

## APPENDIX II. NOTATION

The following symbols are used in this paper:

$A$	=	cross-sectional area of member;
$a$	=	Fourier coefficient;
$B$	=	slope angle;
$E$	=	Young's modulus;
$EI$	=	bending stiffness;
$F_i$	=	tension force in cable $i$ ;
$h_p$	=	height of pier;
$I$	=	cross-sectional moment of inertia;
$[K_c]$	=	stiffness of bridge without tower;
$[K_t]$	=	stiffness of tower alone;
$k_{uu}, k_{uw}, k_{wu}$	=	stiffness of caisson at head;
$L_k$	=	span of bridge beam $k$ ;
$M$	=	end moment, bending moment;
$P$	=	concentrated live load;
$p$	=	intensity of uniformly distributed live load;
$[Q_c]$	=	resultant of horizontal components of cable forces;
$x_f$	=	displacements of caisson;
$y$	=	girder deflection;
$\Delta_i$	=	deflection of tower at point of attachment of cable $i$ ;
$\theta_f$	=	angle of rotation of caisson head; and
$\theta_i$	=	slope angle of cable $i$ .

## BEHAVIOR OF MONOTUBE HIGHWAY SIGN SUPPORT STRUCTURES

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**ABSTRACT:** The design of highway sign support structures is governed by the AASHTO Specifications which limit the dead load deflection of the beam to  $d^2/400$ , where  $d$  is the depth of the traffic sign in feet. In many cases, the more slender monotube sign support structures, which have become more popular in recent years, can not satisfy this deflection criterion. The results of a parametric study of monotube highway sign support structures are presented in this paper. Structures with various column and beam stiffnesses, spans, area and location of traffic signs were analyzed for static and dynamic (wind) loads. Vortex shedding is a prime consideration for such structures; this was accounted for in the dynamic analysis. It was found that stress levels are well below the allowable values, although the deflections exceeded the AASHTO criterion by significant margins. It was also concluded that for typical structures, resonance was not a likely occurrence. Recommendations are made for the analysis and design procedures.

### INTRODUCTION

The development of the nationwide interstate highway system prompted considerable attention to be given to the safety of the roads and their related traffic systems. Thus, improvements were made in the form of roadway alignment, bridge design, and placement of traffic signs, to mention some of the areas. The latter has become a significant construction industry over the years, and it is estimated that the annual national expenditure on manufacturing and maintenance of signs and sign support structures is approximately 250 million dollars.

The design of highway sign support structures is governed by a specification issued by the American Association for State Highway and Transportation Officials (AASHTO 1975). The primary requirement limits the dead load deflection of the beam (for sign bridge) to:

$$\Delta_{DL} = \frac{d^2}{400} \dots \dots \dots (1)$$

where  $\Delta_{DL}$  = dead load deflection (in feet), and  $d$  = vertical dimension (depth of the traffic sign[s] in feet). Although the detailed origins of the above requirement are difficult to confirm, it is believed that the criterion was established to limit or eliminate the possibility of the structure developing resonance due to the vortex shedding of the sign panels in high speed winds (Pelkey 1971; Ehsani and Bjorhovde 1986). While it appears that this deflection criterion has tended to produce very conservative designs, it is also noted that the structures have performed satisfactorily over the years.

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and no failures of such structures due to resonance have been reported.

The higher cost of construction and maintenance, along with difficulties in the dismantling and re-erection of truss type structures have contributed to the increasing usage of monotube pipe frame and monotube span-type structures. The latter has been well received in the United States and Canada, due to construction economies and attractive appearance. Due to the flexibility of monotube structures, it is very difficult to produce designs that satisfy the AASHTO deflection criterion. This has been a major cause for concern among users and designers, although it is recognized that the design criteria were originally developed for different kinds of structures.

The study reported here was undertaken to investigate the validity of the  $d^2/400$  criterion as a design limit in general, and its applicability to the design of monotube sign support structures in particular. A detailed evaluation of the former has been presented elsewhere (Ehsani and Bjorhovde 1986). Together with a full-scale field testing program (Martin et al. 1987), the work that is presented here covers the analytical aspects of the behavior of monotube structures. It is expected that the reader is familiar with the two earlier publications.

#### DESCRIPTION OF MONOTUBE STRUCTURE

The basic structure used for this study is identical to one that is owned by the Arizona Department of Transportation and located within the city of Phoenix, Arizona. The overall dimensions of the structure are shown in Fig. 1, and a detailed description is given in Ehsani et al., 1985.

The columns are approximately 21 ft long, and are linearly tapered circular single (mono) tubes, with external diameters varying from 15 in. at the base to 12 in. at the top. The beam is 100 ft long, and is spliced by means of bolted flanges approximately at the third points. The outside diameter of the beam is 18 in. at the mid-span and 11.1 in. at the ends. The wall thickness of the tubes for the columns and the beam is a constant 0.188 in.

The beam-to-column connection is shown in Fig. 2. The connection provides some moment resistance in the plane of the structure and essentially zero restraint in the out-of-plane direction. The column base plate is attached to the foundation with four 1-3/4 in. diameter anchor bolts, and therefore

