Stochastic Volatility and Jump-Diffusion — Implications on option pricing

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Abstract

This paper conducts a thorough and detailed investigation on the implications of stochastic volatility and random jump on option prices. Both stochastic volatility and jump-diffusion processes admit asymmetric and fat-tailed distribution of asset returns and thus have similar impact on option prices compared to the Black-Scholes model. While the dynamic properties of stochastic volatility model are shown to have more impact on long-term options, the random jump is shown to have relatively larger impact on short-term near-the-money options. The misspecification risk of stochastic volatility as jump is minimal in terms of option pricing errors only when both the level of kurtosis of the underlying asset return distribution and the level of volatility persistence are low. While both asymmetric volatility and asymmetric jump can induce distortion of option pricing errors, the skewness of jump offers better explanations to empirical findings on implied volatility curves.

Key Words: Stochastic Volatility, Jump-Diffusion, Option Pricing.

JEL Classification: G13; C22; C52

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1 Introduction

When modeling financial asset returns, empirical stylized facts drawn from both static behavior (unconditional distribution) and dynamic behavior (conditional distribution) of state variables are the major factors in the consideration of model specification and selection. Since the early sixties it was observed, notably by Mandelbrot (1963) and Fama (1963, 1965) among others, that asset returns have leptokurtic distributions [1] and the volatilities of asset returns have strong intertemporal persistence. A model's ability to reproduce certain empirical stylized facts is often an important criterion to judge whether or not such a model represents the true underlying data generating process (DGP). There is a considerable amount of empirical evidence in the literature suggesting that the classical Black-Scholes (1973) model which assumes asset returns follow continuous diffusion processes with constant conditional volatility is inconsistent with the statistical properties of many asset price processes. It is also well documented that the Black-Scholes option pricing formula generates systematic biases of option prices. Extension of the underlying Black-Scholes model has been mainly along the following two directions in the literature.

One direction, pioneered by Merton (1976a) in deriving more general option pricing formula, is to model the sampling path of underlying asset returns as a mixture of continuous diffusion process and discontinuous jump process through introduction of a jump component.[2] Motivations of using jump-diffusion process to model stock returns were clearly stated in Merton (1976a) in which he distinguishes two types of changes in the stock price: the "normal" vibrations in price and the "abnormal" vibrations in price. The first type of price change can be modeled by a stochastic process with continuous sampling path, e.g. a Wiener process, and the second type of price change can be modeled by a process which explicitly allows for jumps, e.g. a "Poisson driven" process. In the case of foreign currency prices, shifts in interest rate differentials between two countries or monetary and fiscal policy changes are also believed to usually result in a revaluation of foreign currency. As Jorion (1988) points out, the foreign exchange market is characterized by active

[1] Such observation on the unconditional density has led people to propose modeling asset returns as i.i.d. draws from fat-tailed distributions, such as Student-t, Paretian or Lévy.

[2] Press (1967) first proposed a jump-diffusion model for stock price changes which is different from the "random walk model" originally proposed by Bachelier in 1900 and its modified versions. Press assumes that the changes of logarithmic stock prices follow the distribution of a Poisson mixture of normal distributions, that is, the combination of a Wiener process and a compound Poisson process. The model resembles earlier random walk models in that it is a Markov process with discrete parameter space and continuous state space. But the presence of the compound Poisson process produces a non-Gaussian and essentially discontinuous process.

exchange rate management policies which do not exist in other securities' market, e.g. the stock market. [3] Therefore, stochastic processes which incorporate jumps might reflect the change of foreign currency prices better than the pure continuous-time Wiener process, and thus provide models which are more accurate in pricing currency options [4]. Due to the compound distribution of jump component, the jump-diffusion model can offer a formal link between the behavior of asset price changes and the steady state leptokurtic distribution of asset returns.

Another direction is to introduce time-varying conditional volatility or heteroscedasticity into the underlying asset price process through various volatility changing models, for instance, the autoregressive conditional heteroscedasticity (ARCH) models introduced by Engle (1982), the generalized ARCH (GARCH) models extended by Bollerslev (1986) and Taylor (1986), [5] the continuous-time stochastic volatility (SV) models by Hull and White (1987) among others, [6] and the discrete-time SV models introduced by Taylor (1986). [7] It is undisputed that financial time series have time-varying and persistent conditional volatility, i.e. the volatility exhibits different levels over time and is sometimes clustered with bunching of high and low episodes. Such observations on volatility naturally lead to ARCH/GARCH models and stochastic volatility (SV) models. In fact, volatility varying or clustering over time and fat-tailed distributions of asset returns in the steady state are also believed to be intimately related. In the case of stochastic autoregressive volatility (SARV) models, it can be shown that the dynamic (conditional) volatility also implies (unconditional) fat-tailed distribution of asset returns.[8]

Both jump-diffusion and SV processes are exploiting the earlier ideas proposed by Mandelbrot and Taylor (1967) and Clark (1973) among others that asset returns should be explicitly linked

[3] Apart from the formally justified motivations, any casual observation of the sampling paths of most financial asset returns can reveal discontinuity or jumps over time. Moreover, if underlying asset prices are modeled in continuous-time diffusion processes, allowing for time-varying conditional volatility in the model would not be sufficient to reflect discontinuity of asset returns as the underlying processes are continuous. Thus explicit inclusion of jump components, such as Bernoulli jump or Poisson jump, in the model seems to be essential to describe the dynamics of asset returns.

[4] Bodurtha and Courtadon (1987) claim that the systematic biases of the modified Black-Scholes American option pricing model for currency options are partly due to the jumps along the exchange rate processes.

[5] For a survey of the ARCH/GARCH applications to financial time series, see Bollerslev, Chou, and Kroner (1992). Duan (1995) considers the application of GARCH model in pricing options.

[6] Other examples include Johnson and Shanno (1987), Wiggins (1987), Scott (1987, 1991, 1997), Bailey and Stulz (1989), Chesney and Scott (1989), Melino and Turnbull (1990), Stein and Stein (1991), Heston (1993), Bates (1996b), and Bakshi, Cao and Chen (1997).

[7] Other examples include Amin and Ng (1993), Andersen (1994), Taylor (1994), and Kim, Shephard and Chib (1996).

[8] See Shephard (1996) and Ghysels, Harvey and Renault (1996) for Statistical properties of SV models.

to the flow of information arrivals which are non-uniform through time and often are not directly observable. The jump-diffusion model uses the flow of important information shocks to explain discontinuity of asset price paths, while the SV model links the flow of information shocks to volatility. In particular, the Merton (1976a) jump-diffusion process explicitly relies on independent mixture normal distributions over time to account for kurtosis and skewness of asset return distributions. The level of kurtosis is controlled by jump frequency and magnitude, while skewness can be easily modeled by allowing for asymmetric jumps. The SV models in continuous-time often assume an instantaneous lognormal distribution of asset returns and rely on the dynamics of conditional volatility to incorporate kurtosis and asymmetry into the underlying asset return distributions. The discrete time SV models, on the other hand, are similar to the jump-diffusion model in assuming independent fat-tailed asset return distributions over time but further rely on the dynamics of conditional volatility to determine the level of kurtosis and asymmetry. The level of kurtosis is controlled by the coefficients of the stochastic volatility process, while asymmetry is controlled by the correlation between asset returns and stochastic volatility.

Many existing models have explicitly included both jump and stochastic volatility in asset return processes, see e.g. Jorion (1988), Bates (1996b, 1997), Ho, Parraudin and Sørensen (1996), Bakshi, Cao and Chen (1997), and Scott (1997). Empirically, identification of such models or the task of disentangling the jump component from the SV component in asset return processes remains a challenge.[9] First of all, the identification and estimation of a jump-diffusion process as shown in Merton (1976b) essentially relies on a sample of data over a long time period. Such data are not always available in reality. Even though they are, people (especially practitioners) are reluctant to use such data due to either the difficulty involved in the estimation of the underlying models or the unwillingness to assume that the parameters of the model are constant over such a long time period. Secondly, the use of a sample over a relatively short sampling period immediately evokes two serious problems. One is that an estimated statistically significant jump could be a result of misidentification of other random noise factors, such as stochastic volatility. Many empirical results have reported that the jump-diffusions estimated from daily or weekly data typically have high-frequency and low-amplitude jump components. As Bates (1996a) points out,

[9] This is mainly due to the fact that neither conditional volatility nor the jump size or frequency is directly observable. Estimation of the dynamic latent variable models of this type is difficult but has achieved enormous progress in the field of statistics and econometrics, see e.g. Ghysels, Harvey and Renault (1996). However, the newly developed methods are mostly very intensive in computation.

it seems likely that such jump component is picking up lumpy information flows associated with news announcements, as discussed in Ederington and Lee (1993). [10] The other problem is that, as argued in Merton (1976b) and recently in Renault (1995), when the underlying asset returns do have jumps but are misspecified as a pure diffusion process, e.g. the Black-Scholes model, which is estimated using the sample over a relatively short time period, people are often led to infer that the parameters of the process are changing over time and the underlying volatility is also varying over time when it is actually not. It should be noted that the implied estimation method often adopted in the finance literature, which rely on market option prices to estimate (or back-out) the underlying asset return process, cannot resolve these problems either. In general, the implied estimation procedure results in poor identification of the underlying process, which is evidenced in the dramatic day-to-day change of the parameter values, and invalidates the associated statistical tests of model specification.

If differences between the implications of SV and jump on option pricing are not so significant, such misidentification may not matter much in practice. Otherwise, it would be very important to disentangle the jump component from SV in the underlying asset returns. On the other hand, if the implications of SV and jump on option prices are indeed very different, then the information extracted from the observed market option prices would be helpful in identifying the statistical properties of underlying asset returns. In this sense, a thorough and detailed investigation on the implications of SV and jump on option prices seems to be important and necessary. Such a study would be also helpful for choosing the right option pricing formula in the implementation of implied estimation method and improving the identification of the underlying process.

The purpose of this paper is to conduct a thorough and detailed investigation on the implications of stochastic volatility and jump on option prices. Section 2 summarizes the statistical properties of a discrete-time SV model and the approach of pricing options under stochastic volatility; Section 3 discusses the statistical properties of the Merton (1976a) jump-diffusion process and the jump-diffusion option pricing formula; Section 4 investigates the impact of persistent and asymmetric volatility on option prices, the impact of jump and asymmetric jump size on option prices, as well as comparison between the implications of SV and jump on option prices; Section 5 concludes.

[10] Although this problem can be alleviated by explicitly allowing for either SV or more than one jump terms or both in the model as suggested in Bates (1996a), identification of the model will rely on a sample over even a longer time period.

