A.WCDMC), the third line from Lemma 2.2, the final line equality from rearranging terms, and the final line inequality from Lemma C.1(2). Thus, D_j has no profitable deviation.

Since there are no profitable one-shot deviations for any agent in both odd and even periods, the candidate set of strategies comprise an equilibrium. By Lemma C.1(5), prices at this equilibrium converge to Nash-in-Nash prices. □

C.3 Proof of Theorem 3.1 (Necessity)

We now prove that if A.WCDMC does not hold, there is no equilibrium where at every period t and history h^t all open agreements $ij \in \mathcal{C}(h^t)$ immediately form at prices $p^R_{ij,D}$ ($p^R_{ij,U}$) if t is odd (even). We proceed by contradiction: assume that such an equilibrium exists, and that there exists an upstream firm U_i and a set of agreements $\mathcal{K} \subseteq \mathcal{G}_{i,U}$ such that $\Delta\pi_{ij,U}(\mathcal{G},\mathcal{K}) < \sum_{ij\in\mathcal{K}}\Delta\pi_{i,U}(\mathcal{G},\{ij\})$ (the proof is symmetric if A.WCDMC is violated for some downstream firm D_j). Consider again the gain in one-shot surplus from U_i rejecting all $ij \in \mathcal{K}$, denoted $F(\mathcal{K})$, evaluated at period $t_0 = 1$. From the candidate equilibrium, we know that U_i has formed agreements at prices $p^R_{ij,D}$ for all $ij \in \mathcal{G}_{i,U}$. $F(\mathcal{K})$ is given by:

$$\begin{aligned}F(\mathcal{K}) &\equiv -\left[(1-\delta_{i,U})\Delta\pi_{i,U}(\mathcal{G},\mathcal{K}) + \sum_{ij\in\mathcal{K}}\left[p^R_{ij,D} - \delta_{i,U}p^R_{ij,U}\right]\right] \\ &> -\left[\sum_{ij\in\mathcal{K}}(1-\delta_{i,U})\Delta\pi_{i,U}(\mathcal{G},\{ij\}) + \sum_{ij\in\mathcal{K}}\left[p^R_{ij,D} - \delta_{i,U}p^R_{ij,U}\right]\right] = 0,\end{aligned}$$

where the inequality follows from the assumption that A.WCDMC does not hold and the equality from (2). Hence, it is a profitable deviation for U_i to reject the offers in $\mathcal{K}$ at period 1, implying a contradiction. □

D Proof of Theorem 4.1 (Uniqueness for No-Delay Equilibria)

Consider any no-delay equilibrium. Theorem 4.1 follows from the following two claims.

Claim A: In every odd period t with history h^t, each agreement $ij \in \mathcal{C}(h^t)$ has equilibrium price $p_{ij}(h^t) \geq p^R_{ij,D}$. In every even period with history h^t, each agreement $ij \in \mathcal{C}(h^t)$ has equilibrium price $p_{ij}(h^t) \leq p^R_{ij,U}$.

Proof of Claim A. We prove the claim by contradiction. First, suppose that there is an odd period t with history of play h^t for which $p_{ij}(h^t) < p^R_{ij,D}$. Then, U_i has a profitable one-shot deviation: reject ij and accept all its other offers. In this case, at period $t+1$, ij will be the only open agreement (as all other agreements form at t in a no-delay equilibrium). Thus, following this deviation, ij will then form at price $p^R_{ij,U}$ at period $t+1$ by Rubinstein (1982). The gains to U_i from this deviant action will then be:

$$\delta_{i,U} p^R_{ij,U} - p_{ij}(h^t) - (1-\delta_{i,U})\Delta\pi_{i,U}(\mathcal{G},\{ij\}) > \delta_{i,U} p^R_{ij,U} - p^R_{ij,D} - (1-\delta_{i,U})\Delta\pi_{i,U}(\mathcal{G},\{ij\}) = 0,$$

where the inequality follows from the contradictory assumption and the equality follows from (2). Hence this deviation is profitable. The even period proof is symmetric and omitted.

Claim B: Fix $\varepsilon > 0$. For any odd (even) period history of play h^t, $\exists\ \bar{\Lambda} > 0$ such that if $\Lambda \leq \bar{\Lambda}$, then the equilibrium price $p_{ij}(h^t) \leq p^{Nash}_{ij} + \varepsilon$ (for even periods, $p_{ij}(h^t) \geq p^{Nash}_{ij} - \varepsilon$).

Proof of Claim B. Define $\bar{\Lambda}$ as any positive number that is small enough so that: (a) when $\Lambda \leq \bar{\Lambda}$, the maximum absolute value of profits to any firm for any subset of agreements over period length Λ is less than $\varepsilon/2$; and (b) the maximum of the absolute value of the difference between $\delta p^R_{ij,U}$ and p^{Nash}_{ij} across all agreements in $\mathcal{G}$ is also less than $\varepsilon/2$.[4]

We now prove our claim by contradiction. First, suppose that there is an odd period history of play h^t for which, for some $\Lambda \leq \bar{\Lambda}$, there is an agreement ij that is formed where $p_{ij}(h^t) > p^{Nash}_{ij} + \varepsilon$. Consider the following deviation by D_j: at h^t, D_j makes a deviant offer $\tilde{p}_{ij}$ sufficiently low that it is sure to be rejected by U_i. D_j will expect that, at period t, U_i will reject this deviant offer (and potentially some other offers), and that all offers that do not involve U_i will be accepted. Let $\mathcal{K}$ denote the set of offers that U_i rejects following this deviant offer from D_j. At period $t+1$, given that h^{t+1} is the history following D_j's period t deviant action, U_i will propose offers at equilibrium prices $\{p_{ik}(h^{t+1})\}_{\forall ik \in \mathcal{K}}$; furthermore, D_j expects that all agreements will be formed at the end of $t+1$ (given no-delay equilibrium strategies). The gain to D_j from this deviation is:

$$\begin{aligned}\underbrace{p_{ij}(h^t) - \delta_{j,D} p_{ij}(h^{t+1})}_{\text{change in payments}} - \underbrace{(1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\mathcal{K})}_{\text{change in flow profits}} &> p^{Nash}_{ij} + \varepsilon - \delta_{j,D} p^R_{ij,U} - (1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\mathcal{K}) \\ &> \varepsilon - \varepsilon/2 - \varepsilon/2 = 0,\end{aligned}$$

where the first line inequality follows from the contradictory assumption ($p_{ij}(h^t) > p^{Nash}_{ij} + \varepsilon$) and Claim A ($p_{ij}(h^t) \leq p^R_{ij,U}$), and the second line follows from the assumption that $\Lambda < \bar{\Lambda}$ (implying that $|p^{Nash}_{ij} - \delta_{j,D} p^R_{ij,U}| < \varepsilon/2$ and $|(1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\mathcal{K})| < \varepsilon/2$). Thus, this deviation is profitable for D_j, implying a contradiction.

We have thus shown that, for any $\varepsilon > 0$, there is a $\bar{\Lambda}$ such that for any $\Lambda < \bar{\Lambda}$, equilibrium prices in odd periods are bounded above by Nash-in-Nash prices plus ε. The even period proof is symmetric and omitted.

□

E Proof of Theorem 4.2 (Uniqueness Without Assuming Immediate Agreement)

We prove Theorem 4.2 under two sets of conditions. The first set, as stated in the main text, comprises A.GFT, A.SCDMC, A.LNEXT, and restricts consideration to common tie-breaking equilibria. We also prove that our uniqueness result holds under A.GFT, A.SCDMC, and a "no-externalities" assumption (discussed informally in Section 4.2) without restricting attention to common tie-breaking equilibria. The no-externalities assumption states that any firm's profits only depend on agreements that directly involve that firm:

Assumption E.1 (A.NEXT: No Externalities)
For upstream firms: for all $i = 1, \ldots, N$, $\mathcal{A} \subseteq \mathcal{G}_{i,U}$, and $\mathcal{B}, \mathcal{B}' \subseteq \mathcal{G}^U_{-i}$,

$$\pi_{i,U}(\mathcal{A} \cup \mathcal{B}) = \pi_{i,U}(\mathcal{A} \cup \mathcal{B}').$$

[4] Such a $\bar{\Lambda}$ exists, since profits are bounded and $\lim_{\Lambda \to 0} p^R_{ij,U} = p^{Nash}_{ij}$ by Lemma 2.1.

For downstream firms: for all $j = 1, \ldots, M$, $\mathcal{A} \subseteq \mathcal{G}_{j,D}$, and $\mathcal{B}, \mathcal{B}' \subseteq \mathcal{G}^D_{-j}$,

$$\pi_{j,D}(\mathcal{A} \cup \mathcal{B}) = \pi_{j,D}(\mathcal{A} \cup \mathcal{B}').$$

It is straightforward to prove that A.SCDMC and A.NEXT directly imply A.LNEXT.

Thus, in our subsequent proofs, we will assume that A.GFT, A.SCDMC, and A.LNEXT hold, and either restrict attention to common tie-breaking equilibria or assume A.NEXT. For the proofs in this section, we will use equilibrium to refer to common tie-breaking equilibrium when A.NEXT is not employed.

E.1 Inductive Structure and Base Case

For any history h^t with open agreements $\mathcal{C}$ at the start of period t, let $\Gamma_{\mathcal{C}}(h^t)$ be the subgame starting at period t. We prove Theorem 4.2 by induction on the set of open agreements $\mathcal{C}$ in any subgame $\Gamma_{\mathcal{C}}(h^t)$. The base case is provided by analyzing $\Gamma_{\mathcal{C}}(h^t)$ where $|\mathcal{C}| = 1$: i.e., there is only one agreement that has not yet been formed at period t.

Lemma E.2 (Base Case) *Consider any subgame $\Gamma_{\mathcal{C}}(h^t)$ for which $|\mathcal{C}| = 1$. Then $\Gamma_{\mathcal{C}}(h^t)$, where $\mathcal{C} \equiv \{ij\}$, has a unique equilibrium involving immediate agreement at t at prices $p^R_{ij,D}$ if t is odd, and $p^R_{ij,U}$ if t is even.*

Proof. With only one open agreement $ij \in \mathcal{C}$, U_i and D_j engage in a two-player Rubinstein alternating offers bargaining game over joint surplus $\Delta\pi_{i,U}(\mathcal{G}, \{ij\}) + \Delta\pi_{j,D}(\mathcal{G}, \{ij\})$. The result directly follows from Rubinstein (1982). (Note that Rubinstein (1982) only requires A.GFT.) □

We now state the inductive hypothesis and inductive step used to prove Theorem 4.2.

