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Disjunctive Kriging
2. Examples

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The disjunctive kriging (DK) method described in detail in the previous paper of this series is illustrated by two examples. The first example is for the electrical conductivity (EC) which is approximately lognormally distributed. In the second example the natural logarithm of the EC is used to produce an approximately normally distributed data set. The data set was chosen since the Hermite coefficients for a normal and lognormal distribution could be determined analytically as well as to facilitate a comparison between ordinary and disjunctive kriging. The capability of DK to be a nonlinear estimator and an estimator of the conditional probability is explored. The results indicate that DK is a better estimator than linear estimators (i.e., ordinary kriging) in the sense of reduced variance of errors and average kriging variance even when the data are approximately normally or lognormally distributed. Also, using DK one can obtain an estimate of the conditional probability.

INTRODUCTION

In the first paper of this series [*Yates et al.*, 1986] the theory of disjunctive kriging (DK) was reviewed as it pertains to estimating the value of a random function at an unsampled location and estimating the conditional probability that the unknown value of a property at an unsampled location is above a specified cutoff level. In this second paper, we illustrate the use of the DK method with examples.

The first example uses the electrical conductivity data of *Al-Sanabani* [1982] which was sampled at 101 random locations in a 10 ha field. The second example uses the same data set after a natural logarithmic transformation is applied to the data. This gives an example of DK for data sets with approximately normal and lognormal distributions and also allows comparison of the final results between the "two" data sets. Next, a comparison between the DK and ordinary kriging (OK) methods will be given.

EXAMPLE 1: APPROXIMATELY LOGNORMAL DATA

The first example uses data which is approximately lognormal. *Al-Sanabani* [1982] took 101 randomly located electrical conductivity samples in a Typic Haplargid in southwest Arizona. The 1 by 1 m grid system was oriented in a north-south (x direction) and east-west (y direction) direction.

The 1–2 kg air-dried samples collected in the field were placed in a beaker with distilled water and a saturated paste was made. The solution was extracted from the paste and the EC measured. A more detailed description is given by *Al-Sanabani* [1982].

The hypothesis that the data were normally or lognormally distributed was tested using the Kolomogorov-Smirnov (KS) test for goodness-of-fit [*Sokal and Rohlf*, 1981; *Rao et al.*, 1979]. The KS test statistic was calculated and compared to the critical value at the 0.1 probability level. This level was chosen in order to reduce the probability of the type II error, namely the probability of incorrectly selecting the null hypothesis [*Rao et al.*, 1979]. For a sample size greater than 30 the critical value at the $\alpha = 0.1$ probability level can be obtained from the asymptotic expansion [*Rohlf and Sokal*, 1981; *Rao et al.*, 1979]:

$$\mathrm{KScrit} = 0.805/(\sqrt{n}) \tag{1}$$

The results of the KS test indicate that the original and log-transformed data are from lognormal and normal distributions respectively, at the 0.1 probability level. The KS test statistic calculated from the data sets were 0.197 and 0.065 for the original (EC) and transformed (ln EC) data, respectively. The associated critical value is 0.080 and was calculated by (1) for $n = 101$. Figure 1 shows the empirical cumulative frequency distribution for the original and transformed data which are marked on the figure and plotted as points. The normal distribution is plotted as a solid curve. For Figure 1, the abscissa is in terms of the standard normal variable $Y = [x - \mu]/\sigma$ for the normal distribution and $Y = [\ln(x) - \mu_{\ln}]/\sigma_{\ln}$ for the lognormal distribution. From the figure it is evident that there is an improvement in the fit between the theoretical gaussian distribution and the transformed data.

The first step in the DK process is to find the coefficients which define the transform function.

$$Z(x) = \phi[Y(x)] \approx \sum_{k=0}^{K} C_k H_k[Y(x)] \tag{2}$$

For comparison, the C_k's for a lognormal distribution were calculated using the results of Table 2 (paper 1). Also, the actual C_k's appropriate for the data sets were determined by numerical integration using (10) (paper 1). In order to evaluate the integral the transform, $\phi[Y(x)]$, must be known or approximated. The method used here for determining the transform was to order the original data set from smallest to largest in magnitude and then calculate for each $Z(x_i)$ a corresponding $Y(x_i)$ by inverting the probability function

$$Y(x_i) = P^{-1}[(i - 0.5)/n] \tag{3}$$

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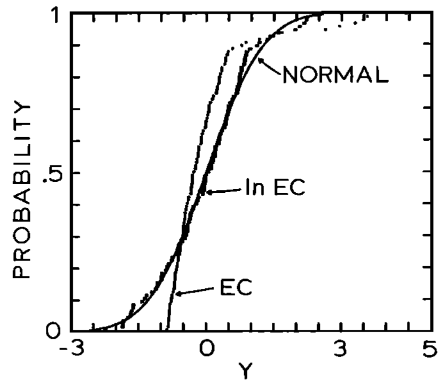