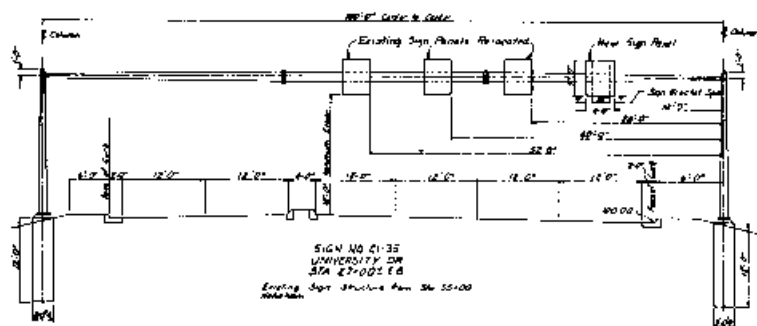


FIG. 1. Dimensions of 100 ft (Base Model) Structure

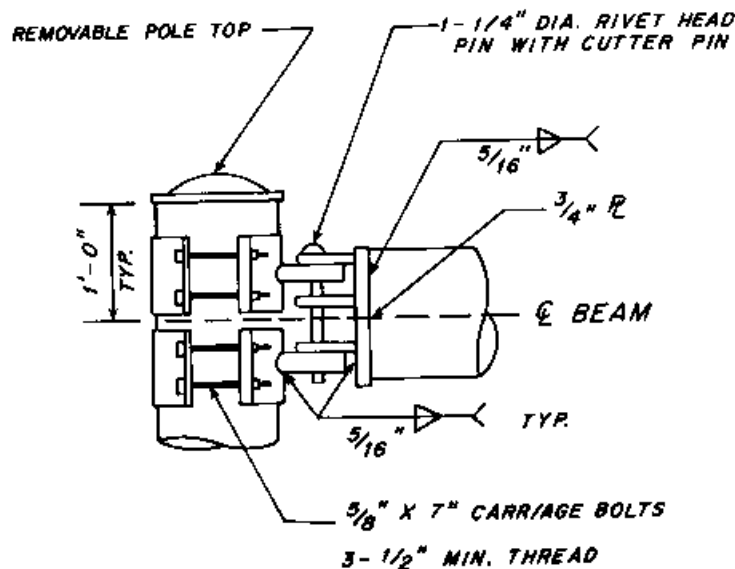


FIG. 2. Beam-to-Column Connection Detail

may be assumed to be fully fixed for all practical purposes. The sign panels are attached to the beam using 5/8 in. diameter U-bolts; these encircle the beam and are bolted into the stiffeners placed on the back of the signs.

#### FINITE ELEMENT MODEL

The structure shown in Fig. 1 was modeled for analysis using the computer program GIFTS (Graphics-Oriented Interactive Finite Element Analysis Time-Sharing System) (Kamel and Nagulpally 1983). The program consists of a group of fully compatible programs (modules). Each module is applicable for a specific function, such as stiffness matrix computation and assembly, or for a class of operations, such as load and boundary condition generation. The program can analyze the structure subjected to static or time dependent forces for different boundary conditions. The output includes nodal point stresses and displacements in three-dimensional space.

Each column was modeled as four elements, as shown in Fig. 3, and the beam consisted of twenty elements. Two additional elements were used to model the connection of the columns to the beam, using short beams of solid rectangular section. The cross sectional area and the orientation of these elements were selected to adequately represent the shear and bending characteristics of the actual connection.

Each element of the structure was assigned three translational and three rotational degrees of freedom at each node. Due to the limitations of the computer program, prismatic elements having cross-sectional properties equal to the average of those at the ends of the actual tapered segments were used. In addition, the lengths of the beam elements were selected such that nodal points were located where the sign panels were attached to the beam.

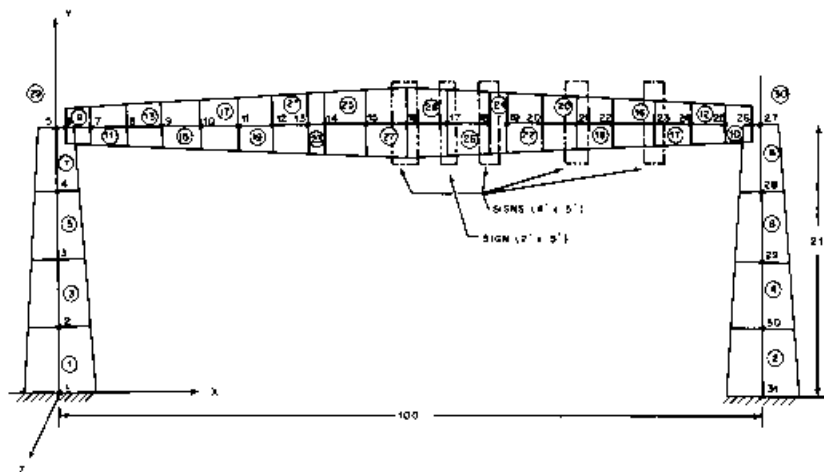


FIG. 3. Finite Element Model of Structure

#### PARAMETRIC STUDY

The behavior of monotube structures depends on such factors as the member stiffnesses, the span, and the location and area of the sign panels. In order to evaluate the effects, eight models were developed in addition to the basic one, which already has been described in detail. The selected parameters for the models are given in Table 1, and Fig. 4 illustrates the data further.

Column Models 1 and 2 are identical to the Base Model, except that the moments of inertia for the columns are 50% and 25% larger, respectively. Similarly, by increasing the moment of inertia of the beams by 50% and 25%, Beam Models 1 and 2 were developed. Because the wind forces acting

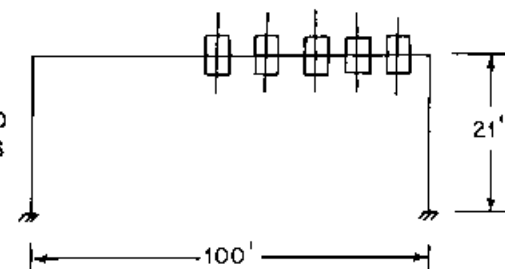
TABLE 1. Properties of Models for Parametric Study

Model (1)	Moment of Inertia		Span* (4)	Wall Thickness (in.)		Outside Diameter (in.)				Sign Location (11)
	Column (2)	Beam (3)		Column (5)	Beam (6)	Base of Column (7)	Top of Column (8)	End of Beam (9)	Midspan of Beam (10)	
Base	$I_c$	$I_b$	100	0.188	0.188	15.00	12.06	11.11	18.00	As shown in Fig. 1
Column #1	$1.5I_c$	$I_b$	100	0.287	0.188	15.00	12.06	11.11	18.00	As shown in Fig. 1
Column #2	$1.25I_c$	$I_b$	100	0.237	0.188	15.00	12.06	11.11	18.00	As shown in Fig. 1
Beam #1	$I_c$	$1.5I_b$	100	0.188	0.286	15.00	12.06	11.11	18.00	As shown in Fig. 1
Beam #2	$I_c$	$1.25I_b$	100	0.188	0.236	15.00	12.06	11.11	18.00	As shown in Fig. 1
Span #1	$I_c$	$I_b$	60	0.188	0.188	15.00	12.06	11.11	15.20	As shown in Fig. 1
Span #2	$I_c$	$I_b$	120	0.188	0.188	15.00	12.06	11.11	19.40	As shown in Fig. 1
Sign #1	$I_c$	$I_b$	100	0.188	0.188	15.00	12.06	11.11	18.00	Two 48-ft wide signs with a 1 ft gap between them at midspan
Sign #2	$I_c$	$I_b$	100	0.188	0.188	15.00	12.06	11.11	18.00	One 33-ft wide sign at midspan