Inductive Hypothesis. *Consider any $\mathcal{C} \subseteq \mathcal{G}$. For any subgame $\Gamma_{\mathcal{B}}(h^t)$ for which $\mathcal{B} \subsetneq \mathcal{C}$, every equilibrium results in immediate agreement for all $ij \in \mathcal{B}$ at prices $p^R_{ij,D}$ if t is odd, and $p^R_{ij,U}$ if t is even.*

The inductive hypothesis is that any subgame of $\Gamma_{\mathcal{C}}(h^t)$ that begins with fewer open agreements than $|\mathcal{C}|$ results in immediate agreement at the Rubinstein prices.

Lemma E.3 (Inductive Step) *Consider any subgame $\Gamma_{\mathcal{C}}(h^t)$ for which $|\mathcal{C}| > 1$. Given the inductive hypothesis, every equilibrium of $\Gamma_{\mathcal{C}}(h^t)$ has immediate agreement for all $ij \in \mathcal{C}$ at prices $p^R_{ij,D}$ if t is odd, and $p^R_{ij,U}$ if t is even.*

The inductive step states that if the inductive hypothesis holds for $\Gamma_{\mathcal{C}}(h^t)$, then $\Gamma_{\mathcal{C}}(h^t)$ also results in immediate agreement for all open agreements at Rubinstein prices. Note that Lemmas E.2 (Base Case) and E.3 (Inductive Step) imply Theorem 4.2 by induction: as we have established that the theorem holds when $|\mathcal{C}| = 1$, the inductive step implies that the theorem will hold for any $\mathcal{C} \subseteq \mathcal{G}$ and history of play h^t.

To prove Lemma E.3 (and by consequence, Theorem 4.2), we first prove the simultaneity of agreements—i.e., if any open agreements are formed in a period, all open agreements are formed—at Rubinstein prices. We employ separate lemmas for two separate cases, depending on whether there are multiple receiving firms (Lemma E.5) or a single receiving firm (Lemma E.6) in a given period. Our proofs of simultaneity use A.SCDMC and restrict attention to common tie-breaking equilibria (or, alternatively, use A.NEXT). We then prove immediacy of agreement—i.e., that all open agreements form in the current period without delay—in Lemma E.7. This proof uses A.SCDMC and A.LNEXT. Establishing Lemmas E.5-E.7 proves our result.

Before proceeding, we state and prove the following lemma that we will use in our proofs:

Lemma E.4 *Assume A.GFT and A.LNEXT. Then $\forall \mathcal{C} \subseteq \mathcal{G}$, $\exists ij \in \mathcal{C}$ such that:*

$$\Delta\pi_{j,D}(\mathcal{G}, \mathcal{C}) > \sum_{hj \in \mathcal{C}_{j,D}} p^R_{hj,U}, \qquad \textit{and}$$

$$\Delta\pi_{i,U}(\mathcal{G}, \mathcal{C}) > -\sum_{ik \in \mathcal{C}_{i,U}} p^R_{ik,D}.$$

Proof. By A.LNEXT, $\forall \mathcal{C} \subseteq \mathcal{G}$, $\exists ij \in \mathcal{C}$ such that:

$$\Delta\pi_{j,D}(\mathcal{G},\mathcal{C}) \geq \sum_{hj \in \mathcal{C}_{j,D}} \Delta\pi_{j,D}(\mathcal{G},\{hj\}), \qquad \text{and}$$
$$\Delta\pi_{i,U}(\mathcal{G},\mathcal{C}) \geq \sum_{ik \in \mathcal{C}_{i,U}} \Delta\pi_{i,U}(\mathcal{G},\{ik\}).$$

By A.GFT, $\Delta\pi_{j,D}(\mathcal{G},\{hj\}) > p^R_{hj,U}$ and $\Delta\pi_{i,U}(\mathcal{G},\{ik\}) > -p^R_{ik,D}$ for all agreements $hj, ik \in \mathcal{G}$ (see Lemma 2.2). The lemma immediately follows. □

E.2 Simultaneity of Agreements

Lemma E.5 (Simultaneity of Agreements: Multiple Receiving Firms.) *Assume that the inductive hypothesis holds.*

1. *Suppose that $\Gamma_\mathcal{C}(h^{\tilde{t}})$ is such that there are at least two upstream firms with open agreements in $\mathcal{C}$. In any equilibrium of $\Gamma_\mathcal{C}(h^{\tilde{t}})$ where the first open agreement in $\mathcal{C}$ is formed at an odd period $t \geq \tilde{t}$, all agreements $ij \in \mathcal{C}$ must form at t and at prices $p^R_{ij,D}$.*

2. *Suppose that $\Gamma_\mathcal{C}(h^{\tilde{t}})$ is such that there are at least two downstream firms with open agreements in $\mathcal{C}$. In any equilibrium of $\Gamma_\mathcal{C}(h^{\tilde{t}})$ where the first open agreement in $\mathcal{C}$ is formed at an even period $t \geq \tilde{t}$, all agreements $ij \in \mathcal{C}$ must form at t and at prices $p^R_{ij,U}$.*

Proof. We prove case 1 using two claims (A and B); the proof of case 2 is symmetric and omitted.

Claim A: In any equilibrium of $\Gamma_\mathcal{C}(h^{\tilde{t}})$ where the first set of open agreements $\mathcal{B} \subseteq \mathcal{C}$, $\mathcal{B} \neq \emptyset$, are formed at an odd period $t \geq \tilde{t}$, then all open agreements in $\mathcal{C}$ also are formed at period t.

Proof of Claim A. By contradiction, assume that there is an equilibrium of $\Gamma_\mathcal{C}(h^{\tilde{t}})$ where $\mathcal{B} \neq \mathcal{C}$ so that a non-empty set of agreements does not form at period t. By the inductive hypothesis, all agreements $hk \in \mathcal{C} \setminus \mathcal{B}$ will form at period $t+1$ at prices $p^R_{hk,U}$. Consider some agreement ij that: (i) is formed at period $t+1$ following equilibrium play under $\Gamma_\mathcal{C}(h^{\tilde{t}})$ and (ii) $\exists h \neq i$ such that U_h has an agreement which forms at period t. Such an ij must exist since $\mathcal{C}$ includes agreements for more than one upstream firm and (by the contradictory assumption) not all agreements form at period t.

Now consider the following deviation by D_j at period t: D_j offers $\tilde{p}_{ij} \equiv p^R_{ij,D} + \varepsilon$ to U_i, where $0 < \varepsilon < p^R_{ij,U} - p^R_{ij,D}$.[5] By passive beliefs, U_i expects at least one agreement to form at period t (e.g., involving U_h) upon receiving this deviant offer from D_j; by the inductive hypothesis, U_i therefore expects that all agreements that do not form at period t will form at period $t+1$. Note that:

1. Such a deviant offer will be accepted by U_i.

   By contradiction, suppose not, and U_i rejects D_j's deviant offer and instead accepted some (potentially empty) set of offers $\mathcal{A} \subseteq (\mathcal{C}_{i,U} \setminus ij)$ at period t. The gain to U_i from adding ij to $\mathcal{A}$ is strictly positive:

$$\begin{aligned}(1-\delta_{i,U})(\pi_{i,U}((\mathcal{G}\setminus\mathcal{C}) \cup \mathcal{A} \cup \mathcal{B}_{-i,U} \cup \{ij\}) - \pi_{i,U}((\mathcal{G}\setminus\mathcal{C}) \cup \mathcal{A} \cup \mathcal{B}_{-i,U})) + \tilde{p}_{ij} - \delta_{i,U} p^R_{ij,U} \\ = (1-\delta_{i,U})\Delta\pi_{i,U}((\mathcal{G}\setminus\mathcal{C}) \cup \mathcal{A} \cup \mathcal{B}_{-i,U} \cup \{ij\}, \{ij\}) + p^R_{ij,D} + \varepsilon - \delta_{i,U} p^R_{ij,U} \\ \geq (1-\delta_{i,U})\Delta\pi_{i,U}(\mathcal{G},\{ij\}) + p^R_{ij,D} + \varepsilon - \delta_{i,U} p^R_{ij,U} = \varepsilon,\end{aligned} \tag{E.10}$$

   where the second line is definitional, the third line inequality follows from A.SCDMC, and the last equality follows from (2). Hence, any best response to this deviant offer by U_i must include accepting ij.

2. Such a deviation is profitable for D_j if accepted by U_i.

[5] By Lemma 2.2, $p^R_{ij,U} > p^R_{ij,D}$, so $\varepsilon > 0$.

If U_i accepts the deviant offer in addition to some set of offers $\mathcal{A} \subseteq (\mathcal{C}_{i,U} \setminus ij)$, D_j's gain from this deviant offer is:

$$\begin{aligned}(1-\delta_{j,D})[\pi_{i,U}((\mathcal{G}\setminus\mathcal{C})\cup\mathcal{A}\cup\mathcal{B}_{-i,U}\cup\{ij\}) - \pi_{i,U}((\mathcal{G}\setminus\mathcal{C})\cup\mathcal{B}_{i,U}\cup\mathcal{B}_{-i,U})] - \tilde{p}_{ij} + \delta_{j,D}p^R_{ij,U} \\ \geq (1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\{ij\}) - p^R_{ij,D} - \varepsilon + \delta_{j,D}p^R_{ij,U} \\ > (1-\delta_{j,D})p^R_{ij,U} - p^R_{ij,D} - \varepsilon + \delta_{j,D}p^R_{ij,U} \\ = p^R_{ij,U} - p^R_{ij,D} - \varepsilon > 0,\end{aligned} \tag{E.11}$$

where the second line follows from A.SCDMC, the third line inequality follows from Lemma 2.2, the last line equality follows from rearranging terms, and the final inequality follows from Lemma 2.2 and the choice of ε.

This is a profitable deviation for D_j, yielding a contradiction. Thus, if the first agreement forms in odd period t, all agreements must form at period t.

Claim B: In any equilibrium of $\Gamma_{\mathcal{C}}(h^{\tilde{t}})$ where all open agreements $\mathcal{C}$ are formed at an odd period $t \geq \tilde{t}$, all agreements $ij \in \mathcal{C}$ are formed at prices $\hat{p}_{ij} = p^R_{ij,D}$.

Proof of Claim B. By contradiction, assume that all open agreements $\mathcal{C}$ are formed at period t, but $\hat{p}_{ij} \neq p^R_{ij,D}$ for some $ij \in \mathcal{C}$. Consider the following two cases:

1. Suppose $\hat{p}_{ij} < p^R_{ij,D}$ for some ij.

   Consider the deviation where U_i rejects only this offer ij at t. Since all other agreements form at period t, from the inductive hypothesis, U_i expects to form ij at $t+1$ at price $p^R_{ij,U}$. U_i's gain from this deviation is positive:

   $$\delta_{i,U}p^R_{ij,U} - \hat{p}_{ij} - (1-\delta_{i,U})\Delta\pi_{i,U}(\mathcal{G},\{ij\}) > \delta_{i,U}p^R_{ij,U} - p^R_{ij,D} - (1-\delta_{i,U})\Delta\pi_{i,U}(\mathcal{G},\{ij\}) = 0,$$

   where the last equality follows from (2), implying a contradiction.