2 The Stochastic Volatility (SV) Model and Option Pricing

2.1 The Discrete-Time Stochastic Autoregressive Volatility Model

Acknowledging the fact that stochastic volatility is manifest in the time-series of asset returns as well as in the empirical variances implied from observed market option prices through the Black-Scholes model, there have been numerous recent studies of option pricing based on stochastic volatility (SV) models.[11] Different from the Merton' (1976a) jump-diffusion which has an exact discrete-time version, the continuous-time SV model is often more difficult for the analysis of its statistical properties. For tractability and to avoid the approximation involved in the continuous-time SV model, this paper focuses only on discrete-time SV models. Since the advantage of the SV model is its flexible distributional structure in which the correlation between volatility and asset returns serves to control the level of asymmetry and the volatility variation coefficients serve to control the level of kurtosis, dealing with the discrete-time model is also helpful to focus on these two major aspects.

Let the time index set be $\mathcal{T} = \{t | t = 0, 1, 2, ..., T\}$ and suppose that $\mathcal{B} = \{(\Omega, \mathcal{F}, P), \mathbb{F}\}$ is a stochastic basis, where $(\Omega, \mathcal{F}, P)$ is a probability space and $\mathbb{F} = \{\mathcal{F}_t | t \in \mathcal{T}\}$ is an increasing, complete filtration satisfying $\mathcal{F}_T = \mathcal{F}$. Traded in the economy is an asset with price path $S_t, t \in \mathcal{T}$, which is adapted to the filtration $\mathbb{F}$. Let μ_t be the conditional mean of the return process which is often assumed to be a constant, i.e. $\mu_t = \mu$, then the demeaned or detrended return process y_t is defined as $y_t = \ln(S_t/S_{t-1}) - \mu$. The discrete-time stochastic volatility (SV) model of the financial asset return may be written as

$$y_t = \sigma_t \epsilon_t, \quad t = 1, 2, ..., T \tag{1}$$

or

$$y_t = \sigma \epsilon_t \exp\{h_t/2\}, \quad t = 1, 2, ..., T \tag{2}$$

where ϵ_t is a iid random noise with a standard distribution, e.g. normal or Student-t distribution. The most popular SV specification assumes that h_t follows an AR(1) process, as proposed by

[11] Reviews of SV models are provided by Shephard (1996) which surveys the current literature on both ARCH/GARCH and SV models with a focus on the comparison of their statistical properties, and Ghysels, Harvey and Renault (1996) which surveys the current literature on SV models with a focus on statistical modeling and inference of stochastic volatility in financial markets. Also see Taylor (1994), Andersen (1992) and Andersen (1994) for review and discussion of SV models.

Taylor (1986), i.e.

$$h_{t+1} = \phi h_t + \eta_t, \quad |\phi| < 1 \tag{3}$$

which is a special case of the general stochastic autoregressive volatility (SARV) model defined in Andersen (1994), where $\eta_t \sim$ iid $(0, \sigma_\eta^2)$ and the constant term is removed due to the introduction of the scale parameter σ in (2). When η_t is Gaussian, this model is called a log-normal SV model. One interpretation for the latent variable h_t is that it represents the random, uneven and yet auto-correlated flow of new information into financial markets, thus the volatility is time-varying and sometimes clustered with bunching of high and low episodes.

The statistical properties of the above SV model are discussed in Taylor (1986, 1994) and summarized in Shephard (1996) for the case that η_t is Gaussian and in Gyhsels, Harvey and Renault (1996) for more general cases. Namely, (i) if η_t is Gaussian and $|\phi| < 1$, h_t is a standard stationary Gaussian autoregression, with $E[h_t] = 0,\ Var[h_t] = \sigma_h^2/(1-\phi^2)$; (ii) y_t is a martingale difference as ϵ_t is iid, i.e. y_t has zero mean and is uncorrelated over time. Furthermore, y_t is a white noise (WN) if $|\phi| < 1$; (iii) as ϵ_t is always stationary, y_t is stationary if and only if h_t is stationary; (iv) if η_t is normally distributed and h_t stationary and ϵ_t has finite moments, then all the moments of y_t exist and are given by

$$E[y_t^s] = \sigma^s E[\epsilon_t^s] E[\exp\{sh_t/2\}] = \sigma^s E[\epsilon_t^s] \exp\{s^2\sigma_h^2/8\} \tag{4}$$

when s is even and $E[y_t^s] = 0$ when s is odd if ϵ_t is symmetric. That is, $Var[y_t] = \sigma^2\sigma_\epsilon^2\exp(\sigma_h^2/2)$, where σ_ϵ^2 is assumed known, e.g. $\sigma_\epsilon^2 = 1$ if $\epsilon_t \sim N(0,1), \sigma_\epsilon^2 = \nu/(\nu-2)$ if $\epsilon_t \sim$ Student-t with $d.f. = \nu$. More interestingly, the kurtosis of y_t is $(E[\epsilon_t^4]/\sigma_\epsilon^4)\exp(\sigma_h^2)$ which is greater than $E[\epsilon_t^4]/\sigma_\epsilon^4$, the kurtosis of ϵ_t, as $\exp(\sigma_h^2) > 1$. When ϵ_t is also Gaussian, then $E[\epsilon_t^4]/\sigma_\epsilon^4 = 3$. In other words, the SV model has fatter tails than that of the corresponding noise disturbance of the return process. This is true even when $\phi = 0$. Since $\sigma_h^2 = \sigma_\eta^2/(1-\phi^2)$ for $|\phi| < 1$, the level of kurtosis is controlled by σ_η^2 and ϕ. It is noted that the above properties of the SV model also hold true even if ϵ_t and η_t are contemporarily correlated.

Dynamic properties of the SV model can be derived under the assumption that the disturbances ϵ_t and η_t are independent of each other. Squaring both sides of the SV process and then take logarithms gives

$$ln y_t^2 = ln\sigma^2 + h_t + ln\epsilon_t^2 \tag{5}$$

which is a linear process with the addition of the iid noise $\ln\epsilon_t^2$ to the AR(1) process h_t. Thus, $\ln y_t^2 \sim$ ARMA(1,1). When ϵ_t is a standard normal distribution, then the mean and variance of $\ln\epsilon_t^2$ are known to be -1.27 and $\pi^2/2$. The distribution of $\ln y_t^2$ is far away from being normal, but with a very long left-hand tail.

When ϵ_t and η_t are allowed to be correlated with each other, the above model can pick up the kind of asymmetric behavior often observed in stock price movements, which is known as the *leverage effect* when the correlation is negative (see Black, 1976). To see this, consider $y_{t+1} = \sigma\epsilon_{t+1}\exp\{h_{t+1}/2\}$, suppose $Corr(\epsilon_t, \eta_t) = \rho$, the random shock of η_t can be written as $\eta_t = \sigma_\eta(\rho\epsilon_t + \sqrt{1-\rho^2}u_t)$ where $u_t \sim$ iid $N(0,1)$ and independent of ϵ_t. Thus, h_{t+1} conditional on time t is explicitly dependent of ϵ_t. In particular, when $\rho < 0$, a negative shock ϵ_t will tend to increase the volatility of the next period and a positive shock will tend to decrease the volatility of the next period. The value of ρ determines the level of asymmetry of conditional volatility.

The continuous-time SV models can be viewed as the limit of discrete-time SV models. A general representation of the continuous-time SV model may be written as

$$\begin{aligned} dS_t/S_t &= \mu_t dt + \sigma_t(h_t) dW_t \\ dh_t &= \gamma_t dt + \delta_t dW_t^h \\ dW_t dW_t^h &= \rho_t dt, \qquad t \in [0, T] \end{aligned} \tag{6}$$

where h_t is an unobserved latent state variable governing the volatility of asset returns and itself also follows a diffusion process, W_t and W_t^h are Wiener processes or standard Brownian motion processes with Cov$(dW_t, dW_t^h) = \rho_t dt$; let $\mathcal{F}_t$ be the natural filtration of the stochastic process which represents all the information available at time $t \in [0, T]$, the coefficient functions $\mu_t, \sigma_t, \gamma_t, \delta_t$ and ρ_t are all assumed to be adapted to $\mathcal{F}_t$. A common specification of the continuous-time SV model resembles that in discrete time, i.e. the logarithmic instantaneous volatility follows an Ornstein-Uhlenbeck process, in e.g. Wiggins (1987), Chesney and Scott (1989), and Melino and Turnbull (1990), [12] i.e.

$$\begin{aligned} \sigma_t(h_t) &= \sigma\exp\{h_t/2\} \\ dh_t &= -\beta h_t dt + \sigma_h dW_t^h \end{aligned} \tag{7}$$

[12] Scott (1987), Stein and Stein (1991), Ball and Roma (1994) assume that $\sigma_t(h_t)$ follows an Ornstein-Uhlenbeck process which allows $\sigma_t(h_t)$ to be negative. Another specification which also guarantees the nonnegativeness of the volatility is the model proposed by Cox, Ingersoll and Ross (1985) for nominal interest rates, used in e.g. Bailey and Stulz (1989) and Heston (1993).

where $\beta > 0$ and the exponential functional specification guarantees the nonnegativeness of volatility. Similar to the discrete-time autoregressive SV model, h_t is also an AR(1) process as

$$h_t = e^{-\beta} h_{t-1} + \int_{t-1}^{t} \sigma_h e^{-\beta(t-\tau)} dW_\tau^h$$

where $\int_{t-1}^{t} e^{-\beta(t-\tau)} dW_\tau^h \sim N(0, \frac{\sigma_h^2}{2\beta}(1-e^{-2\beta}))$. When $\beta > 0$, h_t is stationary with mean zero and variance $\sigma_h^2/2\beta$. Hull and White (1987) specify a geometric Brownian motion process for $\sigma_t(h_t)$ and in particular consider the case that $\rho_t = 0$.

Meddahi and Renault (1995) studied the temporal aggregation of SV models and derived the conditions under which a class of discrete-time SV models are closed under temporal aggregation. For instance, when $\mu_t = 0$, $\sigma_t(h_t) = h_t^{1/2}$, and the state variable h_t follows a CEV process (i.e. a constant elasticity of variance process due to Cox, 1975, and used e.g. in Johnson and Shanno, 1987), Meddahi and Renault (1995) show that, from such a continuous-time SV model, there is a class of discrete-time SV models which is closed under temporal aggregation. Compared to the ARCH/GARCH models, proposed by Engle (1982), Bollerslev (1986) and Taylor (1986), which assume conditional volatility σ_t^2 is driven by past known observations of y_t^2 and σ_t^2, the SV model assumes the conditional volatility driven by an extra random noise.[13] The main advantage of the SV models over ARCH/GARCH models in practical modeling of asset returns is that the SV models can easily incorporate excess kurtosis and *leverage effect* into underlying asset return processes.

