A DIRECT SUM REPRESENTATION(1)

BY DONALD E. MYERS

Tucson, Arizona — U. S. A.

Norms defined by supremums are in general not generated by scalar products. However, using a representation theorem of Bochner; Hardy spaces \((p = 2)\) on tube domains are represented as Hilbert spaces.

Let \(T\) be a tube in \(C^n\) with base \(S\), i.e. \(T = \{Z | Z \in C^n, R(Z) \in S\}\), \(S = \{(x_1, \ldots, x_n) | \sigma_i < x_i < \tau_i\}\). \(H^2(T) = \{f | f\) holomorphic in \(T,\)

\[||f||^2 = \sup_{x \in S} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} |f(x + iy)|^2\ dy_1\ dy_2\ \cdots\ dy_n < \infty\}.

Theorem. \(H^2(T)\) is isomorphic and homeomorphic to the direct sum of \(2^n\) copies of \(H^2(T_0)\) where the base of \(T_0\) is given by \(\sigma_1 = \sigma_2 = \cdots = \sigma_n = 0, \ \tau_1 = \tau_2 = \cdots = \tau_n = +\infty\).

Proof. Because of the invariance of the integral, under affine transformations, used in computing the norm, it is sufficient to show that \(f\) has a unique representation of the form \(f = f_1 + \cdots + f_{2^n}\) where \(f_i \in H^2(T_j), \ T_j\) an octant-shaped tube. \(T_j\) is octant shaped if for each \(i\), either \(\sigma_i = -\infty\) or \(\tau_i = +\infty\) but not both. We must also show that the mapping \(f \rightarrow (f_1, \ldots, f_{2^n})\) is 1:1 and bi-continuous. It is 1:1 and onto if the representation is unique.

The desired unique representation is provided by Bochner’s Theorem [1]. It then remains to establish continuity. Clearly \(||f|| \leq ||f_1|| + \cdots + ||f_{2^n}||\) where \(||f_j||\) is the norm of \(f_j\) in \(H^2(T_j)\),

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hence by the Open Mapping Theorem the inverse map \((f_1, \ldots, f_{2^n}) \mapsto f\) is also continuous.

Since \(H^2(T_0)\) is a Hilbert space, \(H^2(T)\) is isomorphic and homeomorphic to the Hilbert space \(\bigoplus_{j=1}^{2^n} H^2(T_0)\). Using this mapping we may also imbed \(H^2(T)\) as a finite dimensional subspace of the square summable power series on the unit disk with coefficients in \(H^2(T_0)\).

REFERENCES