2. Suppose $\hat{p}_{ij} > p^R_{ij,D}$ for some ij.

   Consider the deviation where D_j lowers its offer from $\hat{p}_{ij}$ to some $\tilde{p}_{ij} \in (p^R_{ij,D}, \hat{p}_{ij})$. We first show that, under either action, *every* best response for U_i must include accepting offer $\tilde{p}_{ij}$ and forming agreement ij at period t. Suppose, by contradiction, that a best response for U_i at t would be to form only agreements $\mathcal{A} \subseteq \mathcal{C}_{i,U} \setminus \{ij\}$. Similar to the logic used to derive (E.10), the gain to U_i from also forming ij in addition to $\mathcal{A}$ is strictly positive:

   $$\begin{aligned}(1-\delta_{i,U})\Delta\pi_{i,U}\big((\mathcal{G}\setminus\mathcal{C}_{i,U})\cup\mathcal{A}\cup\{ij\},\{ij\}\big) + \tilde{p}_{ij} - \delta_{i,U}p^R_{ij,U} \\ > (1-\delta_{i,U})\Delta\pi_{i,U}(\mathcal{G},\{ij\}) + p^R_{ij,D} - \delta_{i,U}p^R_{ij,U} = 0,\end{aligned}$$

   where the last equality follows from (2), implying that forming only agreements in $\mathcal{A}$ was not a best response. An analogous equation but with $\hat{p}_{ij}$ replacing $\tilde{p}_{ij}$ (not shown) also holds, thus implying that any best response for U_i at t given the candidate equilibrium strategies also must involve forming agreement ij.

   We now show that the set of best responses for U_i at t given candidate equilibrium prices coincides to the set of best responses for U_i at t given the deviation by D_j. Consider any best response set of acceptances to D_j's deviation at t. Accepting $\mathcal{A}\cup\{ij\}$, $\mathcal{A} \subseteq \mathcal{C}_{i,U} \setminus \{ij\}$, is a best response for U_i if and only if the value to U_i of accepting this set is weakly greater than the maximum value of accepting any set $\mathcal{A}' \cup \{ij\}$, $\mathcal{A}' \subseteq \mathcal{C}_{i,U} \setminus \{ij\}$. This condition can be written as:

$$\begin{aligned}(1-\delta_{i,U})\pi\big((\mathcal{G}\setminus\mathcal{C}_{i,U})\cup\mathcal{A}\cup\{ij\}\big) + \sum_{ik\in\mathcal{A}}\hat{p}_{ik} + \delta_{i,U}\sum_{ik\in\mathcal{C}_{i,U}\setminus(\mathcal{A}\cup\{ij\})}p^R_{kj,U} \\ \geq \max_{\mathcal{A}'\subseteq\mathcal{C}_{i,U}}\left\{(1-\delta_{i,U})\pi\big((\mathcal{G}\setminus\mathcal{C}_{i,U})\cup\mathcal{A}'\cup\{ij\}\big) + \sum_{ik\in\mathcal{A}'}\hat{p}_{ik} + \delta_{i,U}\sum_{ik\in\mathcal{C}_{i,U}\setminus(\mathcal{A}'\cup\{ij\})}p^R_{kj,U}\right\},\end{aligned}$$

(where we omit the price paid for ij since we have shown that any best response to the deviant offer requires this agreement to be formed at t). This condition is the same as the one determining whether a set $\mathcal{A} \cup \{ij\}$ is a best response for U_i at t under candidate equilibrium strategies (as we have shown earlier in Claim B of the lemma that any best response for U_i under these strategies also must involve agreement ij being formed), implying that the sets of best responses are the same. Because the sets of best responses are the same and we restrict attention to a common tie-breaking equilibrium, U_i must accept the same set of offers—i.e., all offers in $\mathcal{C}_{i,U}$—upon receiving this deviant offer from D_j as under the candidate equilibrium strategies. Thus, the deviant offer will increase profits to D_j by $\hat{p}_{ij} - \tilde{p}_{ij} > 0$, which leads to a contradiction.

If we assume A.NEXT instead of restricting attention to common tie-breaking equilibria, then having shown that U_i accepts the deviant offer $\tilde{p}_{ij}$ is sufficient for D_j to have a profitable deviation, as under A.NEXT, D_j's profits are unaffected by U_i's other acceptances.

Thus, $\hat{p}_{ij} = p^R_{ij,D}$ $\forall ij \in \mathcal{C}$ if the first open agreement in $\mathcal{C}$ forms at an odd period. □

Lemma E.6 (Simultaneity of Agreements: Single Receiving Firm.) *Assume that the inductive hypothesis holds.*

1. *Suppose that $\Gamma_{\mathcal{C}}(h^{\tilde{t}})$ is such that there is exactly one downstream firm, but more than one upstream firm, with open agreements in $\mathcal{C}$. In any equilibrium of $\Gamma_{\mathcal{C}}(h^{\tilde{t}})$ where the first agreement is formed at an even period $t \geq \tilde{t}$, all agreements $ij \in \mathcal{C}$ must form at t and at prices $p^R_{ij,U}$.*

2. *Suppose that $\Gamma_{\mathcal{C}}(h^{\tilde{t}})$ is such that there is exactly one upstream firm, but more than one downstream firm, with open agreements in $\mathcal{C}$. In any equilibrium of $\Gamma_{\mathcal{C}}(h^{\tilde{t}})$ where the first agreement is formed at an odd period $t \geq \tilde{t}$, all agreements $ij \in \mathcal{C}$ must form at t and at prices $p^R_{ij,D}$.*

Proof. We prove case 1 of the lemma; the proof of case 2 is symmetric and omitted.

For this lemma, we cannot apply induction in the case where the single receiving firm rejects all of its offers as the subgame beginning in the following period will have the same set of open agreements. Analyzing this case is more involved and utilizes an argument similar to Rubinstein (1982) and Shaked and Sutton (1984), where bounds on equilibrium prices are obtained by showing that the receiving firm cannot credibly reject a sufficiently generous offer in any equilibrium without the expectation of an even more generous (and infeasible) offer in a future subgame.

We start with two definitions. For any subgame $\Gamma_{\mathcal{C}}(h^t)$ and equilibrium where all agreements in $\mathcal{C}$ are eventually formed, let $\{p_{ij}(\Gamma_{\mathcal{C}}(h^t))\}_{ij\in\mathcal{C}}$ be the equilibrium prices for this game (which need not all form at t), and define $\phi_{\Gamma_{\mathcal{C}}(h^t)} \equiv \sum_{ij\in\mathcal{C}}[p^R_{ij,D} - p_{ij}(\Gamma_{\mathcal{C}}(h^t))]$ to be the *total discount* from prices $p^R_{ij,D}$ that D_j obtains in this equilibrium of this subgame.

We prove the lemma with three claims.

Claim A: For any equilibrium and subgame $\Gamma_{\mathcal{C}}(h^{\tilde{t}})$ where all agreements in $\mathcal{C}$ are eventually formed, $\phi_{\Gamma_{\mathcal{C}}(h^{\tilde{t}})} \leq 0$.

Proof of Claim A. By contradiction, assume that there is an equilibrium where in some subgame $\Gamma_{\mathcal{C}}(h^{\tilde{t}})$, all agreements in $\mathcal{C}$ are eventually formed, and the total discount is strictly positive: $\phi_{\Gamma_{\mathcal{C}}(h^{\tilde{t}})} > 0$. Without loss of generality (since all agreements eventually form), assume that at least one open agreement forms at $\tilde{t}$ in this equilibrium.

Now consider all subgames $\{\Gamma_{\mathcal{C}}(h^t)\}$ of $\Gamma_{\mathcal{C}}(h^{\tilde{t}})$ (including $\Gamma_{\mathcal{C}}(h^{\tilde{t}})$ itself) where $t \geq \tilde{t}$, t is even, there are $\mathcal{C}$ open agreements at t, h^t is consistent with $h^{\tilde{t}}$ (i.e., h^t coincides with $h^{\tilde{t}}$ for all periods $\tilde{t}$ and earlier), and the first open agreement in $\mathcal{C}$ is formed at t given equilibrium strategies. Such subgames, if $t > \tilde{t}$, can be reached from $\Gamma_{\mathcal{C}}(h^{\tilde{t}})$ if all agreements in $\mathcal{C}$ are rejected in periods $\tilde{t}, \ldots, t-1$. In any such subgame $\Gamma_{\mathcal{C}}(h^t)$ where at least one agreement is formed at t, by the inductive hypothesis all agreements will be formed at latest by $t+1$. Let $\overline{\phi}$ denote the supremum of the total discount over all subgames of $\Gamma_{\mathcal{C}}(h^{\tilde{t}})$ satisfying the criteria above.[6]

[6]Note that $\overline{\phi}$ is finite since it cannot be greater than the sum of profits in the game.

Now choose subgame $\Gamma_{\mathcal{C}}(h^t)$ with at least one agreement forming at t that has a total discount very close to the supremum and strictly positive: i.e., for which $\delta_{j,D}\overline{\phi} < \phi_{\Gamma_{\mathcal{C}}(h^t)}$ and $\phi_{\Gamma_{\mathcal{C}}(h^t)} \geq \phi_{\Gamma_{\mathcal{C}}(h^{\bar{t}})} > 0$. At this subgame, fix some U_i for which (i) agreement ij forms at period t and (ii) $\hat{p}_{ij} \equiv p_{ij}(\Gamma_{\mathcal{C}}(h^t)) < p^R_{ij,D}$. Such a U_i must exist by the fact that the agreements that do not form at period t form at period $t+1$ at odd-period Rubinstein prices (which have no discount) by the inductive hypothesis. Finally, denote the agreements that form at period t at this subgame as $\hat{\mathcal{A}} \cup \{ij\}$, $\hat{\mathcal{A}} \subseteq \mathcal{C} \setminus \{ij\}$.