2.2 Option Pricing under Stochastic Volatility

Option pricing under stochastic volatility has received considerable attention in the literature with earlier papers dealing with the case of level dependent SV in e.g. Merton (1973) and Cox (1975), while more recent papers focusing on the case that the stochastic volatility is driven by a different noise than that of the return process. With the extra random noise, holding the stock now involves two sources of risk, the stock return risk and the volatility risk. Constructing a perfectly hedged portfolio is now more difficult since there are two different sources of uncertainty, namely $\{\epsilon_t, \eta_t\}$, and only two securities, the stock and the call option, to hedge these risks. To price the contingent claims of an asset in such an economy, one can follow the risk-neutral approach by Cox and Ross

[13] In this sense, the SV model includes ARCH/GARCH as a special case when the random noise is replaced by the past observation of y_t^2.

(1976) and Harrison and Kreps (1979), or equivalently the state price density (SPD) approach by Constantinides (1992) and Amin and Ng (1993). We consider the simple case that both ϵ_t and η_t are Gaussian, interest rates are nonstochastic and tradings are frictionless. It is known that the exclusion of arbitrage profits or *free lunch* opportunities from trading in the underlying asset is equivalent to the existence of a probability measure Q on $(\Omega, \mathcal{F})$, equivalent to P, under which discounted price processes are martingales (See Harrison and Kreps, 1979). Such a probability is called an equivalent martingale measure and under which the prices of traded claims may be obtained by evaluating the expectation of discounted claim payoffs. Equivalent martingale measure is unique if and only if the markets are complete (see Harrison and Pliska, 1981), and in this case the value of contingent claims is simply the cost of implementing the replication strategy. In a discrete-time economy with stochastic volatility, the market is however incomplete and thus the equivalent martingale measure is not unique.

Since the existence of a state price density (SPD) is equivalent to absence of arbitrage as shown in Dalang, Morton and Willinger (1989). The choice of the equivalent martingale measure is parallel to defining the state price density. The SPD or the pricing kernel is related to the (positive) density process which transforms the original probability measure P to the equivalent martingale measure Q through Radon-Nikodym derivative for the discounted asset price process. Under the assumption that the SV risk is nonsystematic or diversifiable and thus has zero risk premium as in Hull and White (1987), we can assume the following functional form for the SPD [14]

$$\xi_t = \exp\{-rt - \sum_{\tau=1}^{t}(\gamma_\tau \epsilon_\tau + \frac{1}{2}\gamma_\tau^2)\} \tag{8}$$

[14]This SPD is the same as that in Amin and Ng (1993) for stochastic volatility processes and in Duan (1995) for GARCH processes. The formal justification of the choice of this state price density is as following. Consider the continuous-time SV model as in (6), from the integral form of martingale representations (see Karatzas and Shreve, 1988, p.184), the (positive) density process of any probability measure Q equivalent to P can be written as

$$M_t = exp\{-(\int_0^t \gamma_u dW_u + \frac{1}{2}\int_0^t \gamma_u^2 du) - (\int_0^t \gamma_u^h dW_u^h + \frac{1}{2}\int_0^t (\gamma_u^h)^2 du)\}$$

where the processes γ_u and γ_u^h are adapted to the natural filtration $\mathcal{F}_t$ and satisfy the integratability conditions

$$\int_0^t \gamma_u^2 du < +\infty, a.s. \quad \int_0^t (\gamma_u^h)^2 du < +\infty, a.s.$$

By Girsanov's theorem, the process $\widetilde{W}_t = W_t + \int_0^t \gamma_u du$ and $\widetilde{W}_t^h = W_t^h + \int_0^t \gamma_u^h du$ are Brownian motions under Q. With the SV risk premium assumed to be zero, the SPD in (8) is simply the discrete version of M_t adjusted by the discounting bond price process.

with a $\mathcal{F}_{t-1}$-measurable finite random variable $\gamma_t = (\mu + \frac{1}{2}\sigma_t^2 - r)/\sigma_t$. The price of an European call option can be priced as

$$C(S_t, t) = e^{-r(T-t)} E_t^Q[\max(0, S_T - K)] = E_t^P[\frac{\xi_T}{\xi_t} \max(0, S_T - K)|\mathcal{F}_t], \quad 0 \leq t \leq T \quad (9)$$

When $\rho = 0$, i.e. the asset return is uncorrelated with conditional volatility, it leads to the Amin and Ng (1993) option pricing formula with constant interest rate

$$C(S_t, t) = E_t^Q[C_{BS}(S_t, t; K, T, r, V_t^2)] \quad (10)$$

where

$$C_{BS}(S_t, t; K, T, r, V_t^2) = S_t\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2) \quad (11)$$

which is the Black-Scholes price of an European call option with strike price K, expiration date T, where

$$d_1 \equiv \frac{ln(P(t)/K) + (r + \frac{1}{2}V_t^2)(T-t)}{V_t\sqrt{T-t}}, \quad d_2 \equiv d_1 - V_t\sqrt{T-t}, \quad V_t^2 \equiv \sum_{\tau=t+1}^{T} \sigma_\tau^2/(T-t)$$

r is the risk-free rate of return, and $\Phi(\cdot)$ is the cumulative distribution function (CDF) of standard normal distribution. The expectation in (10) is taken w.r.t. the risk-neutral probability and can be calculated from simulations. This option-pricing formula resembles that in Hull-White (1987) with the Black-Scholes (1973) option pricing formula as a special case. As Renault (1995) points out, this option pricing formula shares with many option pricing formulas in the literature the feature that option prices can be expressed as an expectation of the Black-Scholes prices over an heterogeneously distributed conditional volatility.

3 The Jump-Diffusion Process and Option Pricing

3.1 The Merton (1976a) Jump-Diffusion Model

The jump-diffusion process proposed by Merton (1976a) to model asset returns, as a mixture of both continuous diffusion path and discontinuous jump path, can be written as:

$$dS_t/S_t = (\alpha(\cdot) - \lambda\alpha_0)dt + \sigma dW_t + (Y_t - 1)dQ_t(\lambda) \quad (12)$$

where

$\alpha(\cdot)$ —the instantaneous expected return on the asset;

σ^2 — the instantaneous volatility of the asset's return conditional on no arrivals of important new information (i.e. the Poisson jump event does not occur);

$Q_t(\lambda)$ — a Poisson counting process which is assumed to be i.i.d. over time, λ is the mean number of jumps per unit of time, i.e. the intensity parameter of the Poisson distribution with Prob$(dQ_t(\lambda) = 1) = \lambda dt$, Prob$(dQ_t(\lambda) = 0) = 1 - \lambda dt$;

$Y_t - 1$ — the random jump size ($Y_t \geq 0$) representing the percentage change of asset price if the Poisson event occurs, $\int_0^t (Y_\tau - 1) dq_\lambda(\tau)$ is a compound Poisson process, and α_0 is the expectation of the relative jump size, i.e. $\alpha_0 = E[Y_t - 1]$;

$dQ_t(\lambda)$, dW_t — assumed to be statistically independent.

If $\alpha(\cdot)$ is also a constant as assumed in Merton (1976a), following Doléans-Dade exponential formula the random variable ratio of the asset price at time t to the asset price at time $t - \tau$ can be written as $S_t/S_{t-\tau} = \exp\{(\alpha - \sigma^2/2 - \lambda\alpha_0)\tau + \sigma(W_t - W_{t-\tau})\}Y(n)$, where $Y(n) = 1$ if $n = 0$; $Y(n) = \prod_{i=1}^n Y_i$ for $n \geq 1$ and $Y_i, i = 1, 2, ..., n,$ are independently and identically distributed and n is Poisson-distributed with parameter $\lambda\tau$.

The jump-diffusion process defined in (12) can be rewritten in terms of the logarithmic asset prices, i.e., $s_t = lnS_t$, as:

$$ds_t = \mu(\cdot)dt + \sigma dW_t + lnY_t dQ_t(\lambda) \tag{13}$$

where $\mu(\cdot) = \alpha(\cdot) - \lambda\alpha_0 - \frac{1}{2}\sigma^2$. When $\mu(\cdot) = \mu$ and Y_t is assumed to be i.i.d. lognormal, i.e. $lnY_t \sim i.i.d.N(\mu_0, \nu^2)$, the above process is a well defined Markov process with discrete parameter space and continuous state space and the SDE (13) has an explicit solution. [15] The major properties of this process include: (i) It is a nonstationary compounding Poisson process; (ii) However, the first difference of lnS_t or s_t over $\tau(> 0)$-period or the $\tau-$period return of asset $\Delta_\tau s_t = s_{t+\tau} - s_t$ is a stationary process, with density given by

$$f(\Delta_\tau s_t = x) = \sum_{n=0}^{\infty} \frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!}\phi(x; \mu\tau + n\mu_0, \sigma^2\tau + n\nu^2) \tag{14}$$

[15] When Y_t is assumed to be lognormally distributed with $\alpha_0 = E[Y_t - 1]$ and $\mu_0 = E[lnY_t]$, the relation between α_0 and μ_0 is $\mu_0 = ln(1 + \alpha_0) - \frac{1}{2}\nu^2$, where $\nu^2 = Var[lnY_t]$.

which has an infinite series representation, where $\phi(x; \mu, \sigma)$ is the p.d.f. of a standard normal distribution of $(x - \mu)/\sigma$. Let $\varphi_{y_\tau}(u)$ denote the characteristic function of the detrended asset return $y_\tau(t) = \Delta_\tau s_t - \mu\tau$, then $\ln\varphi_{y_\tau}(u) = -\frac{1}{2}\sigma^2\tau u^2 + \lambda\tau(\exp(\mu_0 ui - \frac{1}{2}\nu^2 u^2) - 1)$. It is easy to derive that

$$E[y_\tau] = \lambda\mu_0\tau, \quad Var[y_\tau] = (\sigma^2 + \lambda(\mu_0^2 + \nu^2))\tau$$

$$E[(y_\tau - E[y_\tau])^3] = \lambda\tau\mu_0(\mu_0^2 + 3\nu^2), \quad E[(y_\tau - E[y_\tau])^4] = 3Var[y_\tau]^2 + \phi_0 \tag{15}$$

where $\phi_0 = \lambda\tau\mu_o^4 + 6\lambda\tau\nu^2\mu_0^2 + 3\lambda\tau\nu^4$, and the distribution of y_τ is leptokurtic, more peaked in the vicinity of its mean than the distribution of a comparable normal random variable, asymmetric if $\mu_0 \neq 0$, and the skewness has the same sign as that of μ_0. These features are more consistent with the empirical findings on the unconditional distributions of most financial asset returns; and (iii), Furthermore, $s(t)$ is a process with independent increments, i.e.

$$Cov[y_\tau(t), y_\tau(t - \delta)] = 0 \tag{16}$$

for $\delta \geq \tau$. Special cases of the above model include: Press (1967) with $\mu = 0$, Beckers (1981) and Ball and Torous (1985) with $\mu_0 = 0$.

