Economics 8205-8206:
Applied Econometrics I

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Midterm Examination

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Exam is closed book and closed notes. All questions weighted equally.

1) Consider a model with a constant term and a set of other regressors X, $y = \alpha\iota + X\beta + u$. Let $\hat{\beta}$ denote the estimate of β from this regression. Now consider a similar model without a constant term, $y = X\beta + u$ and let $\tilde{\beta}$ denote the estimate of β from this regression.

a- Under exactly what conditions on X and y is $\hat{\beta} = \tilde{\beta}$?

b- Under exactly what conditions on X and y is $\mathrm{Var}(\hat{\beta}) = \mathrm{Var}(\tilde{\beta})$?

c - Under exactly what conditions on X and y are the (uncentered) R^2's from the two regressions equivalent?

Prove your answers. You may make use of theorems proved in class, as long as you clearly state when you are making use of them.

2) True or false. Prove each assertion or give a counterexample.

a - Let β be a k-dimensional vector and let $\hat{\beta}$ and $\tilde{\beta}$ be two unbiased estimators of β such that $\mathrm{Var}(\hat{\beta}_i) < \mathrm{Var}(\tilde{\beta}_i)$, $i = 1, \ldots, k$. Then $\hat{\beta}$ is always a more efficient estimator of β than $\tilde{\beta}$.

b - Suppose that $\hat{\beta}_1$ is the coefficient vector from a regression $y = X_1\beta_1 + u$ and $\tilde{\beta}_1$ is the coefficient vector of β_1 from a regression $y = X_1\beta_1 + X_2\beta_2 + u$. Then, $\mathrm{EstVar}(\hat{\beta}_1) \le \mathrm{EstVar}(\tilde{\beta}_1)$, where "EstVar" denotes the estimated variance from the least squares regression.

3) Likelihood estimation

a - Consider a likelihood model with data given by y^n and parameter space given by Θ. Suppose that the true parameter vector is θ_0. Show that $E_{\theta_0}\left[\ln L(\hat{\theta}, y^n)\right] \le E_{\theta_0}\left[\ln L(\theta_0, y^n)\right]$, $\forall \hat{\theta} \in \Theta$.

b - Consider a linear model $y = X\beta + u$ where each u_t is independent but that the variance of each u_t is known and potentially different, so that $u_t \sim N(0, \sigma_t^2)$. Compute the maximum likelihood estimator of β. You may use the fact that if $z \sim N(\mu, \sigma^2)$ then the density $f(z) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(z-\mu)^2\right)$.