Now consider the following deviation by U_i at period t: U_i offers $\tilde{p}_{ij} \equiv \hat{p}_{ij} + \varepsilon$, where $\varepsilon > 0$ is sufficiently small such that (i) the total discount realized by D_j if it still forms agreements $\hat{\mathcal{A}} \cup \{ij\}$ is still strictly positive and greater than $\delta_{j,D}\overline{\phi}$ and (ii) $\hat{p}_{ij} + \varepsilon < p^R_{ij,U}$. Thus, by (2.2),

$$p^R_{ij,U} - \tilde{p}_{ij} + \sum_{kj \in \hat{\mathcal{A}}} (p^R_{kj,U} - \hat{p}_{kj}) > p^R_{ij,D} - \tilde{p}_{ij} + \sum_{kj \in \hat{\mathcal{A}}} (p^R_{kj,D} - \hat{p}_{kj}) > \delta_{j,D}\bar{\phi}\,, \tag{E.12}$$

We now show that any best response by D_j at t must include accepting ij. Suppose, by contradiction, that a best response for D_j involves accepting only offers $\mathcal{A} \subseteq \mathcal{C} \setminus \{ij\}$ at t. We consider four potential cases of equilibrium play following this candidate best response:

1. *$\mathcal{A} = \emptyset$, and no further agreements ever form (i.e., D_j rejects all offers in every subsequent even period, and makes sufficiently low offers for all open agreements in every subsequent odd period that all of its offers are rejected).*

   Consider an alternative action for D_j of accepting only agreement ij at price $\tilde{p}_{ij}$ at period t. If D_j accepts only ij, then all other agreements will form at $t+1$ by the inductive hypothesis. The gain (in period t units) to D_j from accepting only ij as opposed to following the candidate action and rejecting all offers at t is:

$$\begin{aligned}(1-\delta_{j,D})&\Delta\pi_{j,D}\big((\mathcal{G}\setminus\mathcal{C})\cup\{ij\},\{ij\}\big) - \tilde{p}_{ij} + \delta_{j,D}\Big(\Delta\pi_{j,D}(G,\mathcal{C}) - \sum_{kj\in\mathcal{C},k\neq i} p^R_{kj,D}\Big)\\ &> (1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\{ij\}) - p^R_{ij,U} + \delta_{j,D}\Big(\Delta\pi_{j,D}(G,\mathcal{C}) - \sum_{kj\in\mathcal{C},k\neq i} p^R_{kj,D}\Big)\\ &= -\delta_{j,D}p^R_{ij,D} + \delta_{j,D}\Big(\Delta\pi_{j,D}(G,\mathcal{C}) - \sum_{kj\in\mathcal{C},k\neq i} p^R_{kj,D}\Big)\\ &= \delta_{j,D}\Big(\Delta\pi_{j,D}(G,\mathcal{C}) - \sum_{kj\in\mathcal{C}} p^R_{kj,D}\Big) > 0,\end{aligned} \tag{E.13}$$

   where the second line follows from A.SCDMC and the definition of the deviant action ($\tilde{p}_{ij} \equiv \hat{p}_{ij} + \varepsilon < p^R_{ij,U}$), the third line follows from (1), and the final line from Lemma 2.2 and A.FEAS. Thus, accepting no offers at t is not a best response in this case.

2. *$\mathcal{A} = \emptyset$, and the first agreement $ij \in \mathcal{C}$ to form does so at an odd period $t+t'$, $t' \geq 1$.*

   In this case, by Lemma E.5, all agreements must form at time $t+t'$ at Rubinstein prices $p^R_{ij,D}$ (as there are multiple receiving (upstream) firms with open agreements). Consider an alternative action for D_j of accepting $\hat{\mathcal{A}} \cup \{ij\}$ at period t (i.e., original equilibrium acceptances at t). Note that the gain from following this alternative action relative to forming all agreements in $\mathcal{C}$ at period $t+1$ at odd-period

Rubinstein prices is:

$$\begin{aligned}
&(1-\delta_{j,D})\Delta\pi_{j,D}\big((\mathcal{G}\setminus\mathcal{C})\cup(\hat{\mathcal{A}}\cup\{ij\}),\hat{\mathcal{A}}\cup\{ij\}\big)-\tilde{p}_{ij}+\delta_{j,D}p^R_{ij,D}-\sum_{kj\in\hat{\mathcal{A}}}(\hat{p}_{kj}-\delta_{j,D}p^R_{kj,D})\\
&\geq(1-\delta_{j,D})\Big(\sum_{kj\in\hat{\mathcal{A}}\cup\{ij\}}\Delta\pi_{j,D}(\mathcal{G},\{kj\})\Big)-\tilde{p}_{ij}+\delta_{j,D}p^R_{ij,D}-\sum_{kj\in\hat{\mathcal{A}}}(\hat{p}_{kj}-\delta_{j,D}p^R_{kj,D})\\
&\geq\underbrace{\Big(\sum_{kj\in\hat{\mathcal{A}}\cup\{ij\}}(1-\delta_{j,D})\big(\Delta\pi_{j,D}(\mathcal{G},\{kj\})-p^R_{kj,U}+\delta_{j,D}p^R_{kj,D})\Big)}_{=0\text{ by (1)}}+p^R_{ij,U}-\tilde{p}_{ij}+\sum_{kj\in\hat{\mathcal{A}}}(p^R_{kj,U}-\hat{p}_{kj})\\
&>\delta_{j,D}\bar{\phi}>0,
\end{aligned}\tag{E.14}$$

where the second line follows from A.SCDMC, the third line follows from rearranging terms, and the last line follows from (E.12) and the assumption that $\bar{\phi}>0$. Thus, the alternative action yields D_j a payoff that is strictly higher than the payoff of forming all agreements at period $t+1$ at odd-period Rubinstein prices.

Next, the gain to D_j from forming all agreements in $\mathcal{C}$ at period $t+1$ at odd-period Rubinstein prices instead of forming all agreements in $\mathcal{C}$ in some odd period $t+t'$, $t'>1$, in period $t+1$ units, is:

$$\begin{aligned}
&(1-\delta^{t'-1}_{j,D})\Delta\pi_{j,D}(\mathcal{G},\mathcal{C})-\sum_{kj\in\mathcal{C}}p^R_{kj,D}+\delta^{t'-1}_{j,D}\sum_{kj\in\mathcal{C}}p^R_{kj,D}\\
&\qquad\geq(1-\delta^{t'-1}_{j,D})\sum_{kj\in\mathcal{C}}\Delta\pi_{j,D}(\mathcal{G},\{kj\})-\sum_{kj\in\mathcal{C}}p^R_{kj,D}+\delta^{t'-1}_{j,D}\sum_{kj\in\mathcal{C}}p^R_{kj,D}\\
&\qquad=(1-\delta^{t'-1}_{j,D})\sum_{kj\in\mathcal{C}}\big(\Delta\pi_{j,D}(\mathcal{G},\{kj\})-p^R_{kj,D}\big)>0,
\end{aligned}\tag{E.15}$$

where the second line follows from A.WCDMC, the third line equality follows by rearranging terms, and last inequality from Lemma 2.2. Thus, accepting no offers at t is not a best response in this case.

3. *$\mathcal{A}=\emptyset$, and the first agreement $ij\in\mathcal{C}$ to form does so at an even period $t+t'$, $t'\geq 2$.*

   Let $\mathcal{B}$ denote the equilibrium set of agreements that forms at time $t+t'$ following D_j's rejection of all offers at t. For any $kj\in\mathcal{B}$, let p'_{kj} denote the equilibrium price at which the agreement forms. By the inductive hypothesis, the remaining agreements, $\mathcal{C}\setminus\mathcal{B}$ form at $t+t'+1$ (odd) at odd-period Rubinstein prices.

   Consider an alternative action for D_j of accepting $\hat{\mathcal{A}}\cup\{ij\}$ at period t. From (E.14), the gain to D_j from following this alternative action as opposed to forming all agreements in $\mathcal{C}$ at period $t+1$ at odd-period Rubinstein prices is strictly greater than $\delta_{j,D}\overline{\phi}$.

   The gain to D_j from forming all agreements in $\mathcal{C}$ at period $t+1$ at odd-period Rubinstein prices relative to forming agreements $\mathcal{B}$ at t' and $\mathcal{C} \setminus \mathcal{B}$ at $t'+1$, is (in period $t+1$ units):

$$
\begin{aligned}
&(1-\delta_{j,D}^{t'-1})\Delta\pi_{j,D}(\mathcal{G},\mathcal{C}) + \delta_{j,D}^{t'-1}\left(\Delta\pi_{j,D}(\mathcal{G},\mathcal{C}\setminus\mathcal{B}) + \sum_{kj\in\mathcal{C}\setminus\mathcal{B}} \delta_{j,D}p^R_{kj,D} + \sum_{kj\in\mathcal{B}} p'_{kj}\right) - \sum_{kj\in\mathcal{C}} p^R_{kj,D} \\
&\geq (1-\delta_{j,D}^{t'-1})\Delta\pi_{j,D}(\mathcal{G},\mathcal{C}) + \delta_{j,D}^{t'-1}\left(\sum_{kj\in\mathcal{C}\setminus\mathcal{B}} \left(\Delta\pi_{j,D}(\mathcal{G},\{kj\}) + \delta_{j,D}p^R_{kj,D}\right) + \sum_{kj\in\mathcal{B}} p'_{kj}\right) - \sum_{kj\in\mathcal{C}} p^R_{kj,D} \\
&\geq (1-\delta_{j,D}^{t'-1})\sum_{kj\in\mathcal{C}}\Delta\pi_{j,D}(\mathcal{G},\{kj\}) + \delta_{j,D}^{t'-1}\left(\sum_{kj\in\mathcal{C}\setminus\mathcal{B}} p^R_{kj,U} + \sum_{kj\in\mathcal{B}} p'_{kj}\right) - \sum_{kj\in\mathcal{C}} p^R_{kj,D} \\
&> (1-\delta_{j,D}^{t'-1})\sum_{kj\in\mathcal{C}} p^R_{kj,D} + \delta_{j,D}^{t'-1}\left(\sum_{kj\in\mathcal{C}} p^R_{kj,D} - \overline{\phi}\right) - \sum_{kj\in\mathcal{C}} p^R_{kj,D} \\
&> -\overline{\phi},
\end{aligned}
$$

where the second line follows from A.WCDMC, the third line follows from A.WCDMC and (1), the fourth line follows from Lemma 2.2 and the definition of $\overline{\phi}$, and the final line follows from the fact that $t' \geq 2$. Thus, the gain from the alternative action relative to the supposed equilibrium strategy—now in period t units—is greater than $(\delta_{j,D} - \delta_{j,D})\overline{\phi} = 0$. Thus, accepting no offers at t is not a best response in this case.

4. *$\mathcal{A} \neq \emptyset$, and D_j forms some agreements in $\mathcal{C} \setminus \{ij\}$ at t.*

   By assumption, $ij \notin \mathcal{A}$. By the inductive hypothesis, the remaining agreements $\mathcal{C} \setminus \mathcal{A}$ form at time $t+1$ at odd-period Rubinstein prices. In this case, the gain to D_j from adding ij to the agreements in $\mathcal{A}$ instead of forming only agreements in $\mathcal{A}$, in period t units, is:

$$
\begin{aligned}
&(1-\delta_{j,D})\Delta\pi_{j,D}\big((\mathcal{G}\setminus\mathcal{C})\cup\mathcal{A}\cup\{ij\},\{ij\}\big) - \tilde{p}_{ij} + \delta_{j,D}p^R_{ij,D} \\
&\quad > (1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\{ij\}) - p^R_{ij,U} + \delta_{j,D}p^R_{ij,D} = 0,
\end{aligned}
\tag{E.16}
$$

   where the inequality follows from A.SCDMC and the definition of the deviant action ($\tilde{p}_{ij} \equiv \hat{p}_{ij} + \varepsilon < p^R_{ij,U}$), and the equality from (1). Thus, forming agreements $\mathcal{A}$ where $ij \notin \mathcal{A}$ is not a best response in this case.