Fig. 1. Comparison of the distributions for the original and transformed data (points), and the gaussian distribution function (solid curve).

where i is the number of data that have a value less than or equal to $Z(x_i)$, n is the total number of data, P is the probability function, and the argument $(i - 0.5)/n$ is an approximation for the probability of datum $Z(x_i)$. A series approximation for $P^{-1}(x)$ is given by *Abramowitz and Stegun* [1965, p. 933]. After $Y(x_i)$ is calculated for all $Z(x_i)$, intermediate values for the relationship $Z(x) = \phi[Y(x)]$ can be determined by either linear interpolation, fitting an nth order polynomial or some equivalent method. This is necessary because the abscissa values for the Hermite integration will not necessarily correspond to a $Y(x_i)$ contained in the data set. Both methods were used here to determine the abscissa values. In general it was found that fitting a 10th order polynomial to the $[Z(x_i), Y(x_i)]$ pairs gave a better representation of the data set compared to linear interpolation and produced better estimates of the mean and variance of the data (see equations (11) and (12) of paper 1). The results of this calculation are given in Table 1.

The plot of EC versus the probability shown in Figure 2 demonstrates the fit between the function $\phi(Y)$ (solid curve) and the original data (points) for several cases. In Figure 2*a* the C_k's corresponding to the theoretical lognormal distribution (based on the mean and variance of the data set) were used to generate $\phi(Y)$ from (2). In Figures 2*b* to 2*d* the C_k's, calculated from the actual data using the polynomial fit, were used to generate the $\phi(Y)$ function, with $K = 5$, 10, and 15 (i.e., C_k's), respectively, as is shown in Figures 2*b*, 2*c*, and 2*d*. These results indicate that the theoretical lognormal distribution fits the data very well in the small EC range (i.e., 0.5 < EC (dS/m) < 8) but there is poorer agreement for an EC above 8 dS/m. When the C_k's are calculated from the data there is an overall improvement in the fit between the model and the data as K increases. Since increases in K are associated with increases in computational time the 10 term representation is deemed adequate.

Since the C_k's calculated above are only approximate (2) must be inverted to find the actual $Y(x_i)$ which corresponds to $Z(x_i)$, given the approximate C_k's, before implementing the DK method. This is necessary to assure that DK will exactly interpolate at sample locations.

CORRELATION FUNCTION

As with all kriging methods some function is necessary which contains the information about spatial correlation structure. For DK the appropriate function is the correlogram, which exists for second-order random functions.

To facilitate comparison with ordinary kriging the correlogram used in subsequent analysis was determined by

$$\rho(h) = 1 - \gamma(h)/C(0) \tag{4}$$

and the variogram, $\gamma(h)$, shown in Figure 3, was fitted to the sample variogram and modeled using (5).

$$\begin{aligned} \gamma(h) &= 0 \qquad h = 0 \\ \gamma(h) &= 20 + 25[1.5h/90 - 0.5(h/90)^3] \qquad 0 < h < 90 \\ \gamma(h) &= 45 = C(0) \qquad h \geq 90 \end{aligned} \tag{5}$$

This variogram model was validated by the jackknifing technique [see *Vauclin et al.*, 1983; *Russo*, 1984] using ordinary

TABLE 1. Actual and Theoretical C_k When Linear Interpolation or a Tenth-Order Polynomial is Used to Interpolate $\phi[Y(x_i)]$

	Lognormal Distribution (Original Data)			Normal Distribution (Transformed Data)		
k	Linearly Interpolated C_k	Tenth Polynomial C_k	Lognormal C_k	Linearly Interpolated C_k	Tenth Polynomial C_k	Normal C_k
0	5.8818	5.8447	5.795	1.3472	1.3996	1.4004
1	5.0164	4.9464	4.893	0.6992	0.8385	0.8487
2	1.9070	1.9881	2.066	−0.1348	0.0177	0
3	0.1541	0.3572	0.581	−0.1038	−0.0125	0
4	−0.2414	−0.0766	0.123	−0.0189	0.0043	0
5	−0.1296	−0.0770	0.0206	3.74*E*-4	−0.0039	0
6	−0.0155	−0.0160	0.0029	0.0035	−0.0014	0
7	0.0110	0.0042	3.34*E*-4	0.0014	2.88*E*-4	0
8	0.0046	0.0021	3.53*E*-5	−1.16*E*-4	1.18*E*-4	0
9	−4.58*E*-4	−1.28*E*-4	2.44*E*-6	−1.19*E*-4	−2.47*E*-5	0
Mean	5.882	5.845	5.795	1.347	1.400	1.400
Variance	37.571	34.453	34.926	0.623	0.710	0.720
Actual mean			5.846			1.400
Actual variance			35.123			0.720

For the lognormal case the mean and standard deviation used to calculate the C_k's was 1.40 and 0.849, respectively.