3.2 The Jump-Diffusion Option Pricing Formula

Similar to the presence of stochastic volatility, in the Merton (1976a) jump-diffusion model it is impossible to construct a riskless portfolio of underlying asset and options due to the presence of "jumps". Under the assumption that the jump component represents only non-systematic risk, or the jump risk is diversifiable,[16] Merton (1976a) derives the call option pricing formula following along the line of the original Black-Scholes derivation which assumes that the CAPM is a valid description of equilibrium asset returns. [17] Let $C_M(S_t, t; K, T, r, \sigma^2)$ be the price of a European call option at time t for the jump-diffusion model with asset price S_t, expiration date T, exercise

[16]Subsequent research by Jones (1984), Naik and Lee (1990) and Bates (1991) shows that Merton's option pricing formulas with modified parameters are still relevant under non-diversifiable jump risk or more general distributional assumptions.

[17]Alternatively, using the equivalent martingale measure approach of Cox and Ross (1976) and Harrison and Kreps (1979) as in Aase (1988) and Bardhan and Chao (1993) for general random, marked point process, or Jeanblanc-Picque and Pontier (1990) for non-homogeneous Poisson jumps, or using a general equilibrium argument as in Bates (1988), the same valuation PDE can also be derived.

price K, the instantaneous riskless rate r, and the constant non-jump instantaneous volatility σ^2, then $C_M(S_t, t; K, T, r, \sigma^2)$ solves the following integro-differential-difference equation

$$\begin{aligned}\frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 C_M(S_t, t)}{\partial P^2(t)} &+ (r - \lambda\alpha_0) S_t \frac{\partial C_M(S_t, t)}{\partial S_t} + \frac{\partial C_M(S_t, t)}{\partial t} \\ &+ \lambda E_Y[C_M(S_t Y_t, t) - C_M(S_t, t)] = r C_M(S_t, t)\end{aligned} \tag{17}$$

subject to the boundary conditions: $C_M(0, t) = 0$, $C_M(P(T), T) = Max[0, P(T) - K]$. If $\lambda = 0$, i.e. there is no jump, this pricing PDE reduces to the Black-Scholes equation and its solution for the option price formula is given by (11). Merton (1976a) showed that the solution of call option price with jumps can be written as:

$$C_M(S_t, t; K, T, r, \sigma^2) = \sum_{n=0}^{\infty} \frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!} E_{Y(n)}[C_{BS}(S_t Y(n) e^{-\lambda\alpha_0\tau}, t; K, T, r, \sigma^2)] \tag{18}$$

where $Y(n) = 1$ for $n = 0$, $Y(n) = \prod_{i=1}^{n} Y_i$, for $n \geq 1$, Y_i, $i = 1, 2, ..., n$, are i.i.d. n jumps. Under further condition that Y_t follows a log-normal distribution as assumed by Press (1967), i.e. $lnY_t \sim i.i.d.N(ln(1 + \alpha_0) - \frac{1}{2}\nu^2, \nu^2)$, thus $Y(n)$ has a log-normal distribution with the variance of logarithm of $Y(n)$, $Var[lnY(n)] = \nu^2 n$, and $E_{Y(n)}[Y(n)] = (1 + \alpha_0)^n$, a closed-form solution is given by

$$C_M(S_t, t; K, T, r, \sigma^2) = \sum_{n=0}^{\infty} \frac{e^{-\lambda'\tau}(\lambda'\tau)^n}{n!} C_{BS}(S_t, t; K, T, \gamma_n, \nu_n^2) \tag{19}$$

where $\lambda' = \lambda(1 + \alpha_0)$, $\nu_n^2 = \sigma^2 + n\nu^2/\tau$ and $\gamma_n = r - \lambda\alpha_0 + nln(1 + \alpha_0)/\tau$. The option price is simply the weighted sum of the Black-Scholes price conditional on knowing that exactly n Poisson jumps will occur during the life of the option with each weight being the probability that a Poisson random variable with intensity $\lambda'\tau$ will take on the value n. Note that the SPD in this case can be defined similarly as in (8) with γ_t being constant.

4 Implications of SV and Jump on Option Prices

In this section, we investigate the implications of SV and jump on option prices with Black-Scholes as a benchmark model. We measure option pricing errors using both absolute differences and relative percentage differences as well as implied Black-Scholes volatility. Empirical evidence suggests systematic biases of the Black-Scholes option pricing model with respect to the call option's exercise prices, its time to expiration, and the underlying common stock's volatility, see e.g. Black (1975), Gultekin, Rogalski and Tinic (1982), MacBeth and Merville (1979), Rubinstein (1985), and Bodurtha and Courtadon (1987). Since there is a one-to-one relationship between volatility and option's price through the Black-Scholes formula, an equivalent measure for the mispricing of Black-Scholes model is thus the implicit or implied Black-Scholes volatility, i.e. the volatility which generates the corresponding option price. The use of implied volatility as a measure of mispricing has a few advantages: First, since volatility is an input factor in the Black-Scholes option pricing formula, the implied volatility can directly measure the magnitudes of input errors; Second, since the Black-Scholes model imposes a flat term structure of volatility cross different degrees of moneyness and terms of expiration, the comparison base of the implied volatility is not sensitive to the calendar time, the moneyness, or the maturity as extremely are the absolute or relative errors of option prices; Third, empirical findings on the patterns of implied volatility are well-documented and more agreeable to each other for certain traded assets. The price distortions, well-known to practitioners, are usually documented in the empirical literature under the terminology of the *smile* effect, referring to the U-shaped pattern of implied volatilities across different strike prices. The following stylized facts are extensively documented in the literature (see for instance Rubinstein, 1985, Clewlow and Xu, 1993, Taylor and Xu, 1993): (i) The U-shaped pattern of implied volatility as a function of moneyness has its minimum centered at near-the-money options; (ii) The volatility smile is often but not always symmetric as a function of moneyness; and (iii) The amplitude of the smile increases quickly when time to maturity decreases. Indeed, for short maturities the smile effect is very pronounced while it almost completely disappears for longer maturities. In this paper, we use a slightly different definition of moneyness for options from the conventional one[18] following Ghysels, Harvey and Renault (1996), we define

$$x_t = ln(S_t/Ke^{-r(T-t)}) \tag{20}$$

If $x_t = 0$, the current stock price S_t coincides with the present value of the strike price K, the option is called at-the-money (ATM); if $x_t > 0$ (respectively $x_t < 0$), the option is called in-the-money (ITM) (respectively out-of-the-money (OTM)).

[18]In practice, it is more common to call an option as at-the-money, in-the-money, or out-of-the-money when $S_t = K$, $S_t > K$, or $S_t < K$ respectively. For American type options with possibility of early exercise, it is more convenient to compare S_t with K, while for European type options and from an economic point of view, it is more appealing to compare S_t with the present value of the strike price K.

4.1 Implications of SV on Option Prices

Implications of changing volatility on option prices have been investigated in the literature with a focus on continuous-time models. It is obvious that such implications depend critically on the specifi cation of the SV processes. For instance, based on the model with lognormal stochastic volatility, Hull and White (1987) show that when the volatility is uncorrelated with the stock price, the Black-Scholes model underprices the ITM and OTM options and overprices the ATM options. The largest absolute price differences occur at or near the money. The actual magnitude of the pricing error, however, is quite small in general. When the volatility is correlated with the stock price, this ATM overprice continues on to ITM options for positive correlation and to OTM options for negative correlation. Stein and Stein (1991) specify a Ornstein-Uhlenbeck process for the stochastic volatility $\sigma_t(h_t)$ with zero correlation between stock price and conditional volatility. Their results suggest that SV has an upward influence on option prices and SV is "more important" for OTM options than ATM options in the sense that the implied volatility corresponding to the SV prices exhibit a U-shaped curve as the strike price is varied. Implied volatility is lowest for ATM options and rises as the strike price moves in either direction. Johnson and Shanno (1987) specify a CEV type SV process and investigate the implications of correlation coeffi cient ρ between stock price and volatility based on Monte Carlo simulations. They show that, when ρ changes from negative to positive, the call prices increase for OTM options, decreases for ITM options, but is rather insensitive for ATM options. Heston (1993) specifi es a squared root SV process and assumes a non-zero correlation between volatility and asset prices. Using the underlying probability density function of spot asset returns, Heston shows that the SV model can induce almost any type of bias to option prices. Various empirical studies on the applications of SV models in pricing options have been conducted based on observed asset returns and/or observed market option prices. We mention Scott (1987), Wiggins (1987), Chesney and Scott (1989), Melino and Turnbull (1990), Bakshi, Cao and Chen (1997), and Jiang and van der Sluis (1998). The joint effect of SV, stochastic interest rates and systematic SV on option prices are also investigated in the literature, see e.g. Bailey and Stulz (1989) and Amin and Ng (1993). For instance, Bailey and Stulz (1989) analyzes the pricing of stock index options by assuming that both the volatility of the stock index and the spot rate of interest are driven by the same state variable.

As we have mentioned, the main advantage of the SV model is its flexible distributional structure in which the correlation between volatility and asset returns serves to control the level of asymmetry and the volatility variation coefficient serves to control the level of kurtosis, this paper will investigate the implications of SV model on option prices with a focus on these two aspects. In addition, since financial data are essentially observed over discrete time and the persistence of conditional volatility is often gauged on the basis of daily or weekly frequency, our investigation will be based on the discrete-time model specified in (2) and (3).

