Economics 8205-8206:
Applied Econometrics I

Instructor: Professor Gautam Gowrisankaran
Teaching Assistant: Mehmet Ozhabes
University of Minnesota, Fall Semester 2000

Midterm Examination
October 24, 2000
Exam is closed book and closed notes. All questions weighted equally.

1) Consider a model with a constant term and a set of other regressors \( X, y = \alpha + X\beta + u \). Let \( \hat{\beta} \) denote the estimate of \( \beta \) from this regression. Now consider a similar model without a constant term, \( y = X\beta + u \) and let \( \tilde{\beta} \) denote the estimate of \( \beta \) from this regression.
   a- Under exactly what conditions on \( X \) and \( y \) is \( \beta = \tilde{\beta} \)?
   b- Under exactly what conditions on \( X \) and \( y \) is \( \text{Var}(\tilde{\beta}) = \text{Var}(\hat{\beta}) \)?
   c - Under exactly what conditions on \( X \) and \( y \) are the (uncentered) R^2's from the two regressions equivalent?
   Prove your answers. You may make use of theorems proved in class, as long as you clearly state when you are making use of them.

2) True or false. Prove each assertion or give a counterexample.
   a - Let \( \beta \) be a k-dimensional vector and let \( \hat{\beta} \) and \( \tilde{\beta} \) be two unbiased estimators of \( \beta \) such that \( \text{Var}(\hat{\beta}_i) < \text{Var}(\tilde{\beta}_i) \), \( i = 1, \ldots, k \). Then \( \hat{\beta} \) is always a more efficient estimator of \( \beta \) than \( \tilde{\beta} \).
   b - Suppose that \( \hat{\beta}_1 \) is the coefficient vector from a regression \( y = X_1\beta_1 + u \) and \( \tilde{\beta}_1 \) is the coefficient vector of \( \beta_1 \) from a regression \( y = X_1\beta_1 + X_2\beta_2 + u \). Then, \( \text{EstVar}(\hat{\beta}_1) \leq \text{EstVar}(\tilde{\beta}_1) \), where “EstVar” denotes the estimated variance from the least squares regression.

3) Likelihood estimation
   a - Consider a likelihood model with data given by \( y^n \) and parameter space given by \( \Theta \). Suppose that the true parameter vector is \( \theta_0 \). Show that \( E_{\theta_0} \left[ \ln L(\theta, y^n) \right] \leq E_{\theta_0} \left[ \ln L(\theta_0, y^n) \right] \forall \theta \in \Theta \).
   b - Consider a linear model \( y = XB + u \) where each \( u_i \) is independent but that the variance of each \( u_i \) is known and potentially different, so that \( u_i \sim N(0, \sigma_i^2) \). Compute the maximum likelihood estimator of \( \beta \). You may use the fact that if \( z \sim N(\mu, \sigma^2) \) then the density \( f(z) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left( -\frac{1}{2\sigma^2} (z-\mu)^2 \right) \).