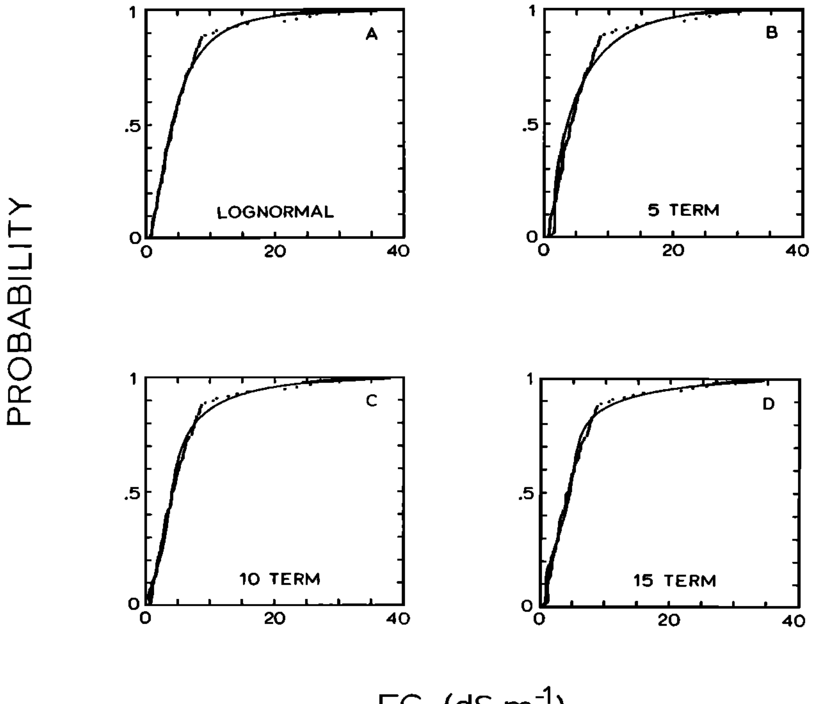

Fig. 2. Illustration of the fit of the transform, ϕ, for the EC data. In Figure 2*a*, the C_k's correspond to the lognormal distribution. Five, 10, and 15 C_k's calculated from the data were used in Figures 2*b*, 2*c*, and 2*d*, respectively.

kriging. An acceptable model variogram is given when the following holds:

$$R_\mu = \sum_{i=1}^{n} [Z(x_i) - Z^*(x_i)]/(\sigma_k n) \approx 0$$

and (6)

$$R_{\sigma^2} = \sum_{i=1}^{n} [(Z(x_i) - Z^*(x_i))/\sigma_k]^2/n \approx 1$$

where $Z^*(x_i)$ is the estimated value at x_i and the known value [i.e., $Z(x_i)$] is not used in the estimation process. For the model given in (5), $R_\mu = 0.011$ and $R_{\sigma^2} = 1.01$. One problem in using (6) is the lack of a means of determining how close to zero and unity R_μ and R_{σ^2}, respectively, should be.

Example 2: Approximately Normal Data

The data set used in this example was obtained by taking the natural logarithm of the EC data described in example 1. The estimates of the mean ($\mu_{\ln}$) and standard deviation ($\sigma_{\ln}$) using the log-transformed data are 1.40 and 0.849, respectively. The results of Figure 1, as well as the KS test, support the hypothesis that the log-transformed data fit a normal distribution.

The theoretical C_k's for a gaussian distribution (i.e., approximated by using the transformed data) are $\mu_{\ln} = C_0$ and $\sigma_{\ln} = C_1$ with C_k for $k > 2$ all zero, since for a normal distribution the standard normal variate is

$$Y(x_i) = [Z(x_i) - \mu_{\ln}]/\sigma_{\ln} \qquad (7)$$

The C_k's were calculated by numerical integration of (10) (paper 1) for two cases described above (i.e., when the intermediate values for $Z(x) = \phi[Y(x)]$ are determined by linear interpolation and by a 10th order polynomial). The resulting C_k's are given in Table 1. As in example 1, the mean and variance are better estimated using the 10th order polynomial. Figure 4 gives the natural logarithm of the EC versus the probability for two models. Figure 4*a* compares the representation when the theoretical normal distribution is used and Figure 4*b* when 10 C_k's are used along with the polynomial fit between the data pairs.