Thus, any best response by D_j must include accepting agreement ij at price $\tilde{p}_{ij}$.

Now consider any best response for D_j to the deviant offer by U_i. Accepting offers $\mathcal{A} \cup \{ij\}$, $\mathcal{A} \subseteq \mathcal{C} \setminus \{ij\}$ is a best response if and only if the value to D_j of accepting this set is weakly greater than the maximum value of accepting any set $\mathcal{A}' \cup \{ij\}$, $\mathcal{A}' \subseteq \mathcal{C} \setminus \{ij\}$. This condition can be written as:

$$
\begin{aligned}
&(1-\delta_{j,D})\pi((\mathcal{G}\setminus\mathcal{C})\cup\mathcal{A}\cup\{ij\}) + \sum_{kj\in\mathcal{A}}\hat{p}_{ij} + \delta_{j,D}\sum_{kj\in\mathcal{C}\setminus(\mathcal{A}\cup\{ij\})} p^R_{kj,D} \\
&\qquad \geq \max_{\mathcal{A}'\subseteq\mathcal{C}}\left\{(1-\delta_{j,D})\pi((\mathcal{G}\setminus\mathcal{C})\cup\mathcal{A}'\cup\{ij\}) + \sum_{kj\in\mathcal{A}'}\hat{p}_{ij} + \delta_{j,D}\sum_{kj\in\mathcal{C}\setminus(\mathcal{A}'\cup\{ij\})} p^R_{kj,D}\right\},
\end{aligned}
$$

where we excluded the price paid for ij since this agreement always forms at period t, and we apply the inductive hypothesis to obtain period $t+1$ prices. Using the same logic as in Lemma E.5, the condition is the same as for a set being a best response under the candidate equilibrium offers implying that the sets of best responses are the same.

Because the sets of best responses are the same and we consider a common tie-breaking equilibrium, D_j accepts the same set of offers under the deviant offer from U_i. Moreover, in both cases, all other agreements will form at period $t+1$. Thus, the deviant offer will increase profits to U_i by $\tilde{p}_{ij} - \hat{p}_{ij} > 0$, which leads to a contradiction.

By the same arguments as in Lemma E.5, assuming A.NEXT instead of restricting attention to common tie-breaking equilibrium also leads to a contradiction.

Claim B: For any equilibrium and subgame $\Gamma_{\mathcal{C}}(h^{\tilde{t}})$ where the first agreement is formed at an even period $t \geq \tilde{t}$, all agreements $ij \in \mathcal{C}$ form at t.

Proof of Claim B. By contradiction, assume that $\hat{\mathcal{A}} \subsetneq \mathcal{C}$ agreements form at period t, and $\hat{\mathcal{A}} \neq \emptyset$. By the inductive hypothesis, all agreements in $\mathcal{C} \setminus \hat{\mathcal{A}}$ are formed at period $t+1$ at prices $p^R_{ij,D}$. Consider a deviation where, at period t, some U_i, $ij \in \mathcal{C} \setminus \hat{\mathcal{A}}$, offers $\tilde{p}_{ij}$, for $p^R_{ij,D} < \tilde{p}_{ij} < p^R_{ij,U}$.

1. Such a deviant offer will be accepted by D_j.

   By contradiction, suppose not, and a best response for D_j is to accept offers $\mathcal{A} \subseteq \mathcal{C} \setminus \{ij\}$. We again consider four cases of equilibrium play following this candidate best response:

   (a) *$\mathcal{A} = \emptyset$ and no further agreements form.*

   Consider an alternative action for D_j of forming only agreement ij at price $\tilde{p}_{ij}$ instead of rejecting all offers at t; the gain (in period t units) from this alternative as opposed to the candidate action is:

$$
\begin{aligned}
&(1-\delta_{j,D})\Delta\pi_{j,D}\big((\mathcal{G} \setminus \mathcal{C}) \cup \{ij\}, \{ij\}\big) - \tilde{p}_{ij} + \delta_{j,D}\Delta\pi_{j,D}(G, \mathcal{C}) - \delta_{j,D} \sum_{kj \in \mathcal{C}, k \neq i} p^R_{kj,D} \\
&\quad > (1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G}, \{ij\}) - p^R_{ij,U} + \delta_{j,D}\Delta\pi_{j,D}(G, \mathcal{C}) - \delta_{j,D} \sum_{kj \in \mathcal{C}, k \neq i} p^R_{kj,D} > 0,
\end{aligned}
$$

   where the logic is identical to (E.13). Thus, accepting no offers at t is not a best response in this case.

   (b) *$\mathcal{A} = \emptyset$, and the first agreement $ij \in \mathcal{C}$ to form does so at an odd period $t + t'$, $t' \geq 1$.*

   In this case, by Lemma E.5, all agreements form at time $t + t'$ at Rubinstein prices, as there are multiple upstream (receiving) firms in an odd period. Consider an alternative action for D_j of forming only agreement ij at price $\tilde{p}_{ij}$ instead of rejecting all offers at t; the gain from following this alternative action as opposed to forming all agreements in $\mathcal{C}$ at period $t+1$ at odd-period Rubinstein prices is:

$$
\begin{aligned}
&(1-\delta_{j,D})\Delta\pi_{j,D}\big((\mathcal{G} \setminus \mathcal{C}) \cup \{ij\}, \{ij\}\big) - \tilde{p}_{ij} + \delta_{j,D} p^R_{ij,D} \\
&\quad > (1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G}, \{ij\}) - p^R_{ij,U} + \delta_{j,D} p^R_{ij,D} = 0,
\end{aligned}
\tag{E.17}
$$

   where the inequality follows from A.SCDMC and the definition of the deviant offer ($\tilde{p}_{ij} < p^R_{ij,U}$), and the equality from (1). By (E.15), the gain to D_j from forming all agreements in $\mathcal{C}$ at period $t+1$ at odd-period Rubinstein prices as opposed to forming all agreements in any future odd period $t + t'$, $t' \geq 1$, is weakly positive. Thus, accepting no offers at t is not a best response in this case.

   (c) *$\mathcal{A} = \emptyset$ and the first agreement $ij \in \mathcal{C}$ to form does so at an even period $t + t'$, $t' \geq 2$.*

   Let $\mathcal{B}$ denote the set of equilibrium agreements that form at time $t + t'$ following D_j's rejection of all offers at t. For any $kj \in \mathcal{B}$, let p'_{kj} denote the equilibrium price at which the agreement forms. By the inductive hypothesis, the remaining agreements, $\mathcal{C} \setminus \mathcal{B}$ form at time $t + t' + 1$ (odd) at odd-period Rubinstein prices.

   Consider an alternative action for D_j of forming only agreement ij at price $\tilde{p}_{ij}$ instead of rejecting all offers at t. From (E.17), the gain to D_j from choosing this alternative action as opposed to forming all agreements in $\mathcal{C}$ at period $t+1$ at odd-period Rubinstein prices is strictly positive.

   The gain to D_j from forming all agreements in $\mathcal{C}$ at period $t+1$ at odd-period Rubinstein prices as opposed to forming agreements $\mathcal{B}$ at t' and $\mathcal{C} \setminus \mathcal{B}$ at $t'+1$, in period $t+1$ units, is:

$$
\begin{aligned}
&(1-\delta_{j,D}^{t'-1})\Delta\pi_{j,D}(\mathcal{G},\mathcal{C}) + \delta_{j,D}^{t'-1}\left(\Delta\pi_{j,D}(\mathcal{G},\mathcal{C}\setminus\mathcal{B}) + \sum_{kj\in\mathcal{C}\setminus\mathcal{B}} \delta_{j,D} p^R_{kj,D} + \sum_{kj\in\mathcal{B}} p'_{kj}\right) - \sum_{kj\in\mathcal{C}} p^R_{kj,D} \\
&\geq (1-\delta_{j,D}^{t'-1})\Delta\pi_{j,D}(\mathcal{G},\mathcal{C}) + \delta_{j,D}^{t'-1}\left(\sum_{kj\in\mathcal{C}\setminus\mathcal{B}} \left[\Delta\pi_{j,D}(\mathcal{G},\{kj\}) + \delta_{j,D} p^R_{kj,D}\right] + \sum_{kj\in\mathcal{B}} p'_{kj}\right) - \sum_{kj\in\mathcal{C}} p^R_{kj,D} \\
&\geq (1-\delta_{j,D}^{t'-1})\sum_{kj\in\mathcal{C}}\Delta\pi_{j,D}(\mathcal{G},\{kj\}) + \delta_{j,D}^{t'-1}\left(\sum_{kj\in\mathcal{C}\setminus\mathcal{B}} p^R_{kj,U} + \sum_{kj\in\mathcal{B}} p'_{kj}\right) - \sum_{kj\in\mathcal{C}} p^R_{kj,D} \\
&> (1-\delta_{j,D}^{t'-1})\sum_{kj\in\mathcal{C}} p^R_{kj,D} + \delta_{j,D}^{t'-1}\left(\sum_{kj\in\mathcal{C}} p^R_{kj,D}\right) - \sum_{kj\in\mathcal{C}} p^R_{kj,D} = 0,
\end{aligned}
$$

where the second line follows from A.WCDMC, the third line follows from A.WCDMC and (1), the fourth line inequality follows from Lemma 2.2 and Claim A, and the final equality follows by rearranging terms. Thus, accepting no offers at t is not a best response in this case.

(d) *$\mathcal{A} \neq \emptyset$, and D_j forms some agreements in $\mathcal{C} \setminus \{ij\}$ at t.*
By assumption, $ij \notin \mathcal{A}$. By the inductive hypothesis, the remaining agreements, $\mathcal{C} \setminus \mathcal{A}$ form at time $t+1$ at odd-period Rubinstein prices. In this case, the gain to D_j from accepting offers in $\mathcal{A} \cup \{ij\}$ instead of accepting only offers in $\mathcal{A}$ at t, in period t units, is:

$$
\begin{aligned}
&(1-\delta_{j,D})\Delta\pi_{j,D}((\mathcal{G}\setminus\mathcal{C})\cup\mathcal{A}\cup\{ij\},\{ij\}) - \tilde{p}_{ij} + \delta_{j,D} p^R_{ij,D} \\
&\quad > (1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\{ij\}) - p^R_{ij,U} + \delta_{j,D} p^R_{ij,D} = 0,
\end{aligned}
\tag{E.18}
$$

where the logic is identical to (E.16). Thus, forming agreements $\mathcal{A}$ where $ij \notin \mathcal{A}$ is not a best response in this case.

