Homework: Dynamics of Differentiated Products
Prof. Gautam Gowrisankaran
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We include a dataset on DVD players. The variables that we include are the average price and seasonally-adjusted shares for DVD players at the level of the model-month from March 1997 to October 2006. For each DVD player, we also include 3 characteristics: indicator variables for whether the player has the ability to play MP3 files, whether it has progressive scan and whether it can play DVD-R discs. We also include a unique identifier for each firm and product. We do not include the actual identities of firms or products. Throughout the exercise, set $\beta = .99$ at the level of the month. Assume that there are 1000 households total.

1) Estimation of the static parameters of a Melnikov model

As in Melnikov, assume that the utility for each product at the time it is purchased is

$$u_{ijt} = x_{jt}\alpha^x + \alpha^p \log\left(p_{jt}\right) + \xi_{jt} + \varepsilon_{ijt}.$$

The price term is paid only at the time of purchase while the utility from the characteristics $x_{jt}\alpha^x + \xi_{jt}$ is earned for every period going forward. There are 4 product characteristics: the 3 characteristics given in the dataset and the constant term.

To estimate the model, set up a Berry-style logit model and estimate the parameters of the model with the addition of dummy variables for each month. As in Melnikov, allow for price to be endogenous. Use the following eight instruments for p_{jt}: the number of products at time t (1), the number of products produced by the firm that produces product j at time t (1), the mean value of the three characteristics in the dataset at time t (3), and the mean value of the three characteristics in the dataset for the firm that produces product j at time t (3).

a - Do you need to use non-linear solution methods to solve this model? Why or why not?

b - Report the means and standard deviations of your 8 instruments.

c - Report your parameter estimates and standard errors.

2) Estimate of the dynamic parameters of a perfect foresight Melnikov model

Let us now consider a simplified version of the Melnikov model where there is perfect foresight. The model is simplified in that all future product attributes are known to the firm at any time. (Future logit errors remain unknown.) For the last time period in the sample, consumers assume that the same products as in the last period will be sold for all future periods.

a - What remaining parameter(s) are left to estimate?

b - Specify the dynamic programming problem with perfect foresight. Solve for the DP problem for the constant term $c = 1$ and your estimated parameters from problem 1. Report your results.

c - Estimate the remaining parameter(s) by maximum likelihood. Report your parameter estimate but do not worry about the standard error.

3) Estimate of the dynamic parameters of a Melnikov model with uncertainty

Now allow for expectations of the future industry value to be determined by a regression: $\delta_{t+1} = \gamma_0 + \gamma_1 \delta_t + e_t$. Compute the non-linear parameter(s) of the model. This involves some additional steps. For each value of the non-linear parameter(s), you need to estimate a linear regression. You then need to pick a number of grid points over which to estimate the Bellman equation. I suggest 100 grid points. Each grid point corresponds to a value of δ at which to estimate the value function. You also need to pick a minimum and maximum value for the Bellman equation. I suggested eyeballing the data and picking low and high values that are sufficiently away from the extremes of the data to not bias the results. Last, you need to simulate the expectation of the value function at each point on the δ grid from the density of e_t. I suggest using 20 evenly spaced grid points, so that $e_{t,1} = \sigma\Phi^{-1}(1/40) =, e_{t,2} = \sigma\Phi^{-1}(3/40), \ldots, e_{t,20} = \sigma\Phi^{-1}(39/40)$, where σ is the standard deviation estimated from the above linear regression.

a - Evaluate the linear regression for this model for the estimated parameter(s) from question 3. At which point will the steady state distribution of δ_t be centered?

b - Compute the Bellman equation for this model for the estimated parameter(s) from question 3. Compare your optimal expected aggregate purchase probability for the first and last period from this model to the one computed in question 3.

c - Estimate the parameter(s) of the model using maximum likelihood. Report your estimated parameter(s) but again do not worry about the standard errors.

4) Further questions about the model and empirical question

a - The data that Gowrisankaran and Rysman use are seasonally adjusted, meaning that for any model in any period, I divided the sales by the total sales in that period and then multiplied all sales by a constant to get back the same total number of sales. As you know, this seriously downweights sales for December and upweights sales for all other months. Why might you be concerned that seasonal adjustment of the data might yield inconsistent parameter estimates? What would be the problems with using unadjusted data and how might you control for these problems?

b) Another problem is the presence of a complementary good, most notably DVD movie titles. The number of available DVD titles varies dramatically over the sample period and may affect the utility from a DVD player. Suppose we were to include the number of DVD titles as a characteristic and treat it as endogenous, like price. Can you think of any good instrument(s) for the number of titles? Explain.

5) Adding random coefficients, à la Gowrisankaran and Rysman

This question asks you to estimate a version of the model with random coefficients. Unlike Gowrisankaran and Rysman, assume consumers do not make repeat purchases, but do have perfect foresight. Assume that there is a random coefficient on the constant term so that utility is now:

$$u_{ijt} = x_{jt}\left(\bar{\alpha}^x + \sigma\alpha_i\right) + \alpha^p \log\left(p_{jt}\right) + \xi_{jt} + \varepsilon_{ijt}$$

where σ is 0 for all characteristics but the constant term and is a parameter to be estimated for the constant term. For the constant term, assume that $\alpha_i \sim N(0,1)$ and use the following 20 quadrature points: $\alpha_1 = \Phi^{-1}(1/40) =, \alpha_2 = \Phi^{-1}(3/40), \ldots, \alpha_{20} = \Phi^{-1}(39/40)$.

To estimate the model, you need to perform a non-linear search over the parameters. For each parameter vector $\alpha^x, \alpha^p, \sigma$, you need to do the following:

- Solve for ξ_{jt} by repeatedly computing the mapping

$$\xi'_{jt} = \xi_{jt} + \log\left(s_{jt}\right) - \log\left(\frac{1}{20}\sum_{i=1}^{20} \hat{s}_{ijt}\left(\alpha^x, \alpha^p, \sigma; \xi\right)\right) \text{ until convergence.}$$

- The calculation of $\hat{s}_{ijt}\left(\alpha^x, \alpha^p, \sigma; \xi\right)$ should be done separately for each of the 20 values of α_i. For each value, you need to compute the dynamic programming problem similar to that done in problem 2 and then use the policy function to figure out the percent who purchase any product at any time period.

- Estimate a moment condition based on the interaction of ξ_{jt} with all included exogenous variables and your 8 instruments. Use as a moment condition: $\xi' Z\left(Z'Z\right)^{-1} Z'\xi$ where Z are the exogenous variables.

You then need to minimize your moment condition over parameter space.

Report the value of the moment condition for the parameter vector $(1,1,1,1,1,1)$. Report your estimated parameter values. Don't worry about standard errors for this question. How important is the inclusion of a random coefficient in this case?