The sample variogram was calculated using the log-transformed data, fitted by a spherical model and validated by the jackknifing procedure. The resulting variogram, shown in Figure 5, is

$$\gamma(h) = 0 \qquad h = 0$$

$$\gamma(h) = 0.3 + 0.6[1.5h/160 - 0.5(h/160)^3] \qquad 0 < h < 160 \qquad (8)$$

$$\gamma(h) = 0.9 = C(0) \qquad h \geq 160$$

For the model given in (8), $R_\mu = 0.020$ and $R_{\sigma^2} = 1.19$.

Ordinary Kriging

To facilitate evaluation of nonlinear DK over linear kriging methods, the ordinary kriging (OK) method was also applied to the data sets. For a full discussion of this method the reader

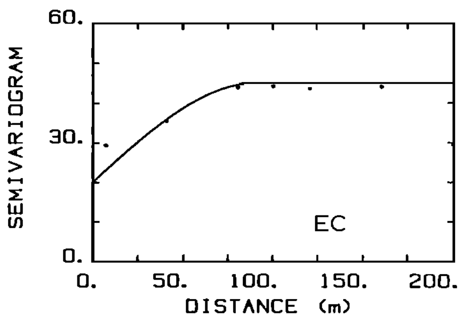

Fig. 3. Sample (points) and model (curve) variogram for the EC.

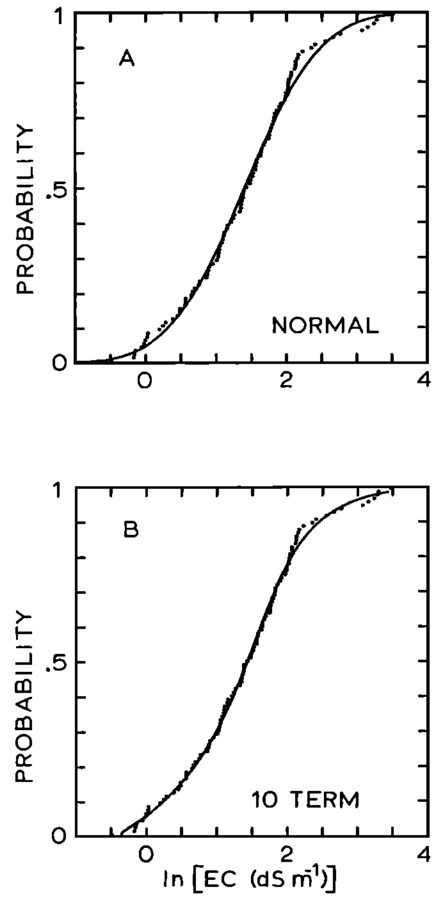

Fig. 4. Illustration of the fit of the transform, ϕ, for the ln (EC) data. In Figure 4*a*, the C_k's correspond to the normal distribution. In Figure 4*b*, the C_k's were calculated from the data.

is referred to *Journel and Huijbregts* [1978], *Burgess and Webster* [1980*a*, *b*], *Rendu* [1978], *Clark* [1979], or other sources.

RESULTS

The first check on the DK method is to verify that the method improves the estimation process compared to linear kriging methods (i.e., ordinary kriging). *Journel and Huijbregts* [1978] and *Kim et al.* [1977] list the hierarchy of estimators (beginning with the best which requires the most restrictive assumptions) as conditional expectation, disjunctive kriging, simple, and ordinary kriging.

Therefore, to show that DK improves the estimation over OK a total of 240 estimates of EC and ln(EC) (using each method) were obtained on a 24 by 24 m grid system superimposed over the field sampled by *Al-Sanabani* [1982]. Along with each estimate a value of the kriging variance was calculated and for DK method a value of the conditional probability. *Bower and Wilcox* [1965] gives 4.0 dS/m as a critical value of EC for many plants therefore this value (or the ln(4.0 dS/m)) was used for z_c in subsequent analyses. For all cases a maximum of 5 nearest neighbors within a radius equal to the range of the variogram were used in the estimation process. Next, the results were contoured to show the spatial distribution of each parameter followed by a calculation of the average kriging variance based on the sampling location was obtained (the known value was not used in the estimation). From the estimate, $Z^*(x_i)$, and actual value, $Z(x_i)$, the sum of squares was calculated from

$$SSQ = \sum_{i=1}^{n} [Z(x_i) - Z^*(x_i)]^2/n \tag{9}$$

This enables one to arrive at an absolute measure of the improvement in the estimation when DK is used over OK.

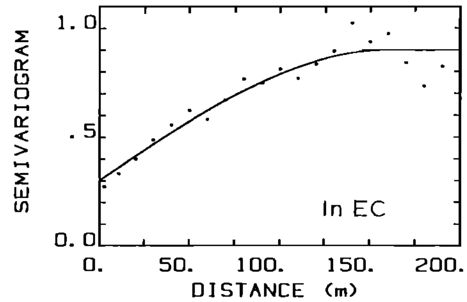

Fig. 5. Sample (points) and model (curve) variogram for the ln (EC).

