

**Economics 2610: Industrial Organization I**  
**Harvard University, Fall Semester 2002**  
**Instructor: Professor Gautam Gowrisankaran**

**Assignment 2**

**Handed out: November 22, 2002**

**Due: December 12, 2002**

You are encouraged to work on the programming in small groups of two or three. However, you must write up the assignments individually. This assignment is based on work in progress by me, Matthew Mitchell and Andrea Moro.

**Model**

Consider the following dynamic model of choice for a senatorial election. A period is six years. Each period, the electorate must choose a senator to represent it for that period. The electorate has a per-period discount factor of  $\beta = 0.925^6 = 0.6$ .

Each candidate  $j$  has a quality level  $q_j$ . Suppose a candidate has served  $t_j$  terms in office. Then, the electorate obtains current period utility  $u(q_j, t_j) = q_j + f(t_j)$  from that period. In this expression,  $f(t_j)$  represents an incumbency effect, wherein the utility from a senator may depend directly on the number of terms she has served in office. One explanation of the incumbency effect is that the ability of a senator to obtain funding for projects in her state varies based on the number of terms that she has served.

At the beginning of each period, the incumbent senator decides whether to retire or run for reelection. The retirement decision is exogenous, based purely on personal reasons, and can be written as  $P(t_j)$ . Following the retirement decision will be an election, where the electorate must choose a senator from among two candidates. If the incumbent runs for reelection, she will face a challenger. If she retires, there will be two challengers facing each other. Quality of the incumbent and current challenger is perfectly observable to the electorate at the time of the election.

Challenger quality is drawn from a common distribution with  $q_j \sim N(\mu, \sigma^2)$ . The quality of a given individual does not change over time. However, in the current period, the electorate has no information about next period's challenger, and hence postulates a prior quality of  $q_j \sim N(\mu, \sigma^2)$ .

Assume that  $P(6) = 1$  so that a senator who has served 6 terms (36 years) always chooses to retire, Ted Kennedy and Strom Thurmond notwithstanding. This makes the state space easier to deal with. Let  $P(1), \dots, P(5) = .2, .2, .4, .4, .6$  be the values that are found in the data. Lastly, let us specify the

incumbency effect  $f(t_j)$  with a fixed effect  $\alpha$  for each level,  $\alpha_1, \dots, \alpha_6$ .

## Data

I have placed a data set on the course web site, that contains senate election returns. Each row of the data represents one observation, which is one sequence of candidates from an open election until the exogenous retirement of a senator, which leads to another open election. Here are two examples:

2 1 0 0 0 0 0 0 0- Candidate elected newly in year 0, reelected in year 6, and lost in year 12. The second candidate retired in year 18.

1 2 1 0 0 0 0 0 0- Candidate elected newly in year 0, and lost in year 6. The second candidate was reelected in year 12, and then lost in year 18. The third candidate retired in year 24.

## Questions

1) Consider the electorate's decision problem. What is the state space? Write down the functional equation. Prove that it is a contraction mapping. Numerically evaluate the dynamic programming problem for the parameters  $(\mu = 0, \sigma = 1, \alpha = 0, 0.8, 1.2, 0.8, 0.2, -0.4)$ . Report the value function and the probability that the electorate reelects the incumbent for six values: incumbent quality of -0.5 and 0.5, both for 1 3 and 5 terms served by the incumbent. For all your computations, you should discretize quality into 100 evenly spaced values from -3.96 to 4.00.

2) Explain intuitively how the assumption of  $\beta = 0$  would result in different behavior from  $\beta = 0.925^6$ . Explain intuitively why it is consistent to estimate the retirement probability without using the dynamic model. Show that  $\mu$  and  $\sigma$  and one of the  $\alpha$ 's (WLOG  $\alpha_1$ ) are not identified from the data. Let us suppose that the  $\alpha$ 's are all non-negative. Could a reduced form regression of the probability of winning on the number of terms served tell us the importance of the incumbency effect? If not, can you think of other reduced form regressions that might shed some light on the importance of this effect?

3) Write down the likelihood function for this model. Explain how you would evaluate the likelihood for this model. Program the likelihood function for this model. Report the likelihood for the parameters given in question 1. Compute and report the maximum likelihood estimates of this model as well as the standard errors of the parameter values.

4) I would like to understand the relative importance of the incumbency effect to the selection effect. Using your estimated parameters, compute how much the expected number of terms served by a senator would change if there were no incumbency effects, and if there were no quality differences across candidates. What does this tell you about the relative importance of the different effects?

5) I would like to understand the effect of term limits. Using your estimated parameters, examine the impact of a 3 term limit for senators on the expected number of terms served by a senator and the expected quality level of a senator. Suppose that the incumbency effect is solely a transfer from

one state to another and hence has no social value. Calculate the social welfare given the term limit and given no term limit. Would a 3 term limit be welfare improving?