4.1.A. Implications of Autoregressive Volatility on Option Prices

To investigate the implications of autoregressive volatility or intertemporally persistent volatility on option prices with the Black-Scholes option pricing formula as a benchmark model, we fix the level of unconditional volatility of asset returns as constant, i.e. $Var[y_t] = \sigma_Y^2$, and allow for ϕ to change from $0.0, 0.3, 0.6$ to 0.9. Thus, the input of Black-Scholes volatility and hence the Black-Scholes option prices are the same cross different models. We further assume that there is no asymmetry between asset returns and conditional volatility, i.e. $\rho = 0$. To calibrate the parameter values of the model, we match relevant statistics to those of real stock returns (namely daily returns over the past 10 years for 3 Com which is traded in NASDAQ). Using the fact that $Var[y_t] = \sigma^2 \exp\{\sigma_h^2/2\}$, $\sigma_h^2 = \sigma_\eta^2/(1-\phi^2)$, $E[\ln y_t^2] = \ln\sigma^2 - 1.27$, and $Var[\ln y_t^2] = \pi^2/2 + \sigma_\eta^2/(1-\phi^2)$, we assign $\sigma_Y = 0.04$, $\sigma = 0.03$, $\phi = 0.0, 0.3, 0.6$, and 0.9, and $\sigma_\eta^2 = 2(1-\phi^2)\ln(\sigma_Y/\sigma)^2$. The basic idea is to generate asset return processes with different conditional volatility process but the same unconditional asset return distribution. The SV option prices are calculated based on Monte Carlo simulation using both (9) and (10). The only approximation error involved is the Monte Carlo error which can be reduced to any desired level by increasing the number of simulations. In our simulation, 100,000 sampling paths are simulated to reduce the Monte Carlo error and to reflect accurately the fat-tail behavior of the asset return distributions, and the antithetic variable technique is used to reduce the variation of option prices (see Boyle, Broadie and Glasserman, 1996). The results show that option prices generated using different methods are almost the same, with the largest differences less than a penny for even long term deep ITM options. The accuracy is further reflected in the small standard derivations of the simulated option prices. The Black-Scholes option prices are calculated based on the unconditional volatility σ_Y^2.

Setting the stock price $P = \$40.00$, strike price K ranging from \$25.00 to \$60.00, maturities τ equal to respectively 7, 14, 30, 91 days, and annualized compound risk-free rate $r = 5\%$, Figure

1 plots the biases of the Black-Scholes model when stock returns follow a SV process as in (2) and (3) with ϕ changing from 0.0 to 0.9 while holding the unconditional volatility, σ_Y^2, constant. When $\phi = 0$, i.e. the conditional volatility is not intertemporally correlated but iid over time, the difference between the Black-Scholes model and the SV model reflects the impact of the moments with orders higher than or equal to three since the first two moments are the same for both models. Figure 1 shows that the Black-Scholes model with lognormal distribution for asset returns tends to overprice near-the-money options while underprice both deep ITM and OTM options. The absolute differences are small but most severe for short-term near the money options, and the percentage differences are most severe for short-term deep OTM options. When ϕ increases to be positive, the differences between the Black-Scholes model and the SV model further reflect the impact of the dynamics of the conditional volatility, i.e. the conditional second moments, on option prices. Figure 1(1), 1(2), & 1(3) show that, overall, both absolute and percentage differences increase as the value of ϕ, i.e. the level of intertemporal persistence, increases. The impact of the change of ϕ, however, is relatively larger on the long-term options than on the short-term options in terms of both absolute and percentage differences. In other words, given the unconditional distribution of asset returns, the level of intertemporal volatility persistence can have significant impact on long-term options. The implied Black-Scholes volatilities exhibit obvious U-shaped patterns (smiles) as the call option goes from deep OTM to ATM and then to deep ITM. The smiles are essentially symmetric as a function of moneyness with the lowest point around the ATM options. Furthermore, the volatility smiles are more pronounced and more sensitive to the time to expiration for short-term options than for medium-term and long-term options. Overall the implied Black-Scholes volatility smiles more as ϕ increases.

Figure 1 about here

4.1.B. Implications of Asymmetric Volatility on Option Prices

To investigate the implications of asymmetric conditional volatility on option prices, we fix the parameter values at $\sigma_Y = 0.04$, $\sigma = 0.03$ and $\phi = 0.9$ which is the case in Figure 1(1)b, 1(2)b and 1(3)b, but let the correlation coefficient ρ change from -0.4, -0.2, 0.2, to 0.4. Option prices are also calculated based on Monte Carlo simulation using (9) with 100,000 simulated sampling paths and the antithetic variable technique. The accuracy is again reflected in the small standard derivations of the simulated option prices. Figures 2 plots the biases of Black-Scholes model when the conditional volatility is asymmetric with the asset returns for $\rho = -0.4$ and 0.4. As expected when $\rho < 0$, i.e. there is a higher conditional volatility associated with a negative price change, the Black-Scholes model tends to underprice ITM options more, underprice OTM options less, and even overprice long-term OTM options. On the other hand, when $\rho > 0$, i.e. there is a higher conditional volatility associated with a positive price change, the Black-Scholes model tends to underprice OTM options more, underprice ITM options less, and even overprice long-term ITM options. These results are consistent with the intuitive explanation offered by Hull and White (1987) in terms of the impact of asymmetric SV on the terminal distribution of stock prices. For instance, when the volatility is negatively correlated with asset returns, which is often observed in the stock price movements known as the "leverage effect", higher stock prices are associated with lower volatility and lower stock prices are associated with higher volatility. High stock prices become more like absorbing states, so that very low price becomes less likely than when the volatility is symmetric. The net effect is that the terminal stock price distribution is more peaked around its mean and may be skewed to the left with thinner but longer left-hand tail than the right-hand tail. In both cases of $\rho > 0$ and $\rho < 0$, the absolute biases are more severe for long-term deep OTM options and the relative biases are more severe for short-term deep OTM options. The change of the value of ρ is shown to have relatively larger impact on long-term options than on short-term options, and larger impact on deep OTM options than on deep ITM options in terms of both absolute and percentage differences. In particular, the change of the value ρ has little effect on the short-term near-the-money options, but significant impact on long-term near-the-money options. That is, similar to the change of the volatility persistence parameter ϕ, the change of the correlation parameter ρ can also have significant impact on long-term options. The implied volatility still exhibits the *smile* patterns, more pronounced for short-term options and almost flat for long-term options. However, even when the level of correlation is as high as 40%, the smiles are only slightly skewed to the direction of ρ for short-term options with both deep OTM and deep ITM options having much higher implied volatility than near-the-money options. This is due to the fact that asymmetry between conditional volatility and asset return, as a dynamic property of the return process, can only be realized through time evolution. It suggests that the asymmetry between

SV and asset returns do not offer good explanations to the empirically documented phenomenon (see e.g. Clewlow and Xu, 1993, and Taylor and Xu, 1993) that the implied volatility of short-term options often tends to either rise more with increasing striking prices and has its minimum at ITM options or rise more with decreasing striking prices and has its minimum at OTM options.

Figure 2 about here

4.2 Implications of Jump on Option Prices

The jump-diffusion option pricing model proposed by Merton (1976a) is an important alternative to and extension of the Black and Scholes (1973) option pricing model. The question raised in Merton (1976a) and answered in details in Merton (1976b) is as following: Suppose an investor believes that the stock price dynamics follows the geometric Brownian motion and therefore uses the Black-Scholes formula to evaluate the options when the true price process is described by the jump-diffusion process (12). How will the investor's appraised value based on a misspecified process for the stock compare with the true value based on the correct process?

To make the comparison feasible and straightforward, Merton (1976b) assumed that $lnY_t \sim iid\ N(-\frac{1}{2}\nu^2, \nu^2)$, or $\alpha_0 = E[Y_t - 1] = 0$. Let $V = \sigma^2\tau + N\nu^2$ be the random volatility of the true jump-diffusion process for the τ-period return, i.e. N is a Poisson-distributed random variable with intensity parameter $\lambda\tau$. So the true volatility observed over τ-period is $V_n = \sigma^2\tau + n\nu^2$ when $N = n$. From Merton's jump-diffusion option price formula, we have the true option price given by (19) as $C_M = E_n[C_{BS}(S_t, t; K, T, r, V_n/\tau)]$. Based on a sufficiently long time series of data, the investor can obtain a true unconditional volatility for τ-period stock return, i.e. $V_{BS} = E[V] = (\sigma^2 + \lambda\nu^2)\tau$ and the incorrect price of the option based on the Black-Scholes model is given by (11) as $C_{BS} = C_{BS}(S_t, t; K, T, r, V_{BS}/\tau)$. The exact magnitude of the difference depends very much on the values of parameters. Merton (1976b) used the following four parameters to gauge the specific patterns of Black-Scholes model biases: (i) $X_t = S_t/Ke^{-r(T-t)}$, i.e. the measure of moneyness; (ii) $V = (\sigma^2+\lambda\nu^2)(T-t)$, the expected variance, or total volatility, of the logarithmic return on the stock over the life of the option; (iii) $\gamma = \lambda\nu^2/(\sigma^2 + \lambda\nu^2)$, the fraction of the total expected variance in the stock's return caused by the jump component which measure the significance of the jump factor in the process and therefore reflects the degree of misspecification of the Black-Scholes model; (iv) $\omega = \lambda(T - t)/V$, the ratio of the expected number of jumps over the life of the option to the expected variance of the stock's return which is also a measure of the degree of misspecification. For given values of the above four parameters, Merton (1976b) showed that: (a) the Black-Scholes model tends to undervalue deep ITM and OTM options, while overvalues the near-the-money options; (b) in terms of percentage difference, there are two local extreme points: one is the largest percentage overvaluation of option price ATM, and the other is the largest percentage undervaluation for ITM options, there is no local maximum for the percentage undervaluation for OTM options, the error becomes larger and larger as the option becomes more OTM; and (c) the magnitude of the percentage error increases as either γ increases or ω decreases. In particular, suppose the value of total conditional volatility $\sigma^2 + \lambda\nu^2$ is fixed, increase of $\lambda\nu^2$ will have a larger impact on the option prices. Suppose $\lambda\nu^2$ is fixed, when λ is relatively small, but ν^2 is relatively large, then the difference between Merton price and Black-Scholes price will be relatively larger, especially for short-maturity and OTM options. Otherwise, if the jump frequency is very high while the variance of the jump becomes very small, applying the Central Limit Theorem (see e.g. Cox and Ross, 1976, for this case), it can be shown that the compounding Poisson jump process approaches a continuous process with a corresponding normal distribution in the limit. Thus, the Merton jump-diffusion process and the Black-Scholes continuous sample process would not be distinguishable and hence the prices of options will not be largely different. Many empirical studies have also been conducted to investigate the impact of jumps in the underlying asset return processes on option prices. As we have mentioned, since most of the empirical results found that the jump components tend to have high frequency but low amplitude, it is not surprising that in these studies the Black-Scholes and Merton prices are virtually not different from each other, see e.g. Beckers (1981), Ball and Torous (1985), and Trautmann and Beinert (1994). Exceptions include Jorion (1988) in which he found substantial differences between the Black-Scholes and Merton prices for less than a month and longer maturity CRSP stock index options.