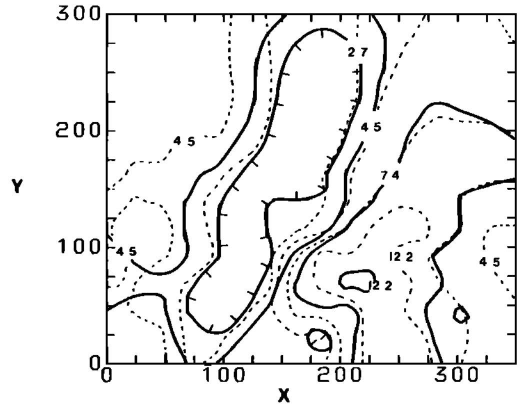

Fig. 6. Contour diagram of the EC. Solid and dashed curves indicate the results from block DK and OK, respectively.

Estimation

Figure 6 is a contour diagram of the 240 estimates of EC generated by disjunctive (solid curve) and ordinary (dotted curve) block kriging using a block size of 24 by 24 m. The block was divided into 2.4 by 2.4 m subblocks which were used for a discrete approximation for the average variogram [*Journel and Huijbregts*, 1979]. Contour maps were also generated by using disjunctive and ordinary punctual kriging which produced similar results. The distribution of EC in this field shows two prominent features, a low EC trough which

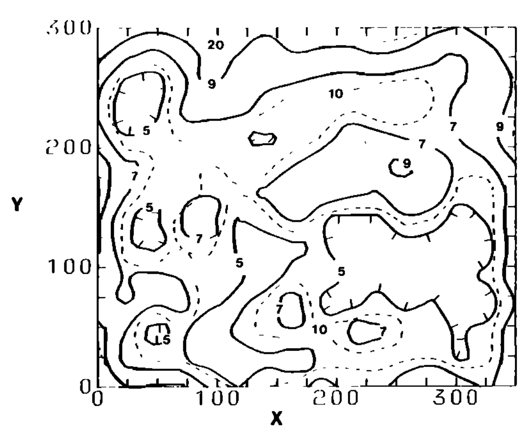

Fig. 7. Contour diagram of the kriging variance for the EC. Solid and dashed curves indicate the results from block DK and OK, respectively.

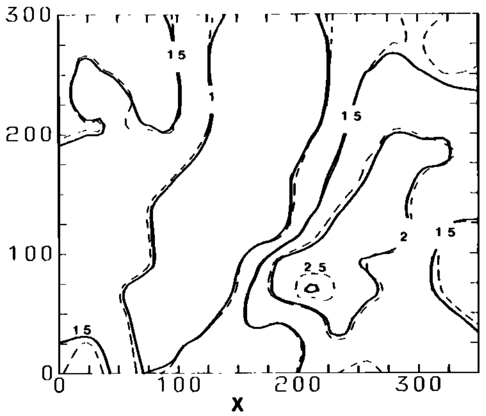

Fig. 8. Contour diagram of the ln (EC). Solid and dashed curves indicate the results from block DK and OK, respectively.

runs from the coordinate (75, 0) to (175, 300) and a zone of high EC in the lower right quadrant.

Figure 7 shows the contours of the kriging variance for disjunctive (solid curve) and ordinary block kriging (dotted curve). The overall pattern gives an indication of where in the field adequate or inadequate sampling occurred. A significant portion of the field has an ordinary kriging variance in excess of 10.0 whereas with DK the variance is generally less than 7.0 $(dS/m)^2$.

In Figure 8 disjunctive (solid curve) and ordinary (dotted curve) block kriging were used to generate a contour map for the ln (EC). Kriging in terms of ln (EC), as opposed to EC, produces a similar pattern of contours. The trough is still present but the zone of high ln (EC) has a somewhat different shape. Note that the contour intervals for ln (EC) are equal to the natural logarithm of the contour intervals given in Figure 6. Also, for the ln (EC), using DK or OK produces similar results compared to the EC (compare with Figure 6).

Contour maps provide a qualitative means for showing the differences between the OK and DK methods. In particular, a comparison of the spatial distribution of EC (or ln (EC)) determined by each method can be made. Also, using contour maps of the kriging variance can help give an indication of the quality of the estimate. The differences between DK and OK appear small when comparing the contour maps given in Figures 6 and 8. However, with the exception of the kriging variance maps, this is a relatively qualitative comparison.