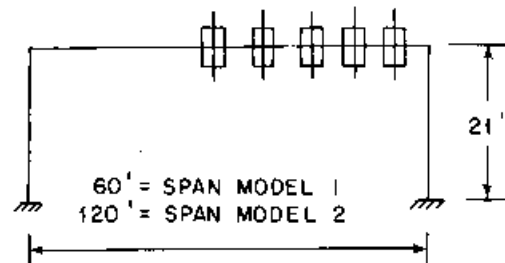
Note: 1 in. = 25.4 mm; 1 ft. = 0.305 m.

\*All structures had the same height, equal to 21 feet.

(a) BASE, BEAM AND COLUMN MODELS



(b) SPAN MODELS



(c) SIGN MODELS

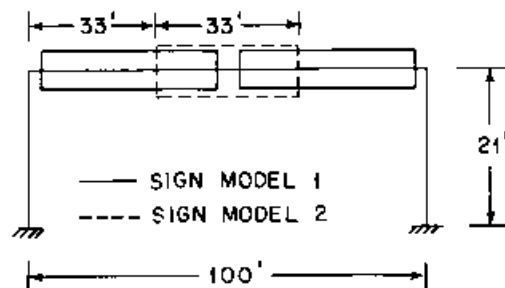


FIG. 4. Dimensions of Different Models; (for Member Sizes see Table 1)

on the structure are a function of the projected area of the elements, the increase in the moments of inertia for the column and beam models was achieved by increasing the thickness of the tube, without changing the outside diameter of the section. As discussed later, this resulted in a significant increase in the static deflection for the two beam models, as would be expected, since the dead load of sign structures is often the most significant gravity loading.

The spans of 60 and 120 ft that were chosen for Span Models 1 and 2 are representative of the range of spans commonly used for monotube structures. The columns of the span models were identical to those of the base model. However, the beams had matching outside diameters at the ends only. Keeping the same tapering ratio as the base model, the outside diameters at the midspan for Span Models 1 and 2 were calculated as 15.2 in. and 19.4 in., respectively. These dimensions are in agreement with those used for actual structures of such spans.

The location and width of the traffic signs were the parameters of Sign Models 1 and 2. These influence both the gravity as well as the wind loads and their point of application on the structure. Thus, for Sign Model 1, 5 foot deep sign panels were placed over the entire span of the structure, with a small opening near midspan. For Sign Model 2, only the middle one-third of the span was used to support the signs. It is noted that the dimensions that were used for these structures were chosen so as to produce the most serious loading conditions possible (Sign Model 1), as well as a realistic sign placement alternative with the signs concentrated in a central region of the span.

### LOADING CONDITIONS

All models were analyzed for static and wind-induced dynamic forces. The static loads are the dead load (self weight) of the structure, ice load, and the static equivalent of the wind pressure. The dynamic forces result from vortices which form alternately on opposite sides of the members of the structure; the vortices are centered in the plane of the structure as the wind acts upon it. This is the phenomenon referred to as vortex shedding (Fung 1955); it will be demonstrated that it constitutes a primary consideration in the analysis and design of highway sign structures.

The dead load of the structure consists of the self weight of the members, equally distributed between the two end nodes of each segment. The weights of the sign panels were also applied as dead loads, specifically, as concentrated loads acting at the nodes to which the signs were attached. In accordance with the specifications (AASHTO 1975), ice loads of 3 psf over the projected area of the structure and the sign panels were calculated and applied at the appropriate nodes.

Based on the applicable isotach map, the equivalent static forces for a wind speed of 70 mph were determined. This wind speed is suitable for design in the Tucson, Arizona, area. The wind load corresponds to the wind blowing in the +z direction (perpendicular to the plane of the structure) of Fig. 3. The structures were analyzed for the following static load combinations:

1. Dead load.
2. Dead load + ice load.
3. Dead load + wind load.
4. Dead load + ice load + wind load.

As explained previously, a steady wind blowing against a structural member produces vortices which alternate on the two sides of the member in a plane perpendicular to the wind direction (i.e. in-plane of the sign structure). The structures were also analyzed for these wind-induced vibrations. The frequency of the vortex shedding,  $\Omega$ , is determined from the expression (Fung 1955):

$$\Omega = \frac{SV}{D} \dots \dots \dots (2)$$

where  $D$  = diameter of the cylindrical member, and  $V$  = wind speed.  $S$  =

non-dimensional Strouhal Number which is related to the Reynolds Number,  $R$ . The Reynolds Number for air is given as:

$$R = 780.5 VD \dots \dots \dots (3)$$

in which  $V$  = wind velocity in miles per hour and  $D$  = diameter of the cylinder in inches.

The generally accepted expression for the alternating time-dependent vortex shedding force,  $F(t)$  is (Mirza et al. 1975)

$$F(t) = \frac{1}{2} \rho V^2 A_p C_L \sin \Omega t \dots \dots \dots (4)$$

in which  $\rho$  = density of dry air,  $A_p$  = projected area of the cylinder,  $C_L$  = force coefficient,  $t$  = time, and the other variables have been defined previously. In order to account for the random force amplitude, Weaver (1961) has experimentally determined and recommends the use of the root mean square value of  $C_L$ , denoted by  $\bar{C}_L$ , as a replacement for the force coefficient. According to Mirza et al. (1975), an appropriate value for  $\bar{C}_L$  is 1.0 for the circular cross section of the members in the monotube structures.