Merton (1976b) focused his comparison on the relative percentage differences of option prices. In this paper, we compare the differences between Merton (1976a) jump-diffusion prices and Black-Scholes (1973) prices in terms of both absolute biases and relative percentage biases as well as implied Black-Scholes volatilities. In addition, we also look at the impact of asymmetric jump on the biases of option prices and the shapes of implied Black-Scholes volatilities.

4.2.A. Implications of Symmetric Jump on Option Prices

To investigate the implications of jump on option prices, we first let γ change from $1/2$ to 3/4 while holding the total volatility $\sigma^2 + \lambda\nu^2$ constant, and then let the jump frequency decrease from $1/4$ to $1/8$ and jump volatility increase from $(0.04)^2$ to $2 \times (0.04)^2$ (thus ω decrease) while holding both total volatility $\sigma^2+\lambda\nu^2$ and the jump volatility $\lambda\nu^2$ (thus γ) constant. Setting the stock price $P = \$40.00$, strike price K ranges from \$25.00 to \$60.00, maturities τ equal to respectively 7, 14, 30, 91 days, and annualized compound risk-free rate $r = 5\%$, Figure 3 plots the biases of the Black-Scholes model when stock returns follow a jump-diffusion for $\gamma = \lambda\nu^2/(\sigma^2+\lambda\nu^2)$ changing from $1/2$, to 3/4 while holding $\sigma^2 + \lambda\nu^2 = (0.04)^2$ constant. Similar to the comparison with the SV model, the Black-Scholes model tends to overprice near-the-money options but underprice both deep ITM and OTM options. The absolute differences are most pronounced for near the money and short-term options, while the percentage differences are most pronounced for short-term deep OTM options. The increase of jump factor, i.e. the increase of the value of γ, has relatively larger impact on the short-term and near the money options in terms of both absolute and percentage differences. The implied Black-Scholes volatilities exhibit obvious U-shaped patterns (smiles) as the call option goes from deep OTM to ATM and then to deep ITM. Furthermore, the volatility smiles are more pronounced and more sensitive to the time to expiration for short-term options than for medium-term and long-term options. Overall the implied Black-Scholes volatility is symmetric as a function of moneyness and smiles more as γ increases. Similar patterns of biases are observed for Black-Scholes model with λ decreasing or ν^2 increasing while holding both $\lambda\nu^2$ and $\sigma^2 + \lambda\nu^2$ (thus γ) as constants. When $\gamma = 1/4$ and λ decreases from $1/4$ to $1/8$ and the jump volatility increases from $(0.04)^2$ to $2 \times (0.04)^2$, both the absolute and percentage biases increase and the volatility smiles become more pronounced.

Figure 3 about here

4.2.B. Implications of Asymmetric Jump on Option Prices

To investigate the implications of asymmetric jump on option prices, we fix $\lambda = 1/4$, $\sigma^2 = 0.25 \times (0.04)^2$, $\nu^2 = 3 \times (0.04)^2$ which is the case in Figure 3(1)b, 3(2)b and 3(3)b, and let the jump size α_0 change from 5%, 2%, -2% to -5%. The Merton option prices are calculated based on (19) and the Black-Scholes option prices calculated based on (11) with volatility given in (15). Figures 4 further shows the biases of Black-Scholes model when the random jump is asymmetric, namely $\alpha_0 = 2\%$ and -2%. As expected when $\alpha_0 > 0$, i.e. there is an expected positive jump, the Black-Scholes model tends to underprice OTM options more, underprice ITM options less, and even overprice ITM options when $|\alpha_0|$ is large. On the other hand, when $\alpha_0 < 0$, i.e. there is an expected negative jump, the Black-Scholes model tends to underprice ITM options more, underprice OTM options less, and even overprice OTM options when $|\alpha_0|$ is large. In both cases, the absolute biases are more severe for moderate OTM and ITM short-term options and for deep ITM or deep OTM long-term options and the relative biases are more severe for deep OTM options. The impact of the change of the value α_0 is shown to be relatively larger on short-term options than on long-term options in terms of both absolute and percentage differences. It is obvious that a negative expected jump induces the implied volatility smile to be skewed to the right and a positive expected jump induces the implied volatility smile to be skewed to the left. Even though there is no clear conclusion on whether the asymmetric smile is a result of skewed underlying return distribution with excess kurtosis, the above evidence shows the other direction is true. It offers a potential explanation to the empirically often observed asymmetric volatility smiles, i.e. the implied volatility of short-term options tends to either rise more with increasing striking prices and has its minimum at ITM options or rise more with decreasing striking prices and has its minimum at OTM options. Furthermore, while as Hull and White (1987) point out that it is difficult to use the asymmetry of conditional volatility to explain the empirically observed reverse of slope from time to time for implied volatilities of short-term options (see Rubinstein, 1985, and also Taylor and Xu, 1993), it may be more reasonable to attribute such phenomenon to the change of the sign of jump sizes. For instance, when the market overreacts to the flow of important information shocks, a positive (respectively negative) jump is often followed by a negative (respectively positive) jump.

Figure 4 about here

4.3 Comparison between SV and Jump

Since the introduction of jump component and stochastic volatility into the underlying asset return process both are to feature the asymmetry and kurtosis of asset return distributions, it is not surprising that they have similar implications on option prices. This makes it difficult to determine whether the biases of the Black-Scholes model are due to the presence of jump or SV or both in the underlying asset return processes. As we have mentioned, in practice it is also difficult to disentangle the jump component from the SV component in the underlying asset return process based on a sample over a relatively short time period. Thus it would be interesting to investigate the interplay between jump and stochastic volatility in pricing options. In other words, it would be important to gauge the risk of misspecification between SV and jump in terms of option pricing errors. To compare the implications of SV and jump on option prices, we assume that the SV process is the true underlying data generating process (DGP) while the Merton (1976a) jump-diffusion process is a misspecified process. Again we only investigate the impact of misspecification on option prices and assume that the jump-diffusion parameters are identified from the true population moments.[19] This is equivalent to assume that the jump-diffusion is estimated based on a large enough sample over long enough sampling period. Since the Merton (1976a) jump-diffusion model has an exact discrete-time version, we can identify the misspecified jump-diffusion from the population distribution of the discrete-time SV process specified in (2) and (3). One way to determine the relationship between the parameters in the misspecified model and the parameters in the true model is to solve for the ML estimators of the parameters of the misspecified model as functions of the parameter values of the true data generating process. Unfortunately there exist no explicit solutions. Here we use the method of cumulants matching, a variant of method of moments, to identify the parameter relationship. [20]

[19]The investigation of the implication of estimation risk would be interesting as well but would be much complicated due to the complicated model structure here.

[20]The method of cumulants matching are often used in the finance literature to estimate jump-diffusion processes, see e.g. Press (1967), and Beckers (1981). As always, since the underlying model is misspecified, strictly speaking such an approach may involve both specification risk and estimation risk in pricing options as the choice of different cumulants may lead to different relationship between the misspecified parameters and the true parameters. However,

Obviously, both the unconditional distribution and the dynamics of conditional volatility of the SV process play a role in the difference between the SV model and jump-diffusion model. Since the Merton (1976a) jump-diffusion assumes no intertemporal persistence of conditional volatility and the unconditional distribution of the SV process is symmetric, for simplicity we restrict our comparison to the case of uncorrelated and symmetric conditional volatility and symmetric jump size, i.e. $\phi = \mu_0 = \rho = 0.0$. When the conditional volatility is indeed autocorrelated and/or asymmetric with asset returns, the combined risk of the jump-diffusion option pricing model can be deduced from the results in sections 4.1 and 4.2. Let $K_i, i = 1, 2, ...,$ be the i-th cumulant of the random variable. Based on the jump-diffusion model specified by the investor as in Merton (1976a) with $\mu_0 = 0$, one can derive that the first six cumulants of the detrended asset return $y_\tau(t)$ are

$$K_1 = K_3 = K_5 = 0,$$

$$K_2 = (\sigma^2 + \lambda\nu^2)\tau, \quad K_4 = 3\nu^4\lambda\tau, \quad K_6 = 15\nu^6\lambda\tau \tag{21}$$

from which we can solve for

$$\lambda = 25K_4^3/3K_6^2\tau, \quad \sigma^2 = K_2/\tau - 5K_4^2/3K_6\tau, \quad \nu^2 = K_6/5K_4 \tag{22}$$

In other words, the jump-diffusion model in Merton (1976a) is fully determined by the first six moments or cumulants. Based on the true SV process in (2), one can also derive the true cumulants $K_i^*, i = 1, 2, ..., 6$ of the detrended asset return y_t (see Appendix). Matching the cumulants, we have the jump-diffusion parameters implied from the SV process.

Figure 5 about here

The difference between the SV and jump-diffusion prices thus reflect the impact of the moments with orders higher than or equal to seven since the first six moments are equal for both

the investor is not aware of the estimation risk as it is believed that the specified process represents the true DGP and the parameters are, as assumed, as if identified from the population instead of the sampling observations.

models. We set the parameters in the SV model as $\sigma = 0.03$, and let the level of kurtosis of the unconditional asset return distribution to change, i.e. kurtosis$/3 = 2, 4, 8$ and 16, where kurtosis$/3 = \exp(\sigma_h^2)$. Figure 5 plots the biases of Merton jump-diffusion option prices when the underlying asset return follows the SV process with iid conditional volatility for kurtosis$/3 = 4$ and 8. It is shown that when the excess kurtosis of the underlying asset return distribution is low, the Merton prices and SV prices are virtually not different from each other. However, as the level of kurtosis increases, the difference increases and becomes no longer negligible. The differences are more pronounced for short-term options than for long-term options and more pronounced for near-the-money options than for ITM and OTM options. Figure 5 further shows that the underlying asset return distribution of the jump-diffusion tends to be more peaked in the vicinity of its mean with thinner but in general longer tails. Combined with the results in sections 4.1 & 4.2, the misspecification risk of stochastic volatility as jump is minimal in terms of option pricing errors only when both the level of volatility persistence and the level of kurtosis of the underlying asset return distribution are low, but increases as either the level of kurtosis or the level of volatility persistence increases. In particular, an increase in the level of kurtosis of the underlying asset return distribution induces a relatively larger risk in pricing short-term options, while an increase in the level of volatility persistence and/or asymmetry induces a relatively larger risk in pricing long-term ITM and OTM options.