A more quantitative measure of the improvement in the estimation process using DK as opposed to OK is shown by the results in Table 2, where a total of 240 estimates of EC and ln (EC) were made and the associated kriging variances were averaged. Also calculated was the reduction in kriging variance from DK which for the data sets used here was about 6%.

TABLE 2. Average Kriging Variance for 240 Estimates Made on a 24 by 24 m Grid System

	Punctual			Block		
Data Sets	OK	DK	Percent Reduction	OK	DK	Percent Reduction
EC	39.47	37.20	5.8	15.05	13.30	11.6
ln (EC)	0.576	0.538	6.9	0.218	0.192	11.8

Block size was 24 by 24 m. OK denotes ordinary kriging; DK denotes disjunctive kriging.

TABLE 3. Sum of Squares Between Estimated and Actual Values for Punctual Kriging With all Data Points Used in Calculations

Data Sets	OK	DK	Percent Reduction
EC	35.13	33.01	6.1
ln (EC)	0.612	0.581	5.1

OK denotes ordinary kriging; DK denotes disjunctive kriging.

Using the jackknife procedure the sum of squares between the estimation and the actual value at each sample location was calculated for EC and ln (EC) and the disjunctive and ordinary punctual kriging. These results are given in Table 3. The reduction in the sum of squares was about 5% for both data sets even though the data sets were approximately normal and lognormally distributed (recall that DK and OK are identical for exactly normal and lognormal data).

Conditional Probability

The DK estimator can also be used to determine the conditional probability that the unknown value is greater than a specified cutoff value. An example of this is given in Figures 9 and 10 where the conditional probability that the EC and the ln (EC) are greater than 4.0 and ln (4.0), respectively is contoured. These results were generated using a 24 by 24 m block size. From these figures one can see that the zones of high probability coincide with areas of high EC. Although the shapes of the high probability zones are similar to the shapes of the EC contours in Figures 6 and 8, two points with the same estimated value of EC do not always produce the same conditional probability. For example, points (240, 24) and (288, 120) both have estimates of approximately 11 dS/m (i.e., 10.99 and 10.77 dS/m, respectively) but the estimated conditional probability have quite different values of 0.65 and 0.91, respectively. This can be explained by consideration of the probability density functions (PDF) that result for each point.

From (44) (paper 1) the conditional probability density functions for the estimated value can be determined. These are plotted in Figure 11*a* for the EC and Figure 12*a* for the ln (EC). In both figures the abscissa is the transformed value, *Y*. The three curves represent the probability density function (PDF) for three points in the field: the solid curve (288, 120), the dashed curve (240, 24), and the dotted curve (96, 96). These three points were chosen to illustrate three features of the

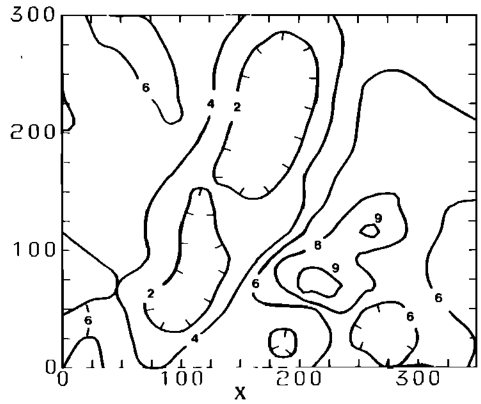

Fig. 9. Contour diagram of the conditional probability that the EC is above 4 dS/m using a 24 by 24 m block size.

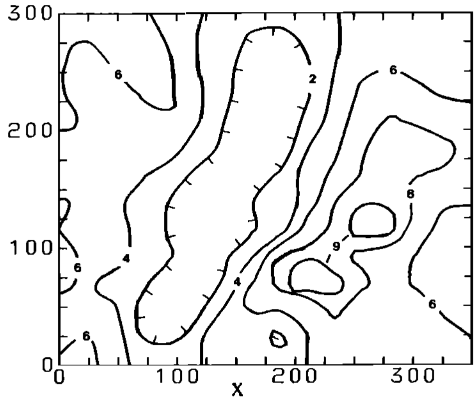

Fig. 10. Contour diagram of the conditional probability that the ln (EC) is above ln (4 dS/m) using a 24 by 24 m block size.

conditional probability. The first is to contrast the PDF for two points with similar mean values (the estimated values at a point) but differing PDF's and vice versa. Lastly, to show how the sample values and the associated weights determine the shape of the PDF and therefore the ultimate conditional probability.

