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DIRECT COMPARISONS BETWEEN KRIGING AND OTHER INTERPOLATORS

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INTRODUCTION

The application of geostatistical techniques is often a search for optimal interpolators. VanKuilenberg et al. (1982), El-Haris (1987) and Laslett et al. (1987) compared kriging to other interpolators for soil properties. Generally speaking, kriging was only marginally better than alternative methods as judged by the root mean squares (RMS) of the error between predicted and measured values. The lack of a clear superiority is somewhat unexpected as the kriging estimation is optimal. However, optimality is defined by the minimum expected error variance with respect to a particular variogram model and not directly in terms of the RMS. Although not pursued here, a closely related topic is equivalence of interpolators, for example of kriging and splines (Myers, 1988).

The objective of this paper is to compare kriging to other estimators, in particular inverse distance weighting vs. kriging. The root mean squares (RMS) of the errors for the different interpolators were calculated and then compared for several data sets. Additionally, rankings and the distribution of errors are compared for the different methods.

DATA SETS

Five data sets are used for the comparisons. Brief descriptions of each follow. Four data sets are for field results with the other being an artificially-generated set. In all but one case, the variograms used were those of the original authors.

1. El-Haris (1987) measured a number of parameters at 188 sites over an 140 hectare area at the University of Arizona's Maricopa Agricultural Center.

Two soil series are represented: the Casa Grande sandy clay loam (fine loamy, mixed, hyperthermic, Typic Natrargids) and Trix clay loam (fine loamy, mixed "calcareous" hyperthermic, Typic Torrifluvents). Parameters considered here represent the 0-25 cm depth and include % sand and Ca on an 1:2 (soil to water) extract. Some of the statistical parameters are given in Table 1.

Variograms were fitted using weighted least squares. The weights were based on couples per class with separations up to half of the largest field dimension. His chosen variogram models are

Table 1. Estimated statistics of the data sets. The mean and coefficient of variation are estimated values.

Source	Property	Mean	CV	Skew	Kurtosis
1. El-Haris (1987)	Sand	60.4 (%)	0.132	-2.08	9.58
	Ca	66.5 (mg/kg)	0.419	0.83	3.31
2. Vieira (1981)	Infiltration rate	6.98 (mm/h)	0.40	0.482	2.95
3. Munoz-Pardo (1987)	Scaling coefficient	1.0 (no dimensions)	0.245	0.153	2.58
4. Samper-Calvete (1987)	Simulated	0.988	0.934	-0.018	3.07
5. Kalamkar (1931)	Potato yield	23.3*	3.94	0.782	3.14

*Pounds for 22 foot length of row.

$$\gamma(h) = 2 + 55[1.5(h/450) - 0.5(h/450)^3], \quad 0 < h \le 450$$

$$= 57 \qquad h > 450 \text{ (sand)} \qquad [1]$$

and

$$\gamma(h) = 495 + 0.5h, \quad h > 0 \text{ (calcium)} \qquad [2]$$

where h is expressed in meters and the parameter dimensions are as in Table 1.

2. Vieira et al. (1981). A total of 1280 steady infiltration values were measured on a 160 m x 55 m plot on the Yolo soil series (Typic Xerothents). The infiltrometer rings were 46 cm in diameter and the infiltration rate measured as the rate at which the water height above the soil surface decreased with time (corrected for evaporation). Sample locations were in parallel rows with the closest spacings of 1 m (in direction 1) or 5 m (in direction 2).

The resulting variogram was of a Langmuir (Michaelis-Menton) type:

$$\gamma(h) = h/[1.2 + 0.105h] \qquad [3]$$

This variogram was fitted by least squares and the interpolated results compared to those obtained by alternative methods with the conclusion that the variogram model was not sensitive to the fitting method.