Thus, any best response by D_j must include accepting the deviant offer $\tilde{p}_{ij}$ from U_i.

2. Such a deviation is profitable for U_i if accepted by D_j.

Suppose that, following this deviant offer, D_j accepts agreements $\mathcal{A}' \cup \{ij\}$ at period t, where $\mathcal{A}' \subseteq \mathcal{C} \setminus \{ij\}$. By the inductive hypothesis, all remaining agreements are formed at period $t+1$ at Rubinstein prices.

The gain to U_i from this deviation is then:

$$
\begin{aligned}
&(1-\delta_{i,U})\pi_{i,U}((\mathcal{G}\setminus\mathcal{C})\cup\mathcal{A}'\cup\{ij\}) - (1-\delta_{i,U})\pi_{i,U}((\mathcal{G}\setminus\mathcal{C})\cup\hat{\mathcal{A}}) + \tilde{p}_{ij} - \delta_{i,U} p^R_{ij,D} \\
&\quad \geq (1-\delta_{i,U})\Delta\pi_{i,U}((\mathcal{G},\{ij\}) + \tilde{p}_{ij} - \delta_{i,U} p^R_{ij,D} \\
&\quad > (1-\delta_{i,U})\Delta\pi_{i,U}((\mathcal{G},\{ij\}) + p^R_{ij,D} - \delta_{i,U} p^R_{ij,U} = 0,
\end{aligned}
$$

where the second line follows from A.SCDMC, the third line inequality follows from Lemma 2.2 and the definition of the deviant action, and the last equality follows from (1). Hence, U_i will find it profitable to make the deviation.

Thus U_i has a profitable deviation, yielding a contradiction.

Claim C: For any equilibrium and subgame $\Gamma_{\mathcal{C}}(h^{\tilde{t}})$ where all agreements in $\mathcal{C}$ are formed at an even period $t \geq \tilde{t}$, they are formed at prices $\hat{p}_{ij} = p^R_{ij,U}$ for all $ij \in \mathcal{C}$.

Proof of Claim C. By contradiction, assume that all agreements in $\mathcal{C}$ are formed at period t, but $\hat{p}_{ij} \neq p^R_{ij,U}$ for some $ij \in \mathcal{C}$.

1. Suppose that $\hat{p}_{ij} > p^R_{ij,U}$ for some ij.

   Consider the deviation where D_j rejects only this offer. Since all other agreements form at period t, by the inductive hypothesis, D_j forms this agreement at price $p^R_{ij,D}$ at $t+1$. Applying (1), D_j's gain from this action are:

   $$-\delta_{j,D} p^R_{ij,D} + \hat{p}_{ij} - (1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\{ij\}) > -\delta_{j,D} p^R_{ij,D} + p^R_{ij,U} - (1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\{ij\}) = 0,$$

   implying a profitable deviation and hence a contradiction.

2. Suppose $\hat{p}_{ij} < p^R_{ij,U}$ for some ij.

   Consider a deviation where U_i raises its offer from $\hat{p}_{ij}$ to some $\tilde{p}_{ij} \in (\hat{p}_{ij}, p^R_{ij,U})$. We now show that any best response set of acceptances for D_j must include accepting ij. Suppose, by contradiction, that D_j has a best response of accepting only agreements in $\mathcal{A} \subseteq \mathcal{C} \setminus \{ij\}$ at t following this deviation. We consider three cases for equilibrium play following this best response:

   (a) *$\mathcal{A} = \emptyset$ and no further agreements form.*
   The gain to D_j from accepting only ij instead of choosing this action, in period $t+1$ units, is:

   $$\begin{aligned}(1-\delta_{j,D})&\Delta\pi_{j,D}((\mathcal{G}\setminus\mathcal{C})\cup\{ij\},\{ij\}) - \tilde{p}_{ij} + \delta_{j,D}\Delta\pi_{j,D}(G,\mathcal{C}) - \delta_{j,D}\sum_{kj\in\mathcal{C},k\neq i} p^R_{ij,D} \\ &> (1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\{ij\}) - p^R_{ij,U} + \delta_{j,D}\Delta\pi_{j,D}(G,\mathcal{C}) - \delta_{j,D}\sum_{kj\in\mathcal{C},k\neq i} p^R_{ij,D} > 0,\end{aligned}$$

   where the logic is identical to (E.13). Thus accepting no offers at t is not a best response in this case.

   (b) *$\mathcal{A} = \emptyset$ and the first agreement $ij \in \mathcal{C}$ to form does so at period $t+t'$, $t' \geq 1$.*
   In this case, by Claim B, all agreements form at time $t+t'$. For any $kj \in \mathcal{C}$, let p'_{kj} denote the equilibrium price at which the agreement forms.
   Consider the alternative action by D_j of accepting only ij instead of rejecting all offers at t. First note that (E.17) applies and so the gain from following this deviant action as opposed to forming all agreements in $\mathcal{C}$ at period $t+1$ at odd-period Rubinstein prices is strictly positive. Next, note that the gain to D_j from forming all agreements in $\mathcal{C}$ at period $t+1$ at odd-period Rubinstein prices as opposed to forming all agreements in $\mathcal{C}$ at period $t+t'$ ($t' \geq 1$), in period $t+1$ units, is:

   $$\begin{aligned}(1-\delta^{t'-1}_{j,D})&\Delta\pi_{j,D}(\mathcal{G},\mathcal{C}) - \sum_{kj\in\mathcal{C}} p^R_{kj,D} + \delta^{t'-1}_{j,D}\sum_{kj\in\mathcal{C}} p'_{kj} \\ &\geq (1-\delta^{t'-1}_{j,D})\sum_{kj\in\mathcal{C}}\Delta\pi_{j,D}(\mathcal{G},\{kj\}) - \sum_{kj\in\mathcal{C}} p^R_{kj,D} + \delta^{t'-1}_{j,D}\sum_{kj\in\mathcal{C}} p^R_{kj,D} \\ &= (1-\delta^{t'-1}_{j,D})\sum_{kj\in\mathcal{C}}\left[\Delta\pi_{j,D}(\mathcal{G},\{kj\}) - p^R_{kj,D}\right] \geq 0,\end{aligned} \tag{E.19}$$

   where the second line follows from A.WCDMC and Lemma E.5 (if t' is odd) or Claim A (if t' is even), the third line equality follows by rearranging terms, and the last inequality from Lemma 2.2. Thus accepting no offers at t is not a best response in this case.

   (c) $\mathcal{A} \neq \emptyset$.
   By assumption, $ij \notin \mathcal{A}$. By the inductive hypothesis, all remaining agreements $\mathcal{C}\setminus\mathcal{A}$ form at time $t+1$ at odd-period Rubinstein prices. Applying (E.18), the gain to D_j from accepting offers in $\mathcal{A}\cup\{ij\}$ instead of accepting only offers in $\mathcal{A}$ at t is positive. Thus, forming agreements $\mathcal{A}$ where $ij \notin \mathcal{A}$ is not a best response in this case.

Thus, any best response by D_j must include accepting the deviant offer $\tilde{p}_{ij}$ from U_i. Now consider any best response set of acceptances, $\mathcal{A}\cup\{ij\}$, to the deviant prices. As in Claim A, the condition is the same as for a set being a best response under the candidate equilibrium agreements implying that the sets of best responses are the same. Because the sets of best responses are the same and we consider a common tie-breaking equilibrium, D_j accepts the same set of agreements—i.e., all agreements in $\mathcal{C}$—under the deviant offer from U_i. Thus, the deviant offer will increase profits to U_i by $\tilde{p}_{ij} - \hat{p}_{ij} > 0$, which leads to a contradiction. Furthermore, as in Claim A, A.NEXT can be used instead of restricting attention to common tie-breaking equilibria in order to establish the claim.

Thus, $\hat{p}_{ij} = p^R_{ij,U} \forall ij \in \mathcal{C}$ for agreements formed at any even period.

Claims A-C prove the lemma. □

E.3 Immediacy of Agreements

Given the inductive hypothesis, Lemmas E.5–E.6 establish that in any equilibrium of any subgame $\Gamma^{\tilde{t}}_{\mathcal{C}}$ where any agreement $ij \in \mathcal{C}$ forms at period $t \geq \tilde{t}$, all agreements in $\mathcal{C}$ form at t at Rubinstein prices. We now prove that, given the inductive hypothesis, there cannot be any delay: i.e., in any equilibrium, all agreements in $\mathcal{C}$ form immediately.

Lemma E.7 (Immediacy of all agreements.) *Assume that the inductive hypothesis holds. Then, any equilibrium of $\Gamma^t_{\mathcal{C}}$ results in all agreements $ij \in \mathcal{C}$ forming at period t.*

Proof. We prove the case where t is odd; the proof of the case where t is even is symmetric and omitted.

By contradiction, consider a candidate equilibrium where no agreements are formed at period t (as, by the previous results, if any agreement is formed at period t, all agreements are formed in that period). Let agreement $ij \in \mathcal{C}$ satisfy the conditions of A.LNEXT. We consider a deviant action by D_j from this candidate equilibrium and then verify that it is profitable for D_j. Suppose D_j offers $\tilde{p}_{ij}$ satisfying $p^R_{ij,D} < \tilde{p}_{ij} < p^R_{ij,U}$ to U_i. We first show that U_i will accept this offer and then show that it will increase D_j's surplus relative to the candidate equilibrium.

Suppose that U_i accepts the offer $\tilde{p}_{ij}$. Then, by passive beliefs, it believes that this is the only agreement to be formed at period t and, by the inductive hypothesis, that the remaining agreements will form at period $t+1$. Hence, its payoffs—in period t units—from accepting the offer are:

$$\underbrace{\tilde{p}_{ij} + (1-\delta_{i,U})\pi_{i,U}((\mathcal{G}\setminus\mathcal{C})\cup\{ij\})}_{\text{Payoff at } t} + \underbrace{\delta_{i,U}\left(\pi_{i,U}(\mathcal{G}) + \sum_{ik\in\mathcal{C}_{i,U}\setminus\{ij\}} p^R_{ik,U}\right)}_{\text{Payoff from } t+1 \text{ on}}$$

$$= \tilde{p}_{ij} + (1-\delta_{i,U})\Delta\pi_{i,U}((\mathcal{G}\setminus\mathcal{C})\cup\{ij\},\{ij\}) + \delta_{i,U}\left(\sum_{ik\in\mathcal{C}_{i,U}\setminus\{ij\}} p^R_{ik,U} + \pi_{i,U}(\mathcal{G})\right) + (1-\delta_{i,U})\pi_{i,U}(\mathcal{G}\setminus\mathcal{C})$$

$$> p^R_{ij,D} + (1-\delta_{i,U})\Delta\pi_{i,U}(\mathcal{G},\{ij\}) + \delta_{i,U}\left(\pi_{i,U}(\mathcal{G}) + \sum_{ik\in\mathcal{C}_{i,U}\setminus\{ij\}} p^R_{ik,U}\right) + (1-\delta_{i,U})\pi_{i,U}(\mathcal{G}\setminus\mathcal{C})$$

$$= \delta_{i,U}\left(\pi_{i,U}(\mathcal{G}) + \sum_{ik\in\mathcal{C}_{i,U}} p^R_{ik,U}\right) + (1-\delta_{i,U})\pi_{i,U}(\mathcal{G}\setminus\mathcal{C}),$$

where the second line adds and subtracts the $(1-\delta_{i,U})\pi_{i,U}(\mathcal{G}\setminus\mathcal{C})$ term, the third line follows from A.SCDMC and the definition of $\tilde{p}_{ij}$, and the final line uses (2) and then combines the $p^R_{ij,U}$ terms in the sum.