The first case is illustrated by the solid (location 288, 120) and the bimodally distributed dashed (location 240, 24) curves in Figure 11*a* (and can also be seen in Figure 12*a*). In Table 4 the estimated mean value and associated probability for an EC cutoff value of 4.0 dS/m for the three points is given. The bimodal behavior for point (240, 24) is due to clustering about two levels of EC in the samples used to estimate the mean and conditional probability (i.e., Z_1, Z_2, and Z_4 versus Z_3 and Z_5 in Table 4). The mean value for each estimate is approximately the same but the conditional probability is vastly different because in one case two distinct levels of EC (with relatively high weights) are used in the estimation and in the other the samples are all large and of more uniform value. This is an important consideration for efficient management when each block can be managed separately.

The second case is illustrated by the solid and dotted curves in Figure 11*a*. For the point (96, 96) the data values used in

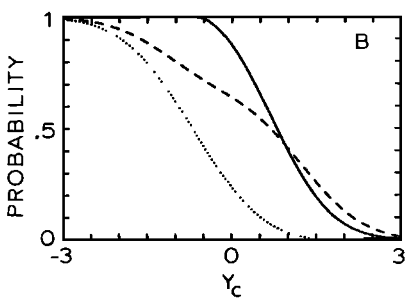

Fig. 11. Conditional probability density function (*a*) and probability function (*b*) for the EC for three points in the field (see text for details).

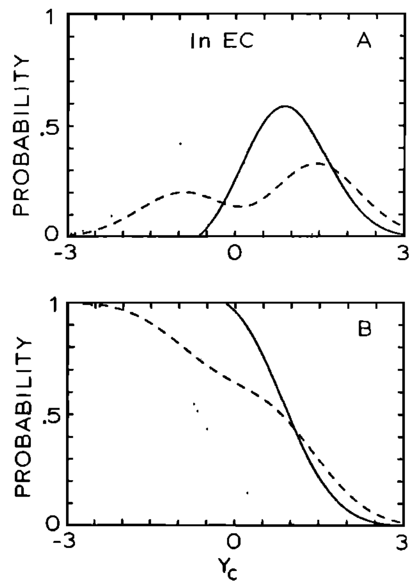

Fig. 12. Conditional probability density function (*a*) and probability function (*b*) for the ln (EC) for three points in the field (see text for details).

the calculation of the PDF (i.e., the b_{ik}'s and $Z(x_i)$'s) are of relatively low magnitude (the range for the observed values of EC were from 0.61 to 32.2). Accounting for the effect of the weights shows that the major contribution is from the data values Z_1, Z_3, and Z_4. Since all the data are of approximately the same magnitude, which is small, this produces a smaller mean value and a strongly peaked PDF. The point (288, 120) behaves in a similar manner but for this point the magnitude of the data is large (i.e., greater than 7 dS/m) and with the exception of Z_5 are between 7 and 13 dS/m. Also, the weights are more uniform.

Punctual disjunctive kriging was also applied to the data sets and produced similar results with respect to the PDF. The distributions tended to be more peaked and were shifted slightly to higher y_c's.

The conditional probability for arbitrary cutoff value is given in Figure 11*b* for the EC and Figure 12*b* for ln (EC). The estimates for both curves are based on block DK on a 24 by 24 m block size. From this figure, one can find the conditional probability for any arbitrary cutoff level. The solid, dashed, and dotted curves correspond to the points (288, 120), (240, 24) and (96, 96), respectively.

The effect of the support on the conditional probability is shown in Figure 13*a* where three block sizes, 25 by 25 m, 100 by 100 m, and 200 by 200 m were generated by building the larger blocks from the smaller blocks. Also, for comparison, the point values located at the center of the boxes are given in Figure 13*b*. The results indicate that the conditional probability for the EC data is relatively insensitive to the block size for block sizes less than 24 by 24 m. For larger blocks the deviation between the point and the block estimates increase.

Computer Requirements

The DK and OK programs used for this analysis were written to run on a Digital Rainbow 100 (not an endorsement but for reader information) microcomputer with 128K internal memory. The programs were run using an 8086 microprocessor and SuperSoft Fortran IV Extended (not an endorsement but for reader information). The compiled version of the DK program (including the Fortran operating system) required

TABLE 4. Weights and Data Values Used in Estimation of Three Points for Block Disjunctive Kriging

Point	k	Z^*	P^*	b_{1k}	Z_1	b_{2k}	Z_2	b_{3k}	Z_3	b_{4k}	Z_4	b_{5k}	Z_5
288, 120	1	10.20	0.868	0.113	7.49	0.170	7.29	0.251	12.82	0.1487	7.52	0.144	21.54
	2	10.58	0.866	0.066		0.104		0.170		0.084		0.088	
	3	10.46	0.887	0.030		0.051		0.094		0.038		0.042	
240, 24	1	7.96	0.688	0.093	0.98	0.154	1.22	0.290	32.20	0.182	5.15	0.090	26.58
	2	10.46	0.677	0.026		0.062		0.155		0.085		0.022	
	3	10.63	0.647	0.006		0.019		0.069		0.032		0.004	
96, 96	1	2.27	0.229	0.364	3.77	0.040	2.01	0.188	0.87	0.110	1.03	0.044	4.75
	2	3.12	0.226	0.197		0.018		0.091		0.044		0.018	
	3	3.00	0.247	0.093		0.005		0.034		0.013		0.005	

Z^* and P^* are the estimated mean and conditional probabilities. Z^* is within 95% of final value for $k = 2$ and 99% for $k = 3$.