The forces calculated from Eq. 4 are valid only for certain ranges of the Reynolds Number. It is suggested that in determining the response of an elastic system with a circular cross section, such as that used in sign support structures, the forcing function is sinusoidal with a deterministic frequency and a random amplitude, but only for Reynolds Numbers between 300 and  $3 \times 10^5$  (Fung 1955). For Reynolds Numbers greater than  $3 \times 10^5$ , the forcing function is sinusoidal, but the frequency and amplitude are both of random character.

The state of the art of flow problems of this kind is such that an exact analysis of structures can be performed only in the deterministic range. For the different models examined in this study, the wind speed corresponding to the upper limit of the deterministic range was between 27 and 29 mph. Consequently, the structures were not analyzed for wind speeds exceeding this limit. It is recognized that this is a limitation; however, on the basis of current knowledge, it appears possible to remove the restriction only through scale model wind tunnel tests. It is noted that the current design criteria appear to have been based on extrapolations of the deterministic model into the random range (Ehsani and Bjorhovde 1986).

Eq. 4 was used to calculate the nodal forces for different wind speeds. It is noted that for the wind blowing along the  $z$  axis, the vortex shedding forces will be in the  $x$ - $y$  plane. Such forces will act along the  $x$ -axis at the column nodes and along the  $y$  axis at the beam nodes.

Wind drag forces were not included in the analyses, partly because it was felt that they would not be significant in the direction perpendicular to the plane of the sign structures, when compared to the static equivalent of the wind pressure. Furthermore, the circular cross section offers little drag effect. The only areas of the structures where drag might be a consideration would be where the signs are located; however, it was decided not to include these effects in the analysis due to their relatively small values compared to the other load influences. Vortex shedding vibration effects for the signs themselves were not examined. Although the sign panels will sometimes vibrate in this fashion, the effects on the tubular members are of torsional

type, and thus less likely to be significant. However, this would have to be determined through further studies.

## STATIC BEHAVIOR OF MONOTUBE STRUCTURES

### Base Model

In the following, detailed data are given for the response of the 100 ft span base model of the sign structure. These consist of the three components of displacements (in-plane vertical, in-plane horizontal and out-of-plane horizontal) as well as the static stresses that were found for the various load combinations. Further, to focus on the most important points of the structure, deflections and stresses are given for the midspan of the beam (nodal point 16), the top of the column (nodal point 27), and the base of the column (nodal point 31). For the latter only stresses are indicated.

Tables 2 and 3 give the deflections and bending stresses at the above locations of the structure for the primary load combinations. It is noted that column axial stresses were negligible in all cases, and are therefore not shown with the stress data. The largest deflection components are the in-plane beam deflection at midspan (gravity load) and the out-of-plane beam deflection at midspan (wind load). The latter is based on a wind speed of 70 mph; it is approximately twice the magnitude of the in-plane component. In terms of deflection-to-span ratios, these gravity and wind load deflections are approximately 1/180 and 1/100, respectively, for the 100 ft span base model.

An important observation is made with respect to the magnitude of the static deflection of point 16 compared to the allowable  $d^2/400$  limit, as required by the current Specification (AASHTO 1975). The signs that were attached to the structure that has been used for this study were 5 ft deep.

TABLE 2. Static Deflections for Different Load Combinations (in.)

Load combinations (1)	Node point (2)	U (3)	V (4)	W (5)
D	16	-0.065	-4.556	0.0
	27	-0.069	-0.003	0.0
D + I	16	-0.105	-6.626	0.0
	27	-0.113	-0.004	0.0
D + W	16	-0.065	-4.556	12.090
	27	-0.069	-0.003	1.928
D + I + W	16	-0.105	-6.626	12.090
	27	-0.113	-0.004	1.928

TABLE 3. Maximum Bending Stresses (ksi)

Load combination (1)	Mid-span of beam (Node 16) (2)	At base of column (Node 31) (3)
D	7.7	5.5
D + I	11.2	8.1
D + W	15.2	17.7
D + I + W	17.3	18.6

This leads to an allowable maximum deflection of 0.75 in at the midspan of the beam. This is almost one order of magnitude smaller than the actual dead load deflection of approximately 4.6 in.

The large out-of-plane deflection of the beam is primarily due to the characteristics of the beam-to-column connection, which provides very little resistance to bending in this direction. It is also noted that this deflection occurs under extreme loading combinations which include the effect of a wind of 70 mph. Since the equivalent wind loads and the resulting deflections are proportional to the square of the wind speed, for a more common speed of 20 mph, the out-of-plane deflection of the beam will be approximately 0.99 in.

The stress data in Table 3 demonstrate that the absolute maximum of the static stress occurs at the column base for the dead + ice + wind load combination, and is equal to 18.6 ksi. The same load combination gives the maximum beam stress (at midspan) as 17.3 ksi.

As far as the static behavior of the structure is concerned, these stresses provide ample margins of safety. In particular, it is observed that the steel that is commonly used in monotube structures has a yield stress of 55 ksi. Using the AISC Specification (AISC 1978), for example, this gives basic allowable stresses for circular prismatic tubular members as:

1. Gravity loads only:  $F_{all} = 0.6F_y = 33$  ksi.
2. Gravity + Wind loads:  $F_{all} = 44$  ksi.

and it is seen that the allowable values are well in excess of the actual stresses. It is therefore obvious that it is not difficult to meet the *strength* requirements of the specifications.

### Parametric Studies

As indicated previously, a total of eight additional sign structure models were devised to study different influences such as beam stiffness, etc. Details of the dimensions and other relevant factors for these structures are given in Table 1. Only the most important findings are discussed in the following; detailed data are contained in Ehsani et al. 1985.

The change in column stiffness had a negligible effect on the deflections and the stresses in the beam. However, the column base stresses were decreased in direct proportion to the increase in the column moment of inertia, as expected. For example, the load combination that produced a maximum column base stress of 18.6 ksi for the original model gives a stress of 12.6 ksi for the stiffest (1.5  $I_c$ ) column model.