5 Conclusion

This paper conducts a thorough and detailed investigation on the implications of stochastic volatility and random jump on option prices. First of all, since both the stochastic volatility and jump-diffusion processes admit asymmetric and fat-tailed distributions of asset returns, they have similar impact on option prices compared to the Black-Scholes model; Second, the impact of stochastic volatility on option prices depends on both the unconditional distribution and the dynamics of conditional volatility, i.e. its intertemporal persistence and asymmetry with asset returns. While the unconditional distribution is shown to have relatively larger impact on short-term options, the dynamic properties of stochastic volatility have relatively larger impact on long-term options; Third, the impact of jump-diffusion on option prices is shown to be relatively larger for short-term near-the money options. The potential risk of a simple jump-diffusion model is the assumption of intertemporal independence of volatility, and its advantage is the flexibility in inducing short-term option pricing errors; Fourth, compared to each other, the difference between SV model and jump-diffusion model or the misspecification risk of SV as jump is negligible when both the level of kurtosis of the underlying asset return distribution and the level of volatility persistence are low, but increases with the increase of either the level of kurtosis or the level of volatility persistence. While an increase in the level of volatility persistence and/or symmetry tends to induce larger risk in pricing long-term options, an increase in the level of kurtosis tends to induce larger risk in pricing short-term near-the-money options; Finally, while both asymmetric stochastic volatility and asymmetric jump can induce distortion of option pricing errors, the skewness of jump-diffusion offers a better explanation to the empirical findings that the implied volatility of short-term options often tends to either rise more with increasing striking prices and has its minimum at ITM options or rise more with decreasing striking prices and has its minimum at OTM options. In addition, the skewness of jump-diffusion offers a more reasonable explanation to the empirically observed phenomenon that the slope of implied volatility for short-term options reverses from time to time.

Implications of the above results are straightforward and important for the implementation of option pricing models. In the area of statistics and econometrics as well as in the area of finance when implementing option pricing models using the implied estimation method, when dealing with a data generating process (DGP) of complicated structure, a common approach of model specification and selection is to start with a complicated model and then test down to a simpler one. The evidence from option pricing errors as studied in this paper can lead to a different approach of model specification and selection in pricing asset options, i.e. to start from a simple model and then expand to a more complicated one *if necessary*. For instance, when a simple jump-diffusion model is implemented, certain patterns of long-term option pricing errors may suggest that the model is oversimplified and the dynamics of conditional volatility should be incorporated. On the other hand, if a simple SV model is implemented, certain patterns of short-term option pricing errors may suggest that the underlying asset return process is indeed discontinuous and thus a jump component should be included in modeling the dynamics of asset returns. From this perspective, the SV model and jump-diffusion model can be viewed as competing ones on the one hand and complimentary to each other on the other.

While we assume a constant term in our study for the conditional mean of asset returns, the impact of a non-constant drift term or predictability of conditional asset returns can also be investigated as in Lo and Wang (1995). In addition, in this paper we base our study on models specified in the objective distribution of the data generating process and investigate their implications on option prices. Since the risk-neutral distribution under the equivalent martingale measure and the objective distribution of the data generating process are closely linked to each other as investigated in Grundy (1991) and Bates (1996a), it is also possible to base our comparison on risk-neutral distributions with non-zero risk premium for both jump risk and stochastic volatility risk. Finally, even though we deal with explicitly the discrete-time SV models in our study, the method and major results can be extended to the continuous-time models which can be viewed as limit of discrete-time models.

Appendix

1. Cumulants of Merton (1976a) Jump-Diffusion Process: Using the characteristic functions in section 3.1, one can easily derive the cumulants of $y_\tau(t)$ as following (for definition of cumulants as well as their relationship with moments, see Kendall and Stuart (1977, pp69-71)) with $\mu_0 = 0$;

$$K_1 = K_3 = K_5 = 0$$

$$K_2 = (\sigma^2 + \lambda\nu^2)\tau, \quad K_4 = 3\lambda\nu^4\tau, \quad K_6 = 15\lambda\nu^6\tau$$

2. Cumulants of SV Process: Since the s^{th} moment of the detrended asset return y_t is given by

$$m_s = E[y_t^s] = \sigma^s E[\epsilon_t^s] \exp\{s^2\sigma_h^2/8\}$$

when s is even and $m_s = 0$ when s is odd. The cumulants of the SV process can be derived using the following relations

$$K_1^* = K_3^* = K_5^* = 0$$

$$K_2^* = m_2, \quad K_4^* = m_4 - 3m_2^2, \quad K_6^* = m_6 - 15m_4m_2 + 30m_2^3$$

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Figure 1. SV Option Prices versus Black-Scholes (1973) Prices

Setting stock price $P = \$40.00$, strike price $K = [\$25.00, \$60.00]$, time to maturity $\tau = 7, 14, 30, 91$ days, annualized compound risk-free rate $r = 5\%$. Parameter values: $\sigma_Y = 0.04, \sigma = 0.03, \rho = 0.0, \sigma_\eta^2 = 2(1-\phi^2)\ln(\sigma_Y/\sigma)^2$.

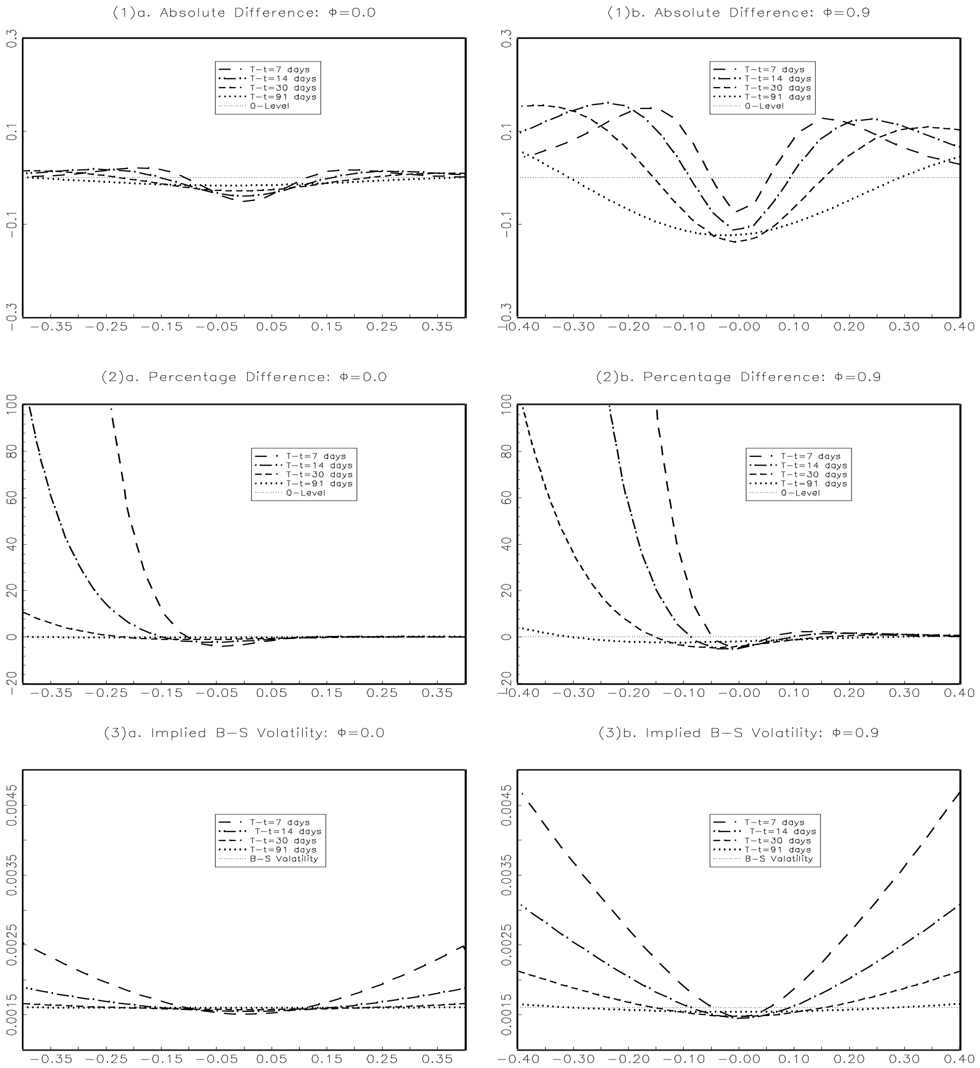

Figure 2. SV Option Prices versus Black-Scholes (1973) Prices

Setting stock price $P = \$40.00$, strike price $K = [\$25.00, \$60.00]$, time to maturity $\tau = 7, 14, 30, 91$ days, annualized compound risk-free rate $r = 5\%$. Parameter values: $\sigma_Y = 0.04, \sigma = 0.03, \phi + 0.9, \sigma_\eta^2 = 2(1-\phi^2)\ln(\sigma_Y/\sigma)^2$.