3. Munoz-Pardo (1987). A total of 77 values of hydraulic conductivity on a 7 x 11 grid of 20 m spacing is presented based on results attributed to A. Gonzalez in 1985. The parameter modeled is the scaling coefficient α defined for a site i as

$$\alpha_i^2 = K_i/K^*. \qquad [4a]$$

with K_i the permeability and K^* an average value defined as

$$K^* = (1/n^2)[\sum_1^n K_i^{0.5}]^2 \qquad [4b]$$

with n the total number of points. The α_i were approximately normally distributed and a spherical model was used, namely

$$\gamma(h) = 0.015 + 0.0615[1.5(h/148) - 0.5(h/148)^3], \quad h < 148$$

$$= 0.0765, \qquad h > 148 \qquad [5]$$

4. Samper-Calvete (1987). This is the only synthetic set. The data is for 200 random sites within a 30 x 30 unit area with a semi-variogram given as

$$\gamma(h) = 1 - \exp(-h/3) \qquad [6]$$

The data were generated by the "Turning-Bands" method and represent a subset of a larger data set given by Samper-Calvete.

5. Kalamkar (1932). The data is for potato yields from 96 rows, each 132 ft. long, with 3 ft. between rows. Each row was harvested in six units, a unit being 22 ft. long. There are 576 one-row plots one unit long. The variogram chosen is the linear model:

$$\gamma(h) = 2.757 + 0.114h \qquad [7]$$

NUMERICAL PROCEDURE AND RESULTS

Three different interpolation methods were used: arithmetic mean (of nearest neighbors), inverse distance weighting and punctual kriging.

For the inverse distance weighted average, each survey point receives a weight proportional to the inverse of the squared distance from test site. That is, the estimate $Z^*(x_o)$ is by

$$Z^*(x_o) = \sum_{i=1}^m w_i Z(x_i) / \sum_{i=1}^m w_i \qquad [8]$$

with

$$w_i = 1/(h_i)^2 \qquad [9]$$

where h_i is the separation distance of interpolated and measured points and m is the number of points used in the interpolation.

The kriged values were obtained using only a specified number of nearest neighbors. Two different computer codes were used (Statpak, U.S. Geological Survey, 1984, Open-File Report 84-522) and a separate program written by the senior author. The motivation for using 2 codes was to cross-check for errors.

For each of the 3 methods and for each of the 5 data sets, the root mean square (RMS) error was calculated, where

$$RMS = \left[(1/n) \sum_i^n (Z_i^* - Z_i)^2 \right]^{0.5} \qquad [10]$$

In Eq.10, n is the total number of data points, Z_i is the observed value and

Z_i^* is the interpolated value found by the appropriate method at the same position as based on the other n-1 data values.

The RMS values are presented in Table 2. Interpolation by arithmetic mean was found to give better results when only the four closest points were used than for more points. For inverse weighting and kriging, the 8 closest points were used. An examination of Table 2 reveals the RMS is nearly the same for the three methods. The main exception is for the data of Vieira for which kriging is considerably better than the other two methods, (0.424 compared to 0.727 and 0.685).

Additionally, the average error rank R_j was calculated for each method (Laslett, 1987); by

$$R_j = (1/n) \sum_{i}^{n} r_{ij}, \; j = 1,2,3 \qquad [11]$$

where r_{ij} is the rank at a specific point. If method "j" is the closest at point i then $r_{ij} = 1$; if it is second best it is 2; and if it is 3rd best the value is 3. Also, the standard deviation S_j of the ranks is defined:

$$S_j = \left\{ [1/(n-1)] \sum_{i}^{n} (r_{ij} - R_j)^2 \right\}^{0.5} \qquad [12]$$

The best method should have the lowest rank (close to 1) and small S_j indicating a consistently good ranking. If the three methods, were equally good, the r_{ij} would occur as 1, 2, and 3 resulting in $R_j = 2$ and S_j approaching $(2/3)^{0.5} = 0.82$.

In terms of ranks, kriging has the lowest value for 4 of the 6 data sets as shown in Table 3. The most clear-cut advantage is on the Vieira and Munoz-Pardo data.

Table 2. Root mean square (Eq. 10) for interpolated values using means, inverse weighting and punctual kriging.