For the beam models, it is emphasized that the increases in beam stiffness were achieved by using larger wall thicknesses of the tubular members. This was done to maintain the same exposed areas of the members for wind load considerations. The dead weight of the beam was consequently larger for the beam models. As a result, the in-plane deflections increased slightly. The out-of-plane deflections were reduced in proportion to the increases in the moments of inertia. The beam stresses were reduced, as expected; the column base stresses were insignificantly affected by the changes in beam stiffness. Therefore, changing the column or beam stiffnesses (either through overall dimensions or wall thickness or both) to satisfy the AASHTO deflection limit are neither efficient nor economical.

Table 1 indicates that the span models had spans of 60 and 120 ft. The smaller one exhibited very low stress and deflection levels, although its dead load deflection is 0.86 in. still did not satisfy the AASHTO criterion for structures with 5 ft deep signs. The stresses were far below the allowable values.

As expected, the in-plane and out-of-plane deflections of the long span structure increased significantly over the base model. For instance, the in-plane dead load deflection was approximately 8 in., as compared to about 4.6 in. for the 100 foot structure. Thus, the AASHTO serviceability requirement is violated by more than a factor of 10. It is important to observe, however, that the maximum stress levels are still well below the allowable values. For example, the maximum bending stress at mid-span of the beam for the dead + ice + wind load combination was 21.7 ksi; the column base stress was 19.3 ksi.

The sign models that were devised were intended to represent an extreme (signs along the entire beam span) and a more representative form of sign placements (signs covering the middle one third of the span). Only the 100 ft span structure was used for these analyses. Large deflections were found for both sign models; in the case of the extreme sign coverage, very large out-of-plane deflections were determined. In spite of these conditions, the maximum stresses were maintained below the allowable values with a 15% margin. This represents an extreme combination of loads and structural configuration; the actual stresses were significantly more than 15% below the allowable values for all of the other cases.

#### DYNAMIC BEHAVIOR OF MONOTUBE STRUCTURES

The finite element discretization that was used for the static evaluation was also utilized to examine the dynamic behavior of the structures. Fig. 3 shows the finite element model. The mass of the structure is lumped at the nodal points; in addition, the masses of the traffic signs are lumped at the nodes where the signs are attached. The masses are assumed to have only translational degrees of freedom.

In the analysis it was assumed that the structure had no damping. This is conservative insofar as the dynamic response is concerned, especially when the question of structural resonance is addressed.

#### Free Vibration Analysis

The natural vibration characteristics of a structure are particularly important when it is being subjected to dynamic loading. The frequencies of the loads and those of the bare structure may be such that the actual response is a magnification of the natural vibrations. In the most unfavorable case there is agreement between the loading frequency and that of one or more of the natural vibration modes, which constitutes resonance.

In theory it is possible to attain resonance, and there are a few recorded instances of actual structural failures where resonance at least played a certain part. The celebrated failure of the Tacoma-Narrows Bridge is one such example; another case is that of the wind-induced vibrations known as flutter, that can be encountered in some flight structures or others where the mass-to-stiffness ratio is very low.

As will be shown, complete two- and three-dimensional vibration analyses have been carried out for the structure. With a total of 31 nodal points in the frame, the resulting number of degrees of freedom will be very large. However, Clough and Penzien (1975) and others have observed that for most civil engineering structures, only the first few modes of vibration are practically significant. For this reason it was decided to determine the properties only of the first 10 natural modes of vibration of the monotube structures for each of the 3D and 2D analysis schemes.

The natural frequencies for the base model (100 ft span) ranged from 0.78 cps for the first two-dimensional (2D) mode of vibration to 44.30 cps for the tenth 2D mode. For the three-dimensional response, the corresponding frequencies were 0.47 and 10.15 cps, respectively. Certain 3D-modes were dominated by in-plane (2D) responses, and the frequencies of 2D-modes 1 through 5 were identical to the second, third, fifth, eighth and tenth 3D-mode frequencies. Thus, the first, fourth, sixth, seventh and ninth 3D-modes were dominated by out-of-plane behavior.

These findings indicate in part why it was eventually decided to propose design recommendations based on independent, individual analyses of the structure in the in-plane and the out-of-plane directions. Due to the static gravity loads that act on the structure, along with the in-plane vortex shedding, which is presented in the following section, the in-plane loading conditions tend to dictate the strength of the overall structure. On the other hand, some of the 3D modes do cause in-plane vibrations that correspond to natural frequencies of the structure, which could lead to in-plane resonance for certain narrow and sustained ranges of wind speed. This is discussed in more detail in the following. However, a static out-of-plane evaluation of the beam member, subjected to wind loads, is also necessary. It has been shown (Ehsani et al. 1985) that this can be accomplished by a simple, independent check of the bending stress and the deflection produced by the static equivalent of the wind load.

#### Forced Vibration Analysis

The dynamic response of the monotube structures has been determined for the full range of deterministic wind speeds, as explained previously. In addition to obtaining the vibration data for wind speeds where the natural and vortex shedding frequencies match, complete data have been developed for a large number of velocities. Some of the results that are presented in the following are but an example of what has been done, such as the displacement-time relationships for certain points in the structure. Similar data have been developed for all velocities, but only the essence of the results are presented. That is, the relationship between the maximum in-plane displacement due to vortex shedding and the corresponding wind speed is the most useful output as far as design evaluations are concerned.