68K memory allocation. The program allowed up to 150 samples, 15 coefficients (C_k's), 10 samples for use in the estimation, and up to a 15th order polynomial fit of the sample distribution (i.e., between $Z(x)$ and $Y(x)$).

In order to improve the program's computational efficiency, an array 150 by 15 was used to store the $H_k[Y(x_i)]$ values for $i = 1, 2, \ldots, 150$ and $k = 1, 2, \ldots, 15$. This allowed only one calculation of the Hermite polynomials associated with the samples (as opposed to calculating them for each estimate). Also, the program was written so that the kriging matrix was formed only once for each estimate (for $k = 1$) was saved and then a working matrix (raised to the appropriate power) was used to determine the b_{ik}'s. Using this strategy requires only one determination of the closest samples per estimate.

The computer time necessary for punctual OK of 240 estimates and estimation variances when 5 nearest neighbors are used in the estimation process was approximately 5 min. DK, on the other hand, required approximately 20 and 45 min when 5 and 10 coefficients, respectively, were used in (2).

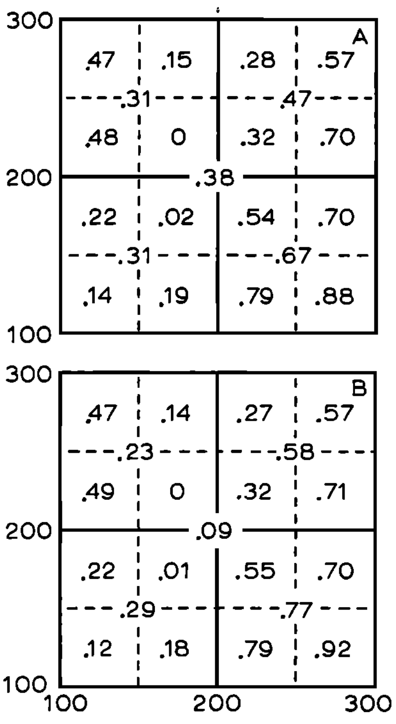

Fig. 13. Effect of support on the conditional probability for EC. In Figure 13a and 13b the values were calculated using block and punctual DK, respectively.

(Note, for DK, an estimate, estimation variance and conditional probability were obtained.)

CONCLUSIONS

The disjunctive kriging (DK) method has been explored in terms of its ability to be an estimator of the conditional probability distribution of the unknown value at a point. Estimates of the conditional probability density (CPDF), its mean value (i.e., the estimate), and the conditional probability (CP) that an unknown value is greater than a specified cutoff level have been obtained.

For the data sets used in this paper, DK has been shown to be a better estimator than ordinary kriging in terms of reduced kriging variance and sum of squares deviation between an estimated and actual value. This result is expected since theoretically a nonlinear estimator should be equal to or better than a linear estimator.

The CPDF at three locations (Figures 11 and 12) have been calculated and show the expected dependence on the samples used in the estimation. Two cases have been discussed, CPDF with approximately the same mean values but different shapes and vice versa, which explains some of the factors that affect the CPDF produced by the DK estimator.

The CP that the unknown value is above an arbitrary cutoff value has been determined for various support. For the data sets used here it is seen that the conditional probability is relatively insensitive to block size for block sizes less than or equal to approximately 24 m. For larger block sizes, differences between the point estimate and the block estimate become larger.

Overall there is an increase computational time necessary to solve the DK equations. This is compensated for by being able to estimate the CP and/or CPDF at the estimation site.

In terms of management decisions, one is able to utilize the conditional probability feature of DK to help in making management decisions. The DK method offers a quantitative method for assessing the level of an indicator variable in space and the probability that the indicator is above an arbitrarily set cutoff level. As illustrated, the CP of the estimate is affected by the samples used in the estimation process. When one can manage a given unit area separately from the surrounding units, management efficiency can be improved by utilizing the CP. An example of this has been given where the mean values for two blocks are approximately the same but only in one case is there a high likelihood that the estimated value is greater than the cutoff level (i.e., P^* of 0.91 versus 0.65 for the other block). DK, along with other kriging methods, has the advantage in that an estimate (of the mean value or the

CPDF) can be made at any location even at unsampled locations.

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