Source	Property	Arithmetic1/ Mean	Inverse Distance	Kriging
1. El-Haris	sand	3.65	3.81	3.65
	Ca	25.4	24.4	24.1
2. Vieira	Infiltration rate	0.727	0.685	0.424
3. Munoz-Pardo	Scaling coefficient	0.171	0.176	0.172
4. Samper-Calvete	Simulated	0.683	0.657	0.650
5. Kalamkar	Potato yield	2.15	2.19	2.13

Table 3. Rankings (Eq. 11) and standard deviations of rankings (Eq. 12). The standard deviations are in parenthesis.

Source	Property	Arithmetic1/ Mean	Inverse Distance	Kriging
1. El-Haris	sand	2.06 (0.792)	2.03 (0.895)	1.90 (0.754)
	Ca	2.06 (0.905)	1.95 (0.781)	1.98 (0.742)
2. Vieira	Infiltration rate	2.39 (0.745)	2.13 (0.630)	1.48 (0.780)
3. Munoz-Pardo	Scaling coefficient	2.23 (0.944)	1.97 (0.707)	1.79 (0.732)
4. Samper-Calvete	Simulated	2.08 (0.850)	1.95 (0.781)	1.98 (0.817)
5. Kalamkar	Potato yield	2.08 (0.829)	2.05 (0.845)	1.87 (0.758)

1/Four nearest points

Other comparisons were also made, but only for the sand data of El-Haris. Scatter plots for Z_i^* and Z_i are given as Figure 1 for each of the three methods. The correlations were 0.89 in all cases and the calculated slopes were 0.74, 0.68, and 0.75 for arithmetic mean, inverse distance and kriging, respectively. The slopes clearly favor the kriging results, primarily because of closer fits for the smaller measured values.

Additional scatter plots are given as Figure 2 for Z_i-Z_i^* vs. Z_i, again for the sand data. The results all show a tendency for the Z_i-Z_i^* to increase with Z_i. The correlation coefficients are 0.57, 0.67 and 0.54 for Figure 2A, B and C, respectively; the slopes are 0.26, 0.32 and 0.25. (Similar plots for Z_i-Z_i^* vs. Z_i^* not shown here revealed correlations of 0.14, 0.26 and 0.09, respectively and slopes of 0.08, 0.16 and 0.05.)

As a final test, frequency histograms of Z_i-Z_i^* for the sand were prepared as Figure 3. The results are similar, with all methods resulting in a slight skew to the right. The kriging and arithmetic mean results are somewhat "better" in that the "zero class" is higher than for inverse weighting.

DISCUSSION

Overall results show the mean squared values (MSE) for interpolation by kriging are as good or better than the arithmetic mean or inverse distance weighting. In the case of one of the 6 comparisons, namely the infiltration rates of Vieira (1981), the kriging results were markedly better. For the other 5 data sets, differences were very slight. The ranking values followed the same pattern, with a tendency towards a more clear advantage for kriging compared to the other 2 methods and with perhaps a more clear advantage of the arithmetic mean over inverse distance weighting. The other comparisons were only for the sand data, but showed similar results, with perhaps a small advantage for kriging.

The kriging estimator is optimal with respect to the estimation variance; this minimized variance is determined by the sample location pattern and the particular variogram model but does not depend on the data itself. Since the inverse distance