As will be seen in the presentation of the results, a choice had to be made for the length of time that the wind would be blowing at a constant speed. A duration of 32 seconds was arbitrarily chosen as a large multiple of the longest natural period of the structure. This period is 2.13 seconds, and pertains to the first 3D mode. It is noted that a constant wind speed duration of 32 seconds is very long, since the wind tends to gust, and therefore only attains specific velocities for short periods of time. However, the long du-

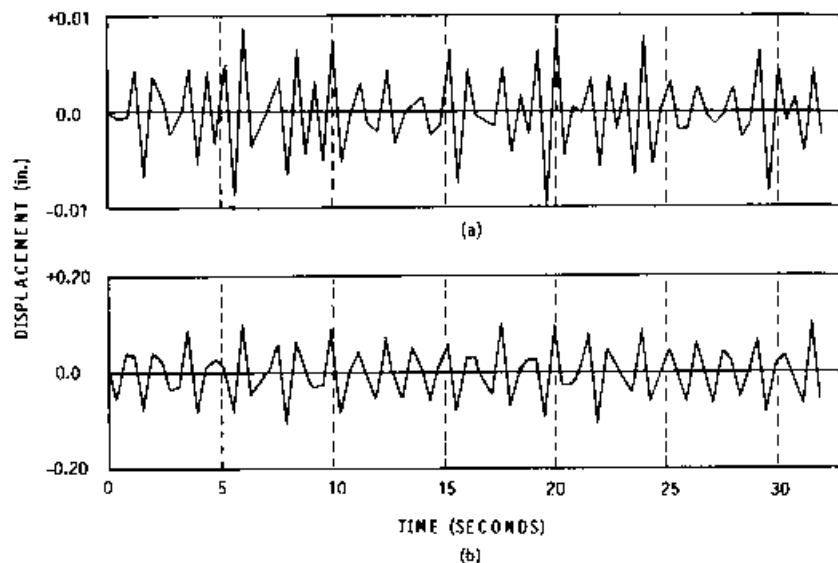


FIG. 5. Displacement Histories for (a) Nodal Point 27 (Top of Column) and (b) Nodal Point 16 (Midspan of Beam)

ration is a conservative choice, especially when evaluating the resonance condition.

As an example, Fig. 5 shows the displacement histories for two points on the structure, using a wind speed of 15 mph. The points are Nodal Point 16 = Midspan of beam, and Nodal Point 27 = Top of column. The displacement history for point 16 reflects vertical (in-plane) deflection of this point in relation to time, and the displacement history for point 27 exhibits horizontal (in-plane) movement. The maximum vertical deflection (at point 16) equals approximately 0.1 in.; it occurs first after about 6 seconds of load duration, and reoccurs roughly every 2 seconds. The response is stable, i.e. the maximum deflection does not increase with time.

As expected, the maximum horizontal deflection (at point 27) is one order of magnitude smaller than the vertical one. It reaches a value of 0.01 in. after approximately 6 seconds, and then occurs under stable conditions every 10 to 15 seconds thereafter. It is clear that the structural significance of the horizontal in-plane deflections is limited; the only aspect of the behavior that might be affected by this may be the fatigue behavior of some of the structural details (beam-to-column connection, column base).

The maximum vertical deflection to the dynamic load must be considered in relation to the value of the maximum static one at the same point. The static displacements are repeated here for comparison:

1. Dead load:  $\Delta_{SD} = 4.56$  in.
2. (Dead + ice) load:  $\Delta_{SG} = 6.63$  in.
3. Dynamic (vortex-shedding) load:  $\Delta_{dm} = 0.1$  in.

The increase of the deflection due to the dynamic effect is therefore less

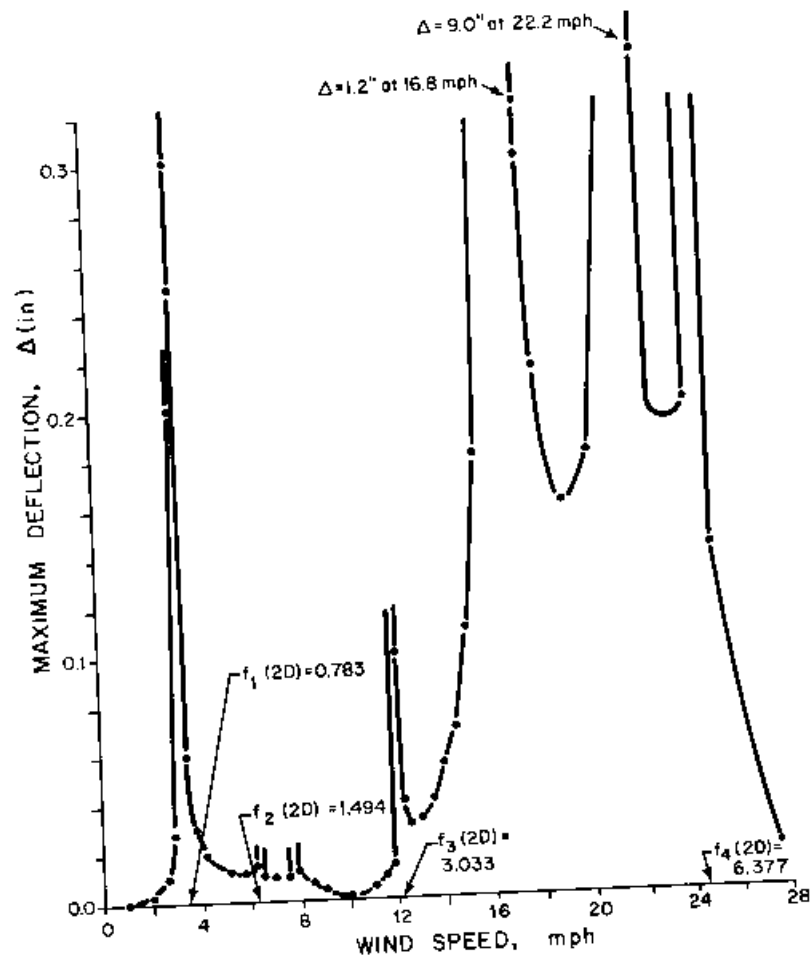


FIG. 6. Maximum Vertical Dynamic Deflections at Midspan of Beam of Base Model for Different Wind Speeds

than 5% for both dead and (dead + ice) loads. Since the allowable stresses (AISC 1978) are increased by 1/3 for the load cases that incorporate wind, it is obvious that gravity load will govern the in-plane design. This is further demonstrated by the small dynamic stress increases of 0.17 ksi and 0.22 ksi at the column base and beam midspan, respectively.

The statically equivalent out-of-plane deflection of the beam is proportional to the square of the wind speed. Using the deflection of 12.09 in. that was computed for the code wind of 70 mph, the beam deflection at a wind speed of 15 mph is 0.56 in. For this wind speed, therefore, the gravity load combination will govern.