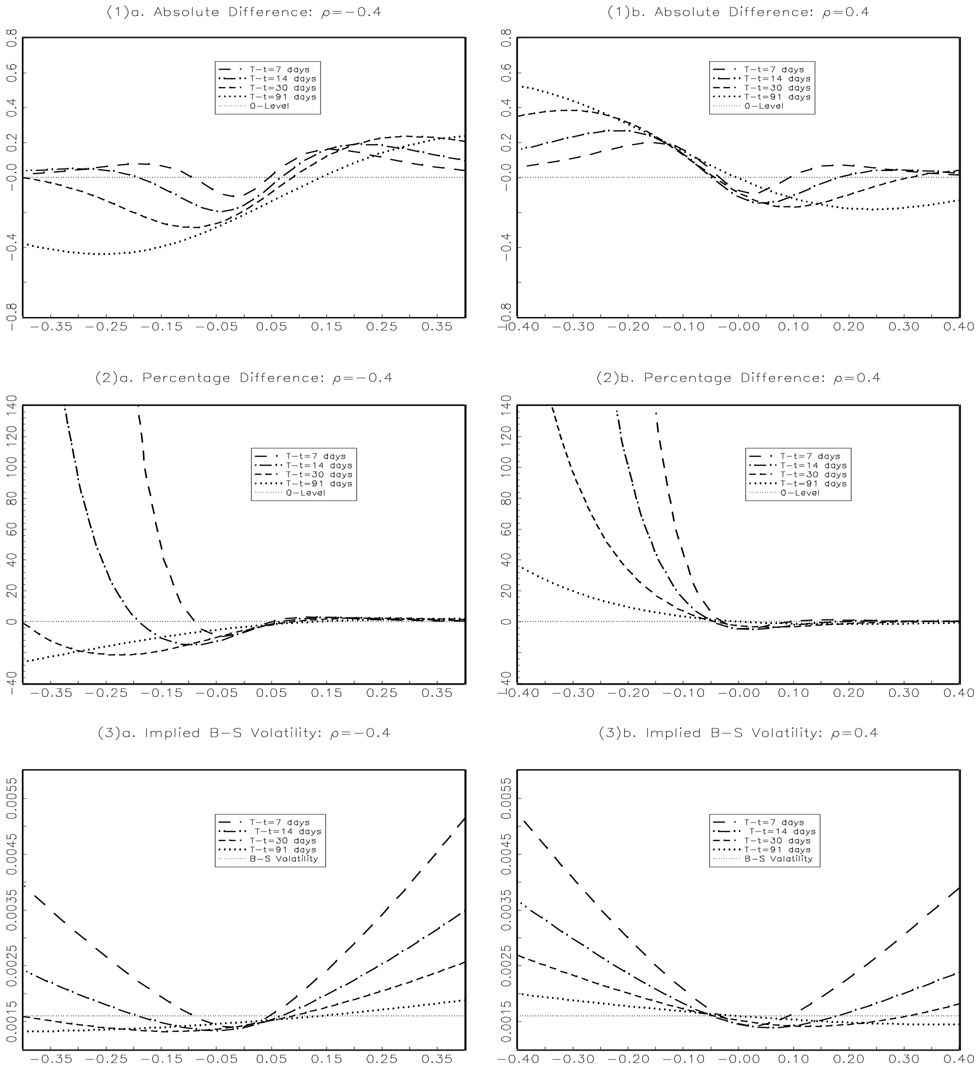

Figure 3. Merton (1976a) Prices versus Black-Scholes (1973) Prices

Setting stock price $P = \$40.00$, strike price $K = [\$25.00, \$60.00]$, time to maturity $\tau = 7, 14, 30, 91$ days, annualized compound risk-free rate $r = 5\%$. Parameter values: $\gamma = \lambda\nu^2/(\sigma^2 + \lambda\nu^2)$, *and* (a) $\lambda = 1/4, \sigma^2 = 0.5 \times (0.04)^2, \nu^2 = 2 \times (0.04)^2, \gamma = 1/2$; (b) $\lambda = 1/4, \sigma^2 = 0.25 \times (0.04)^2, \nu^2 = 3 \times (0.04)^2, \gamma = 3/4$.

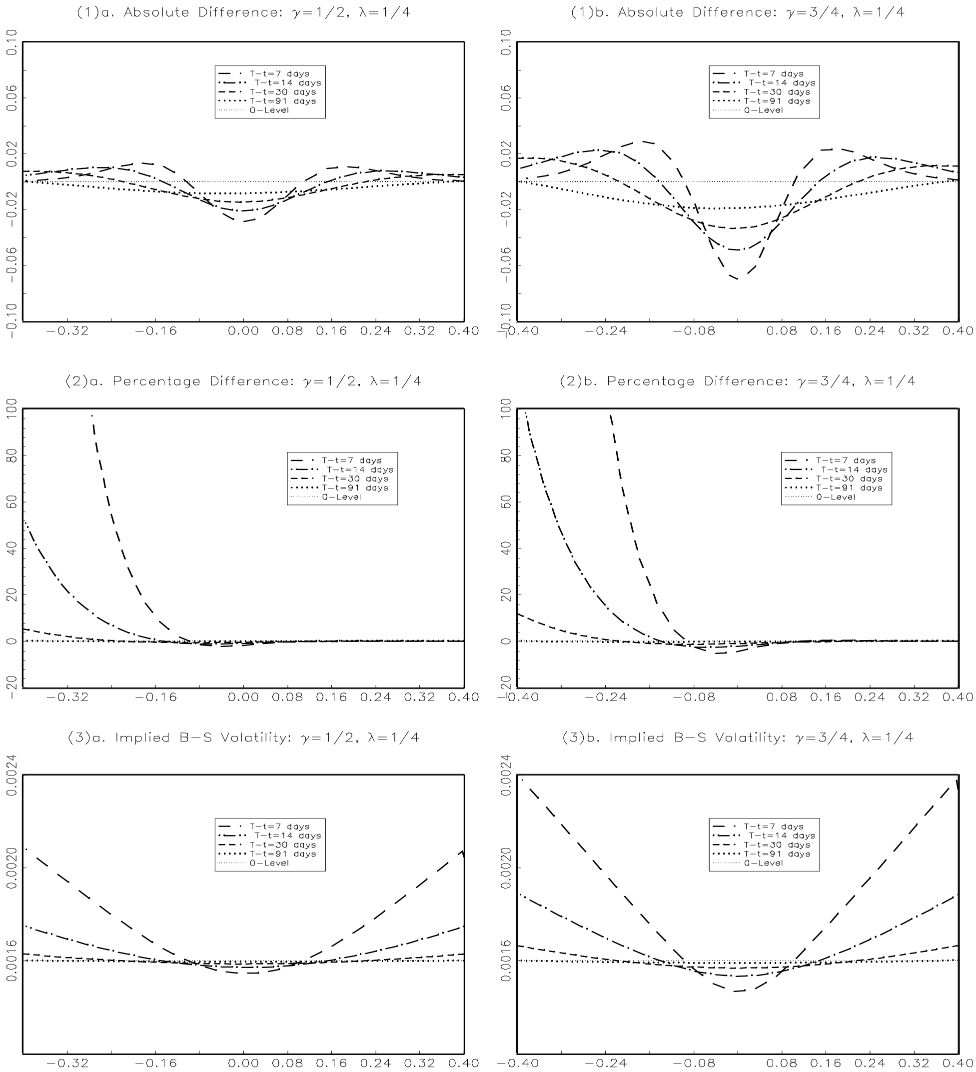

Figure 4. Merton (1976a) Prices versus Black-Scholes (1973) Prices

Setting stock price $P = \$40.00$, strike price $K = [\$25.00, \$60.00]$, time to maturity $\tau = 7, 14, 30, 91$ days, annualized compound risk-free rate $r = 5\%$. Parameter values: $\lambda = 1/4, \sigma^2 = 0.25 \times (0.04)^2$, $\nu^2 = 3 \times (0.04)^2$, $\sigma_{BS}^2 = \sigma^2 + \lambda(\mu_0^2 + \nu^2)$, $\mu_0 = ln(1 + \alpha_0) - \frac{1}{2}\nu^2$.

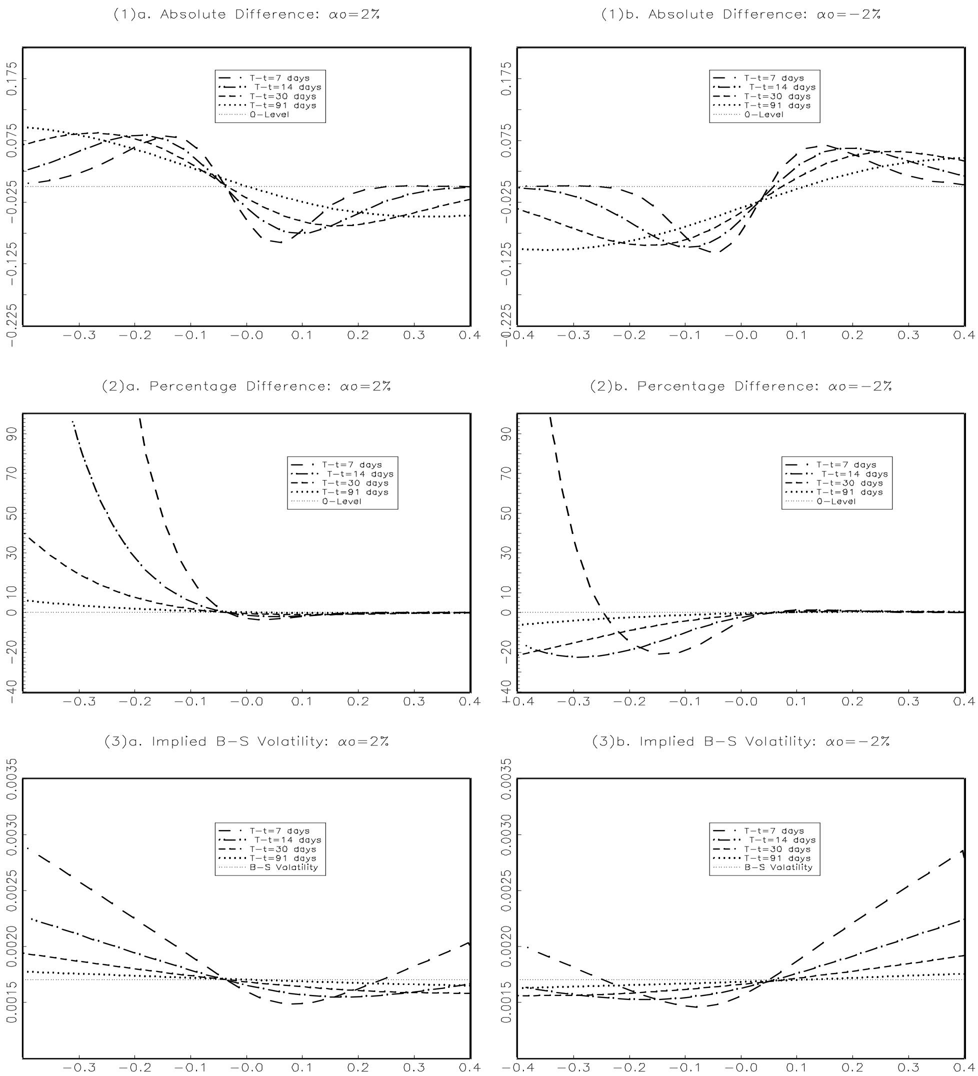

Figure 5. SV Option Prices versus Merton (1976a) Prices

Setting stock price $P = \$40.00$, strike price $K = [\$25.00, \$60.00]$, time to maturity $\tau = 7, 14, 30, 91$ days, annualized compound risk-free rate $r = 5\%$. Parameter values: $\sigma = 0.03, \phi = 0.0, \rho = 0.0$, Kurtosis/3 $= \exp(\sigma_h^2)$.