We next show that any best response for U_i must include accepting ij. Suppose, by contradiction, that a best response for U_i involves accepting only offers $\mathcal{B} \subseteq \mathcal{C}_{i,U} \setminus \{ij\}$ at t. We consider the following four cases of equilibrium play following this candidate best response:

1. *$\mathcal{B} = \emptyset$, and no agreements in $\mathcal{C}$ are ever formed.*

In this case, the payoffs to U_i are:

$$\begin{aligned}\pi_{i,U}(\mathcal{G}\setminus\mathcal{C}) &= \delta_{i,U}\pi_{i,U}(\mathcal{G}\setminus\mathcal{C}) + (1-\delta_{i,U})\pi_{i,U}(\mathcal{G}\setminus\mathcal{C})\\ &< \delta_{i,U}\left(\pi_{i,U}(\mathcal{G}) + \sum_{ik\in\mathcal{C}_{i,U}} p^R_{ik,D}\right) + (1-\delta_{i,U})\pi_{i,U}(\mathcal{G}\setminus\mathcal{C})\\ &< \delta_{i,U}\left(\pi_{i,U}(\mathcal{G}) + \sum_{ik\in\mathcal{C}_{i,U}} p^R_{ik,U}\right) + (1-\delta_{i,U})\pi_{i,U}(\mathcal{G}\setminus\mathcal{C}),\end{aligned}$$

where the second line follows from Lemma E.4 (which uses A.LNEXT) and the third line follows from Lemma 2.2. Since the payoffs to U_i from rejection are less than from accepting D_j's deviant offer, rejecting all offers is not a best response in this case.

2. *$\mathcal{B}=\emptyset$, and all agreements in $\mathcal{C}$ are formed in some even period $t+t'$ for $t'=1,3,5,\ldots$.*

   If U_i accepts no other offers at period t (and by passive beliefs, U_i believes that no agreements in $\mathcal{C}_{-i,U}$ are formed at t), the payoffs to U_i are:

$$\begin{aligned}&(1-\delta^{t'}_{i,U})\pi_{i,U}(\mathcal{G}\setminus\mathcal{C}) + \delta^{t'}_{i,U}\left(\pi_{i,U}(\mathcal{G}) + \sum_{ik\in\mathcal{C}_{i,U}} p^R_{ik,U}\right)\\ &= (1-\delta_{i,U})\pi_{i,U}(\mathcal{G}\setminus\mathcal{C}) + (\delta_{i,U}-\delta^{t'}_{i,U})\pi_{i,U}(\mathcal{G}\setminus\mathcal{C}) + \delta^{t'}_{i,U}\left(\pi_{i,U}(\mathcal{G}) + \sum_{ik\in\mathcal{C}_{i,U}} p^R_{ik,U}\right)\\ &< (1-\delta_{i,U})\pi_{i,U}(\mathcal{G}\setminus\mathcal{C}) + (\delta_{i,U}-\delta^{t'}_{i,U})\left(\pi_{i,U}(\mathcal{G}) + \sum_{ik\in\mathcal{C}_{i,U}} p^R_{ik,U}\right)\\ &\qquad + \delta^{t'}_{i,U}\left(\pi_{i,U}(\mathcal{G}) + \sum_{ik\in\mathcal{C}_{i,U}} p^R_{ik,U}\right)\\ &= (1-\delta_{i,U})\pi_{i,U}(\mathcal{G}\setminus\mathcal{C}) + \delta_{i,U}\left(\pi_{i,U}(\mathcal{G}) + \sum_{ik\in\mathcal{C}_{i,U}} p^R_{ik,U}\right),\end{aligned}$$

   where the second and fourth lines follow by rearranging terms and the third line follows from Lemma E.4. Since the payoffs to U_i from rejecting all offers at t are less than from accepting D_j's deviant offer, rejecting all offers is not a best response in this case.

3. *$\mathcal{B}=\emptyset$, and all agreements in $\mathcal{C}$ are formed in some odd period $t+t'$ for $t'=2,4,6,\ldots$.*

In this case, the payoffs to U_i are:

$$\begin{aligned}
&(1-\delta_{i,U}^{t'})\pi_{i,U}(\mathcal{G}\setminus\mathcal{C})+\delta_{i,U}^{t'}\left(\pi_{i,U}(\mathcal{G})+\sum_{ik\in\mathcal{C}_{i,U}}p_{ik,D}^{R}\right)\\
&<(1-\delta_{i,U}^{t'})\pi_{i,U}(\mathcal{G}\setminus\mathcal{C})+\delta_{i,U}^{t'}\left(\pi_{i,U}(\mathcal{G})+\sum_{ik\in\mathcal{C}_{i,U}}p_{ik,U}^{R}\right)\\
&=(1-\delta_{i,U})\pi_{i,U}(\mathcal{G}\setminus\mathcal{C})+(\delta_{i,U}-\delta_{i,U}^{t'})\pi_{i,U}(\mathcal{G}\setminus\mathcal{C})+\delta_{i,U}^{t'}\left(\pi_{i,U}(\mathcal{G})+\sum_{ik\in\mathcal{C}_{i,U}}p_{ik,U}^{R}\right)\\
&<(1-\delta_{i,U})\pi_{i,U}(\mathcal{G}\setminus\mathcal{C})+(\delta_{i,U}-\delta_{i,U}^{t'})\left(\pi_{i,U}(\mathcal{G})+\sum_{ik\in\mathcal{C}_{i,U}}p_{ik,U}^{R}\right)\\
&\qquad+\delta_{i,U}^{t'}\left(\pi_{i,U}(\mathcal{G})+\sum_{ik\in\mathcal{C}_{i,U}}p_{ik,U}^{R}\right)\\
&=(1-\delta_{i,U})\pi_{i,U}(\mathcal{G}\setminus\mathcal{C})+\delta_{i,U}\left(\pi_{i,U}(\mathcal{G})+\sum_{ik\in\mathcal{C}_{i,U}}p_{ik,U}^{R}\right),
\end{aligned}$$

where the second line follows from Lemma 2.2 and the remaining logic is identical to case 2. Since the payoffs to U_i from rejecting all offers at t are less than from accepting D_j's deviant offer, rejecting all offers is not a best response in this case.

4. *$\mathcal{B}\neq\emptyset$, and U_i forms some agreements in $\mathcal{C}_{i,U}\setminus\{ij\}$ at t.*

   In this case, by the inductive hypothesis, all remaining agreements $\mathcal{A}\equiv\mathcal{C}\setminus\mathcal{B}$ form in the following (even) period $t+1$ at Rubinstein prices. Thus, we can express the payoff to U_i from this action as $(1-\delta_{i,U})\pi_{i,U}((\mathcal{G}\setminus\mathcal{C})\cup\mathcal{B})+\sum_{ik\in\mathcal{B}}\hat{p}_{ik}+\delta_{i,U}\left[\pi_{i,U}(\mathcal{G})+\sum_{ik\in\mathcal{A}_{i,U}}p_{ik,U}^{R}\right]$, where $\hat{p}_{ik}\forall ik\in\mathcal{B}$ are the period t candidate equilibrium prices offered to U_i. But,

$$
\begin{aligned}
&(1-\delta_{i,U})\pi_{i,U}((\mathcal{G}\setminus\mathcal{C})\cup\mathcal{B}) + \sum_{ik\in\mathcal{B}}\hat{p}_{ik} + \delta_{i,U}\left[\pi_{i,U}(\mathcal{G}) + \sum_{ik\in\mathcal{A}_{i,U}} p^R_{ik,U}\right] \\
&= (1-\delta_{i,U})\pi_{i,U}((\mathcal{G}\setminus\mathcal{C})\cup\mathcal{B}) + \sum_{ik\in\mathcal{B}}\hat{p}_{ik} + \delta_{i,U}\left[\pi_{i,U}(\mathcal{G}) + p^R_{ij,U} + \sum_{ik\in\mathcal{A}_{i,U}\setminus\{ij\}} p^R_{ik,U}\right] \\
&= (1-\delta_{i,U})\pi_{i,U}((\mathcal{G}\setminus\mathcal{C})\cup\mathcal{B}) + \sum_{ik\in\mathcal{B}}\hat{p}_{ik} \\
&\qquad + p^R_{ij,D} + (1-\delta_{i,U})\Delta\pi_{i,U}(\mathcal{G},\{ij\}) + \delta_{i,U}\left[\pi_{i,U}(\mathcal{G}) + \sum_{ik\in\mathcal{A}_{i,U}\setminus\{ij\}} p^R_{ik,U}\right] \\
&< (1-\delta_{i,U})\pi_{i,U}((\mathcal{G}\setminus\mathcal{C})\cup\mathcal{B}) + \sum_{ik\in\mathcal{B}}\hat{p}_{ik} + \tilde{p}_{ij} + (1-\delta_{i,U})\Delta\pi_{i,U}(\mathcal{G},\{ij\}) + \delta_{i,U}\left[\pi_{i,U}(\mathcal{G}) + \sum_{ik\in\mathcal{A}_{i,U}\setminus\{ij\}} p^R_{ik,U}\right] \\
&\leq (1-\delta_{i,U})\pi_{i,U}((\mathcal{G}\setminus\mathcal{C})\cup\mathcal{B}) + \sum_{ik\in\mathcal{B}}\hat{p}_{ik} + \tilde{p}_{ij} + (1-\delta_{i,U})\Delta\pi_{i,U}((\mathcal{G}\setminus\mathcal{C})\cup\mathcal{B}\cup\{ij\},\{ij\}) \\
&\qquad + \delta_{i,U}\left[\pi_{i,U}(\mathcal{G}) + \sum_{ik\in\mathcal{A}_{i,U}\setminus\{ij\}} p^R_{ik,U}\right] \\
&= (1-\delta_{i,U})\pi_{i,U}((\mathcal{G}\setminus\mathcal{C})\cup\mathcal{B}\cup\{ij\}) + \sum_{ik\in\mathcal{B}}\hat{p}_{ik} + \tilde{p}_{ij} + \delta_{i,U}\left[\pi_{i,U}(\mathcal{G}) + \sum_{ik\in\mathcal{A}_{i,U}\setminus\{ij\}} p^R_{ik,U}\right],
\end{aligned}
$$

where the second and sixth lines follow by rearranging terms, the third line follows from (2), the fourth line follows from the the definition of the deviant offer, and the fifth line follows from A.SCDMC.

