Economics 8205-8206:
Applied Econometrics I

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Assignment 1

Handed out: September 12, 2000
Due: September 21, 2000

Problem 1 – 5 can be done largely using pen and paper. You may want to use Maple, or something similar, to invert matrices.
Problem 6 should be done using Matlab.

1) Consider the following regression:

$$\begin{pmatrix} 136 \\ 144 \\ 157 \\ 160 \\ 172 \end{pmatrix} = \begin{pmatrix} 1 & 11 \\ 1 & 12 \\ 1 & 13 \\ 1 & 14 \\ 1 & 15 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \text{residuals}$$

$$(y = X\beta + \text{residuals})$$

a) Compute P_X, M_X, P_{X_1} and $P_{M_{X1}X_2}$. (Note that X_i is defined to be the i^{th} column of X.) Show that $P_X = P_{X_1} + P_{M_{X1}X_2}$. Why is this true?

b) Compute the least squares coefficient $\hat{\beta}$, fitted values $P_X y$ and fitted residuals $M_X y$ for this regression. Show that the fitted values and fitted residuals are orthogonal.

2) Consider the following regression:

$$\begin{pmatrix} 3 \\ -3 \\ 5 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 6 \\ 1 & 0 & 1 \\ 0 & 2 & 6 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} + \text{residuals}$$

$$(y = X\beta + \text{residuals})$$

Find the unique fitted values for the least squares projection $P_X y$, and give two sets of coefficients $\hat{\beta}$ that generate these fitted values. Explain all your work.

3) Recall that $0 \le R^2 = \frac{\|P_X y\|^2}{\|y\|^2} = \frac{\|X\hat{\beta}\|^2}{\|y\|^2} \le 1$. Is it possible to find $\tilde{\beta}$ such that $\frac{\|X\tilde{\beta}\|^2}{\|y\|^2} > 1$? Prove or give counterexample.

4) Use the X matrix from the regression in Problem 1. Find a vector $y \neq 0$ such that the least squares coefficient $\hat{\beta} = 0$. Explain how you chose y.

5) Recall that we defined the centered R^2, as $R_c^2 = \frac{\|P_X M_\iota y\|^2}{\|M_\iota y\|^2}$ where $\iota = (1, \ldots, 1)^T$.

a) Show that if X contains a constant term, then $R_c^2 = 1 - \frac{\|M_X y\|^2}{\|M_\iota y\|^2}$. Call this the second definition of R_c^2.
b) Suppose that X does not contain a constant term. Show by counterexample that the two definitions of R_c^2 can be different.
c) Is it ever possible for either of the two definitions of R_c^2 to be less than 0 or greater than 1?

6) You should have received a Matlab data set containing 30 observations and 5 regressors. Using this data set, compute least squares coefficient $\hat{\beta}$, fitted values $P_X y$ and fitted residuals $M_X y$. Compute also the same three values for the restricted regression where $\beta_1 + \beta_4 = 2$ and $\beta_4 + 2\beta_5 = -1$. Explain all your steps. What can you conclude about the fitted residuals?