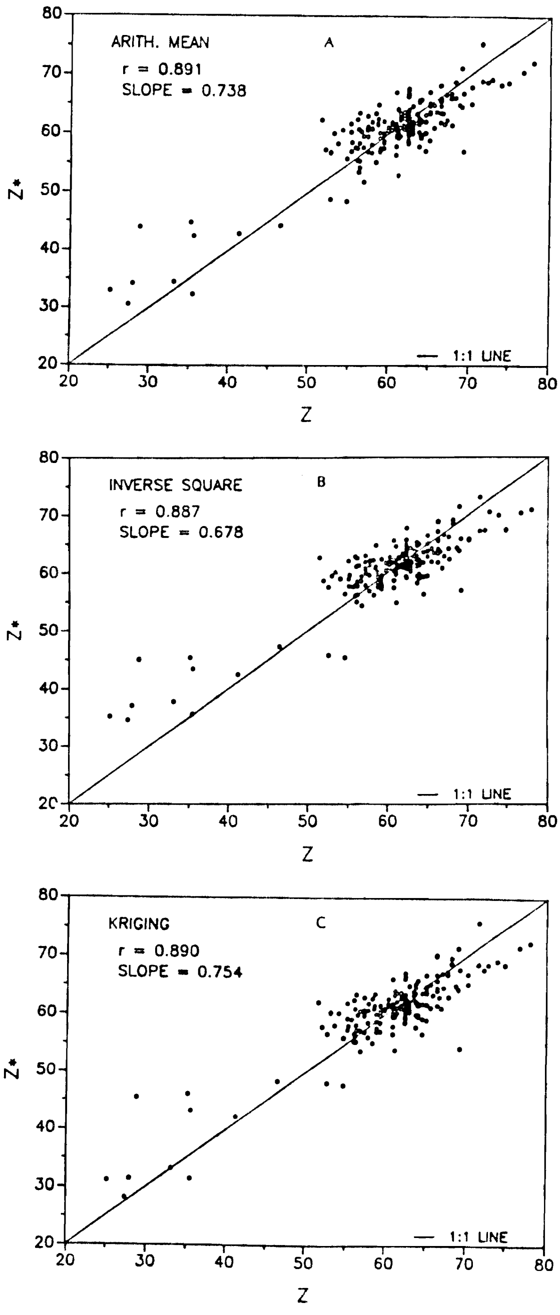

Figure 1. Estimates of sand Z_i^* compared to original values Z_i for the three interpolation methods.

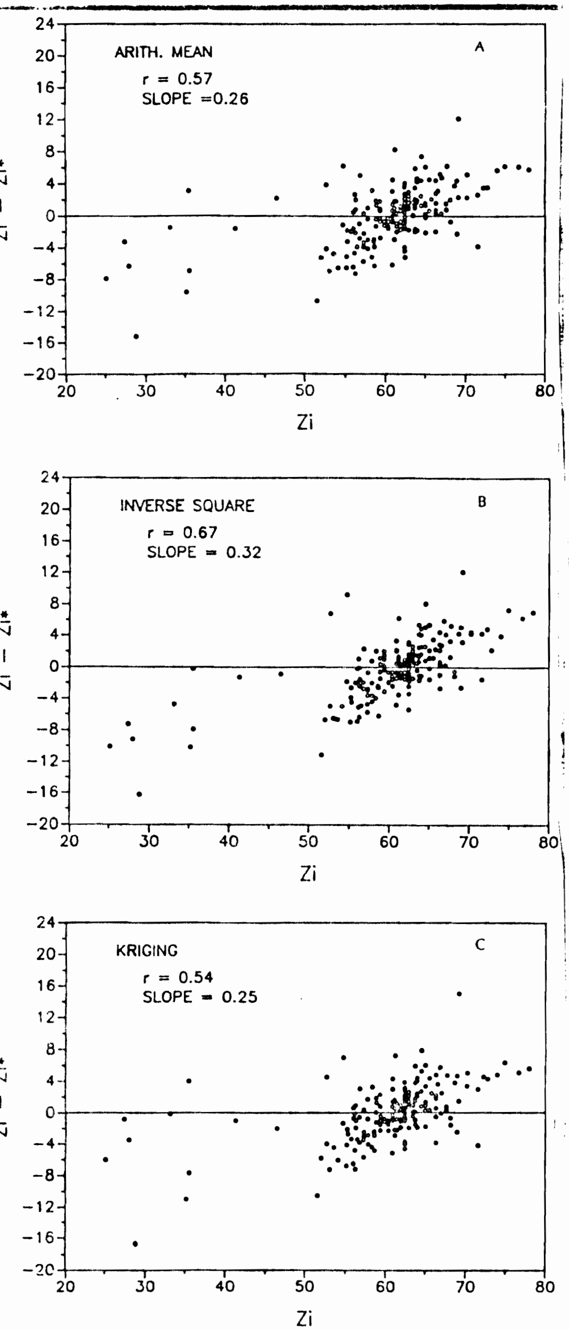

Figure 2. Scatter diagram of error of interpolated values Z_i-Z_i^* vs. original sand value for the three interpolation methods.