Since the final line is the value of accepting D_j's deviant offer and all agreements in $\mathcal{B}$, the payoff to U_i from accepting D_j's deviant offer and all agreements in $\mathcal{B}$ is higher than the payoff from accepting just the offers in $\mathcal{B}$. Thus, forming agreements $\mathcal{B}$, where $ij \notin \mathcal{B}$ is not a best response in this case.

Thus, any best response by U_i must include accepting the deviant offer $\tilde{p}_{ij}$ from D_j. Note that we have not ruled out the possibility that U_i may also choose to accept additional offers in $\mathcal{C}_{i,U}$ at period t upon accepting deviant offer $\tilde{p}_{ij}$; we return to this below.

Having verified that the $\tilde{p}_{ij}$ offer will be accepted by U_i, we now check that the acceptance of this deviant offer will be profitable for D_j. D_j knows that U_i is the only firm that will form agreement(s) at period t and, by the inductive hypothesis, that the remaining agreements will form at period $t+1$. However, it is possible that upon receiving the deviant offer, U_i will also accept some other offers $\mathcal{B} \subseteq \mathcal{C}_{i,U} \setminus \{ij\}$. Hence,

D_j's payoff—in period t units—from making the deviant offer satisfies:

$$\underbrace{-\tilde{p}_{ij} + (1-\delta_{j,D})\pi_{j,D}((\mathcal{G}\setminus\mathcal{C})\cup\mathcal{B}\cup\{ij\})}_{\text{Payoff at } t} + \underbrace{\delta_{j,D}\left(\pi_{j,D}(\mathcal{G}) - \sum_{kj\in\mathcal{C}_{j,D}\setminus\{ij\}} p^R_{kj,U}\right)}_{\text{Payoff from } t+1 \text{ on}}$$

$$\geq -\tilde{p}_{ij} + (1-\delta_{j,D})\Delta\pi_{j,D}((\mathcal{G},\{ij\}) + \delta_{j,D}\left(\pi_{j,D}(\mathcal{G}) - \sum_{kj\in\mathcal{C}_{j,D}\setminus\{ij\}} p^R_{kj,U}\right) + (1-\delta_{j,D})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C})$$

$$> -p^R_{ij,U} + (1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\{ij\}) + \delta_{j,D}\left(\pi_{j,D}(\mathcal{G}) - \sum_{kj\in\mathcal{C}_{j,D}\setminus\{ij\}} p^R_{kj,U}\right) + (1-\delta_{j,D})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C})$$

$$= -\delta_{j,D}p^R_{ij,D} + \delta_{j,D}\left(\pi_{j,D}(\mathcal{G}) - \sum_{kj\in\mathcal{C}_{j,D}\setminus\{ij\}} p^R_{kj,U}\right) + (1-\delta_{j,D})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C})$$

$$> (1-\delta_{j,D})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C}) + \delta_{j,D}\left(\pi_{j,D}(\mathcal{G}) - \sum_{kj\in\mathcal{C}_{j,D}} p^R_{kj,U}\right),$$

where the second line applies A.SCDMC, the third line follows from the definition of $\tilde{p}_{ij}$, the fourth line uses (1), and the final line uses Lemma 2.2 and then combines the $p^R_{kj,U}$ terms in the sum.

Next, we show that the lower bound on payoffs from this deviant offer being accepted (given by the last line of the previous set of equations) is higher than the payoff from equilibrium play under the candidate equilibrium. If D_j does not deviate from equilibrium play with the deviation $\tilde{p}_{ij}$, there are three possibilities for subsequent equilibrium play with no agreements formed at t:

1. *No further agreements are formed.*

   In this case, the payoffs to D_j from the candidate equilibrium are:

   $$\begin{aligned}\pi_{j,D}(\mathcal{G}\setminus\mathcal{C}) &= \delta_{j,D}\pi_{j,D}(\mathcal{G}\setminus\mathcal{C}) + (1-\delta_{j,D})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C})\\ &< (1-\delta_{j,D})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C}) + \delta_{j,D}\left(\pi_{j,D}(\mathcal{G}) - \sum_{kj\in\mathcal{C}_{j,D}} p^R_{kj,U}\right),\end{aligned}$$

   where the inequality follows from Lemma E.4. Thus, the payoffs to D_j from the candidate equilibrium are less than from accepting U_i's deviant offer in this case.

2. *All open agreements are formed in some even period $t+t'$, for $t'=1,3,5,\ldots$.*

In this case, the payoffs to D_j from the candidate equilibrium are:

$$
\begin{aligned}
&(1-\delta_{j,D}^{t'})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C})+\delta_{j,D}^{t'}\left(\pi_{j,D}(\mathcal{G})-\sum_{kj\in\mathcal{C}_{j,D}}p_{kj,U}^{R}\right)\\
&=(1-\delta_{j,D})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C})+(\delta_{j,D}-\delta_{j,D}^{t'})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C})+\delta_{j,D}^{t'}\left(\pi_{j,D}(\mathcal{G})-\sum_{kj\in\mathcal{C}_{j,D}}p_{kj,U}^{R}\right)\\
&<(1-\delta_{j,D})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C})+(\delta_{j,D}-\delta_{j,D}^{t'})\left(\pi_{j,D}(\mathcal{G})-\sum_{kj\in\mathcal{C}_{j,D}}p_{kj,U}^{R}\right)\\
&\qquad\qquad+\delta_{j,D}^{t'}\left(\pi_{j,D}(\mathcal{G})-\sum_{kj\in\mathcal{C}_{i,U}}p_{ik,U}^{R}\right)\\
&=(1-\delta_{j,D})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C})+\delta_{j,D}\left(\pi_{j,D}(\mathcal{G})-\sum_{kj\in\mathcal{C}_{j,D}}p_{kj,U}^{R}\right),
\end{aligned}
$$

where the second and fourth lines follow by rearranging terms and the third line follows from Lemma E.4. Thus, the payoffs to D_j from the candidate equilibrium are less than from making the deviant offer in this case.

3. *All open agreements are formed in some odd period $t+t'$, for $t'=2,4,6,\ldots$.*

In this case, the payoffs to D_j from the candidate equilibrium are:

$$
\begin{aligned}
&(1-\delta_{j,D}^{t'})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C}) + \delta_{j,D}^{t'}\left(\pi_{j,D}(\mathcal{G}) - \sum_{kj\in\mathcal{C}_{j,D}} p_{kj,D}^{R}\right)\\
&= (1-\delta_{j,D}^{t'-1})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C}) + \delta_{j,D}^{t'-1}\left((1-\delta_{j,D})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C}) + \delta_{j,D}\pi_{j,D}(\mathcal{G}) - \delta_{j,D}\sum_{kj\in\mathcal{C}_{j,D}} p_{kj,D}^{R}\right)\\
&= (1-\delta_{j,D}^{t'-1})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C}) + \delta_{j,D}^{t'-1}\left(\pi_{j,D}(\mathcal{G}) - (1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\mathcal{C}) - \delta_{j,D}\sum_{kj\in\mathcal{C}_{j,D}} p_{kj,D}^{R}\right)\\
&< (1-\delta_{j,D}^{t'-1})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C}) + \delta_{j,D}^{t'-1}\left(\pi_{j,D}(\mathcal{G}) - \sum_{kj\in\mathcal{C}_{j,D}} \left[(1-\delta_{j,D})\Delta\pi_{j,D}(\mathcal{G},\{kj\}) + \delta_{j,D}p_{kj,D}^{R}\right]\right)\\
&= (1-\delta_{j,D}^{t'-1})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C}) + \delta_{j,D}^{t'-1}\left(\pi_{j,D}(\mathcal{G}) - \sum_{kj\in\mathcal{C}_{j,D}} p_{kj,U}^{R}\right)\\
&= (1-\delta_{j,D})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C}) + (\delta_{j,D} - \delta_{j,D}^{t'-1})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C}) + \delta_{j,D}^{t'-1}\left(\pi_{j,D}(\mathcal{G}) - \sum_{kj\in\mathcal{C}_{j,D}} p_{kj,U}^{R}\right)\\
&< (1-\delta_{j,D})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C}) + (\delta_{j,D} - \delta_{j,D}^{t'-1})\left(\pi_{j,D}(\mathcal{G}) - \sum_{kj\in\mathcal{C}_{j,D}} p_{kj,U}^{R}\right)\\
&\qquad + \delta_{j,D}^{t'-1}\left(\pi_{j,D}(\mathcal{G}) - \sum_{kj\in\mathcal{C}_{j,D}} p_{kj,U}^{R}\right)\\
&= (1-\delta_{j,D})\pi_{j,D}(\mathcal{G}\setminus\mathcal{C}) + \delta_{j,D}\left(\pi_{j,D}(\mathcal{G}) - \sum_{kj\in\mathcal{C}_{j,D}} p_{kj,U}^{R}\right),
\end{aligned}
$$

where the second, third, and sixth lines follow by rearranging terms, the fourth line follows from A.WCDMC, the fifth line from (1), the seventh line from Lemma E.4, and the final line also by rearranging terms. Thus, the payoffs to D_j from the deviant offer are greater than its equilibrium payoffs in this case.

Thus D_j has a profitable deviation, leading to a contradiction. Hence, any equilibrium involves immediate agreement for all $ij \in \mathcal{C}$ at t. □

References

Binmore, Ken, Ariel Rubinstein, and Asher Wolinsky. 1986. "The Nash Bargaining Solution in Economic Modelling." *The RAND Journal of Economics* 17 (2):176–188.

Björnerstedt, Jonas and Johan Stennek. 2007. "Bilateral oligopoly – The efficiency of intermediate goods markets." *International Journal of Industrial Organization* 25 (5):884–907.

Chipty, Tasneem and Christopher M Snyder. 1999. "The Role of Firm Size in Bilateral Bargaining: A Study of the Cable Television Industry." *Review of Economics and Statistics* 81 (2):326–340.

Rubinstein, Ariel. 1982. "Perfect Equilibrium in a Bargaining Model." *Econometrica* 50 (1):97–109.

Shaked, Avner and John Sutton. 1984. “Involuntary Unemployment as a Perfect Equilibrium in a Bargaining Model.” *Econometrica* 52 (6):1351–64.