Fig. 6 shows the relationship between the maximum dynamic vertical deflection (at midspan of beam) and the wind speed, for the complete deterministic velocity range. Peaks are reached for velocities that correspond to

either 2D (1st through 4th) or 3D (2nd, 3rd, 5th, and 8th) natural frequencies. Large deflections also appear to take place for wind speeds around 7.7, 16.2 to 17.0 and 21.0 to 23.0 mph. These peaks correspond to the 4th, 6th and 7th 3D natural frequencies of the structure, for which the out-of-plane motion of the columns cause in-plane vertical displacement of the beam ends. This sets up the in-plane resonance tendency for the structure.

Very few data points are available for wind speeds larger than about 20 mph. These parts of the curves that are shown in Fig. 6 are therefore estimates, beyond the specific data that are given.

However, it is emphasized here that the ranges of velocities for which resonance appears to be taking place are very narrow. Furthermore, the results in Fig. 6 have been based on a sustained wind duration of 32 seconds, which is unlikely to occur. It is also recalled that it has been assumed that the structure possesses no damping capability. In consequence, although large deflections are indicated for certain wind speeds, their practical impact is questionable.

Analysis of the other structural models for dynamic wind loads showed responses that were similar to that of the base model that has been described above. It was found that there were displacement maxima corresponding to the first 2D mode frequency of each model. Each model exhibited large displacements at wind velocities which corresponded to specific 2D or 3D natural frequencies of the structure.

#### DESIGN IMPLICATIONS OF RESULTS

In order to make use of the data on the relationship between displacement and time, an example of which is given in Fig. 6, it is necessary to define an allowable maximum displacement due to vortex shedding at the midspan of the beam. Two factors were taken into account in determining this deflection. First, it has been shown previously that the static stresses in these structures are low and well within the elastic range. Secondly, when wind forces are considered, some specifications permit a 1/3 increase in the allowable stresses (AISC 1978). Based on these factors, it was decided to define the maximum allowable dynamic deflection as the deflection which occurs when the maximum stress equals the allowable one. Therefore, a dynamic deflection equal to one third of the static vertical deflection at midspan was selected as the allowable dynamic value. For wind speeds causing displacements smaller than this, the combined gravity and wind stresses will not govern, having taken advantage of the 1/3 increase in the allowable stresses.

Based on the above criterion, the maximum allowable dynamic deflection for each model was calculated, and the results are presented in Table 4 for the base model and the first column, beam and span models. The ranges of wind speeds that are given were determined from displacement histories of the respective models. It is important to note that the ranges are very small; the largest is 1 mph.

From the data in Table 4, which summarize the major results for typical structures, it is felt that resonance is a phenomenon that will not occur under actual service conditions. This is due to the fact that in the analyses, damping was set equal to zero. Secondly, the wind has to be maintained for a duration of more than 30 seconds, and finally, the wind speed must be kept

TABLE 4. Static and Dynamic (Wind Vortex Shedding) Response Data for Midspan

Model (1)	Max. Static Deflection (in.)		Resonance wind speed (mph) (4)	H at resonance wind speed (in.) (5)	1/3 $\Delta_{30}$ (in.) (6)	Range of wind speed (mph) corresponding to 1/3 $\Delta_{30}$ (7)	4/3 $\Delta_{30}$ (in.) (8)
	$\Delta_N$ (2)	$\Delta_{30}$ (3)					
Base Column	12.09	6.63	22.0	1.19	2.21	21.8-22.5	8.84
Model #1 (1.5 l col.)	11.56	6.20	22.25	1.17	2.07	22.0-22.5	8.27
Beam							
Model #1 (1.5 l beam)	8.69	6.19	22.5	0.9	2.06	22.2-22.8	8.25
Span	3.5	1.26	13.1	0.12	0.42	12.6-13.6	1.68
Model #1 (60' span)			24.1	0.41		23.6-24.6	

within very narrow ranges for the 30 second duration in order to produce resonance.

As far as the static deflections of the structures are concerned, moderate increases in the stiffness of the columns or the beam do not result in a significant reduction of the deflections. Changing the span of the structure, however, does affect the midspan deflections considerably, both for the in-plane as well as out-of-plane cases. The larger the area of the signs and the closer the signs are placed to the midspan, the larger the deflections. It must be noted that the sign location has considerable effect on the deflections due to ice loads, as the signs will accumulate ice and further increase the midspan deflection.

The static bending stresses were also affected by the changes in the parameters. Increasing the column stiffness resulted in a reduction of the stresses in the column, while increasing the beam stiffness reduced the stresses in the beam. Again, significant changes in the stresses were noticed as a result of changing the span of the structure. Similarly, the location of the signs had a major role in the calculated stresses in the beam and the column. As discussed in the parametric study section of this paper, the stresses in structures supporting large areas of traffic signs can be high and must be investigated carefully. It is a question whether monotube structures are suitable for large groups of signs.

The parameters did not appear to have any influence on the dynamic displacements at the midspan of the beam, as long as resonance was not occurring. The structures exhibited large displacements at higher wind speeds due to resonance-like conditions. However, the absolute magnitudes of these displacements for different models is meaningless, since in theory, undamped resonance leads to infinite deflections. Thus, near the indicated resonance wind speeds, a slight change in wind velocity could lead to a significantly larger deflection.

The change in the parameters did not have any appreciable effect on the dynamic stresses. These stresses were found to be less than 1 ksi, and therefore negligible for all practical purposes. As far as the natural frequencies of the structures are concerned, they were found to be insensitive to the

changes in the column and beam stiffness.

On the basis of the preceding evaluations, it is shown that a static analysis of the monotube structure will be sufficient to ensure that deflections and levels of stress are satisfactory. The  $d^2/400$  requirement, which stems from dynamic analyses of other sign structures, can neither be satisfied nor is it a rational design criterion for monotube structures. The gravity load deflections do not produce excessive levels of stress in the members; if a user feels that a certain dead load deflection level should not be exceeded, this can be satisfied by specifying a certain camber, or a beam splice detail that produces the requisite camber by a suitable splice plate angle.