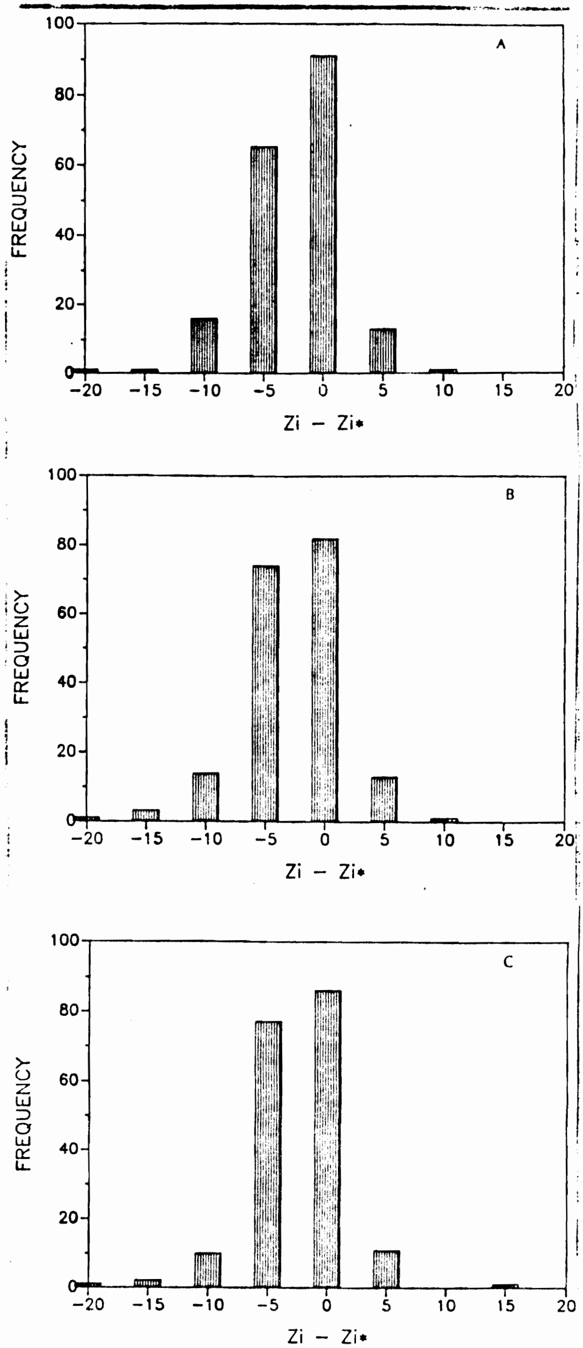

Figure 3. Frequency histogram of errors of interpolation (Z_i-Z_i^*) for the three interpolation methods.

weighting estimator is not theoretically derived it does not have a corresponding estimation variance. It would be possible to attempt to optimize the choice of the variogram model or the exponent on the distance in the case of inverse weighting by using the comparison between estimated values and measured values. In the case of kriging, this is the well-known "cross-validation" technique although unfortunately the behavior of the various statistics are not well enough known to be sure that the optimal variogram model has been selected. Kane et al. (1982) used a similar approach for inverse weighting with the emphasis on the normalized RMS and a sub-selected training set to optimize the choice of the exponent. It is instructive to note that the sensitivity of the RMS to the exponent varied considerably for different chemical elements and different regions. Consequently it is difficult to draw conclusions concerning the relative efficacy of different interpolators that would be applicable in general, this indicates that more must be known about the use of cross validation as used for selecting an optimal estimator.

A natural question is "Which method should be used in a given situation?" The answer is not totally clear. As software (and hardware) becomes readily available and relatively inexpensive, kriging is not much more tedious than the alternatives. Additionally, the kriging variance is indicative of quality of the estimates. Thus, kriging would seem the logical choice, even though in many cases little advantage would be gained over the simpler methods.

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