The structural evaluations have also demonstrated that the in-plane (due to gravity loads) and out-of-plane (due to wind load) deflections may be determined separately. That is, gravity load displacements due to dead load alone and due to dead + ice load can be computed for the in-plane response of the monotube structure. The out-of-plane behavior of the beam can then be found by analyzing this member as simply supported at the ends, due to the very small out-of-plane bending stiffness of the connection. However, simple support conditions are only realistic for the type of connection that has been used for the monotube structures evaluated here.

## CONCLUSIONS

The following conclusions represent some of the major findings of this study of monotube span-type sign structures:

1. The static response of the structure can be determined separately for in-plane and out-of-plane loads. That is, the deflections and stresses due to gravity loads are not influenced by those due to wind loads.
2. The monotube structures are not able to satisfy the current AASHTO dead load deflection requirement. The actual in-plane deflections are significantly larger than those indicated by  $d^2/400$ .
3. The stress levels associated with the actual deflections are well below the magnitudes of the allowable stresses, even though the maximum deflection is well above the  $d^2/400$ -level.
4. The out-of-plane deflections associated with the maximum wind speed, which was set equal to 70 mph for this study vary from 3.5 to 12.1 in. for monotube structures with spans from 60 to 100 ft. The largest deflection-to-span ratio (horizontal) was approximately 1/100.
5. The combined effects of the in-plane and out-of-plane bending did not produce stresses larger than the allowable value for any load combination and any structure.
6. The design of monotube structures is governed by stiffness rather than strength criteria. Allowable stresses were never exceeded, even though large deflections were recorded.
7. The study investigated the influence of varying beam and column stiffness by 25% and 50% over the base model. The beneficial effects of this were limited, and the cost of same appears to rule out such a device if reduced deflections are sought.
8. The dynamic deflection increments that were produced by the vortex shedding were one order of magnitude (or more) smaller than the static values for almost all wind speeds.

9. Resonance-like conditions were found to occur for frequencies of vortex shedding that matched either 2D or 3D natural modes of vibration of the structures. However, this was found to occur only when the wind speed was maintained at a constant value for a longer period of time. In addition, the resonance-producing wind speeds were all within very narrow ranges: the largest one was 1 mph.

10. It is believed that the resonance condition is not serious, for several reasons, as follows:

- (a) Wind speed must be maintained within a narrow range, and for a prolonged period, in order for resonance to occur.
- (b) The structural analysis was performed on the assumption that the monotube structure does not possess any damping capability.
- (c) The apparent strength of the material increases with the rate of loading.

11. Further studies should be conducted on the question of the ranges of wind speeds that may cause resonance, as well as the influence of structural damping. Finally, the response under high wind conditions needs to be examined in wind tunnel tests.

## ACKNOWLEDGMENTS

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## APPENDIX II. NOTATION

The following symbols are used in this paper:

- $A_p$  = projected area;
- $C_L$  = force coefficient;
- $C_L$  = root-mean-square value of  $C_L$ ;
- $D$  = diameter of a tube;
- $F(t)$  = time-dependent force;
- $R$  = Reynolds Number;
- $S$  = Strouhal Number;
- $V$  = wind velocity;
- $d$  = vertical dimension (depth) of traffic sign;
- $t$  = time;
- $\Delta_{DL}$  = dead load deflection;
- $\Delta_H$  = horizontal deflection at midspan of beam ( $x$ -direction);
- $\Delta_{SD}$  = static dead load deflection;
- $\Delta_{SG}$  = static deflection due to (ice + dead) load;
- $\Delta_{dyn}$  = dynamic deflection component;
- $\rho$  = density of air; and
- $\Omega$  = frequency of vortex shedding.

## OUT-OF-PLANE BUCKLING FORMULAS FOR BEAM-COLUMNS/TIE-BEAMS

By Sritawat Kitipornchai<sup>1</sup> and Chien-Ming Wang<sup>2</sup>

**ABSTRACT:** This paper shows that the closed form expression for the elastic out-of-plane buckling of doubly symmetric beam-columns under uniform moment can be expressed in form similar to that of beams by introducing a beam-column parameter,  $\eta = 4r_o^2/h^2$  in which  $r_o$  is the radius of gyration about the centroid and  $h$  is the distance between flange centroids. The closed form solution is equally applicable to tie-beams. For beam-columns/tie-beams under other loading conditions, accurate solutions are determined using the Rayleigh-Ritz energy approach. Approximate formulas to predict the out-of-plane buckling capacities of the beam-columns/tie-beams under moment gradient, uniformly distributed load, and central concentrated load are presented. The proposed moment modification factor formulas furnish reasonably accurate predictions of the buckling capacities of beam-columns/tie beams and may thus be adopted by code rules.

### INTRODUCTION

Relatively few studies have been made on the out-of-plane buckling behavior of thin-walled members subjected to combined bending and axial forces although they commonly occur in practice; for example, a column in a portal frame under side sway wind loading (Woolcock and Kitipornchai 1987). The present design specifications (AISC Specification 1987; Bradford et al. 1987; Trahair 1977) for out-of-plane buckling of beam-columns are based on the closed form solution of a beam-column under equal and opposite end moments (Fig. 1a), modified by the so-called "moment modification factor" to allow for the nonuniform in-plane moment distribution. The adopted modification factors were originally derived for beams under unequal end moments and are therefore highly conservative, as the influences of axial forces have been completely ignored. Cuk and Trahair (1981) rectified this situation by developing an empirical formula containing the axial force factor, based on a curve fitting exercise of some finite element results. The formula is intended for end moments and axial compression loadings only. Apparently, for other loading conditions (see Fig. 1b) and axial force, be it compressive or tensile in nature, no out-of-plane buckling formulas are presently available. Interestingly, quantitative effects of the tensile forces on the buckling capacities have not been fully explored, even though it is clear that the tensile forces enhance the buckling capacities of the member. This lack of information has led designers to neglect the influence of the tensile forces, thereby obtaining an unnecessarily overconservative design.

This paper briefly reviews existing moment modification factors for beam-columns and shows that the elastic out-of-plane buckling formula for doubly symmetric beam-columns/tie-beams under uniform moment can be ex-

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