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Author(s): P. K. Bhattacharya, J. Kimball Dietrich, Donald G. Heckerman, Donald E. Myers

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VARIABLE WAGES AND PRICES AND THE DEMAND
FOR CAPITAL WITH DISCRETE AND
CONTINUOUS ADJUSTMENTS*

BY P. K. BHATTACHARYA, J. KIMBALL DIETRICH,
DONALD G. HECKERMAN AND DONALD E. MYERS¹

1. INTRODUCTION

Managers must be concerned with the effects of uncertain prices and wages on the investment decisions of firms. Suppose, for instance, that more erratic macroeconomic policies cause managers to anticipate greater uncertainty about future price-wage ratios. In this paper we identify the effects of increased uncertainty on the demand for capital by a small firm. Our results indicate that for firms which do not continuously adjust their capital stock, increased uncertainty about future price-wage ratios increases the demand for capital. The results also reveal that the effects of uncertainty on the demand for capital depend on the length of the interval between capital acquisition decisions. That is, the effects of uncertainty on the demand for capital depend on whether investment decisions are revised continuously, monthly, yearly, or at more infrequent intervals.

We consider a multiperiod model of a small firm. The firm views prices, wages, and the discount rate as exogenous variables given by the economy. Managers are assumed to adjust the firm's demand for labor continuously and to adjust the capital stock at discrete time intervals with the objective of maximizing the present discounted value of cash flows. To assure a firm of determinate size, we postulate a Cobb-Douglas production function with decreasing returns to scale.

The analysis employs specific assumptions about present and future prices, wages, and the firm's cost of capital (i.e., discount rate). Future prices and wages are assumed to be determined by a stochastic bivariate log-normal process. This exogenous process permits random changes in the price-wage ratio.² Such a stochastic price-wage process could be generated by an erratic monetary policy if prices and wages adjust at different rates. Similar price-wage dynamics might ensue if the monopolistic labor sector experiences random changes in its bargaining power. This assumed price-wage process is not necessarily consistent with the changes in production and in the demand for labor which will be induced in

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¹ We wish to thank the referees of this Review for comments which have improved the accessibility of our results.

² See Sheshinski and Drèze [1976] for a one period model in which prices and the size of the industry are endogenous. The authors assume that input costs are certain.

response to wage and price changes. The analysis also assumes that the firm's discount rate is constant over time. While this greatly simplifies the analysis, it rules out the possibility of a nonzero covariance between the firm's opportunity cost of funds and the economic conditions likely to be associated with particular wage-price ratios. It should also be understood that to derive the demand for capital given the discount rate is a partial equilibrium approach. Changes in uncertainty surely also affect the supply of equity funds and thereby likely change the expected value of the firm's discount rate. Despite the obvious shortcomings of our partial equilibrium approach, we have elected to utilize the assumptions for they have the virtue of allowing tractible results for the joint effects on the demand for capital of uncertainty and the interval between capital acquisition decisions.

In Section 2 we develop the optimal values of the firm's decision variables, labor and capital, and the firm's market value for the case in which investment decisions are made at discrete intervals. These results are summarized in Theorem 1. In Section 3 we identify the joint effects upon the demand for capital of wage-price uncertainty and of the length of time between investment decisions. These effects are summarized by the elasticities reported in equations (15) and (16). In Section 4 we derive solutions for optimal decisions when investment decisions are revised continuously. These results are summarized in Theorem 2. In Section 5 we summarize the results contained in the paper and identify unresolved problems.

The results of this paper extend those of Hartman [1972]. That paper examined the effects of increased uncertainty of future output prices, wage rates and investment costs on the quantity of investment undertaken by the firm. More specific assumptions about the production function and the stochastic process make it possible to show how those results depend upon the interval between capital acquisition decisions. Our analysis also makes it clear that it is uncertainty about the price-wage ratio which affects investment decisions.

2. THE MODEL SOLUTION

We assume a small firm with diminishing marginal returns. The firm's management views the rate of discount, r , which is applied to future cash flows by investors evaluating the firm, as an exogenously determined parameter. Furthermore, the firm views the process generating future product and capital equipment prices as well as wages as a given stochastic process, with known parameters. The firm at time 0, with planning horizon of length T into the future, initially has no assets. The problem for management is to choose an investment strategy which maximizes the expected value of the future rents from production.

The time interval $[0, T]$ contains $n = T/h$ equal subperiods of length h , which is assumed to be the institutionally determined length of time between capital acquisition decisions. In each of these intervals, the capital outlay remains constant, although labor may be adjusted continuously. For a capital outlay of the amount k_j for the interval $[jh, (j+1)h]$ and for the choice of a labor function L , the net cash flow for the interval would be

$F_j(k_j, L_j)$

$$= -p(jh)k_j + e^{-(r+\delta)h}p((j+1)h)k_j + \int_{jh}^{(j+1)h} e^{-r(s-jh)}R(k_j, L(s), p(s), w(s))ds,$$

where r is the interest rate, δ is the rate of depreciation, $p(s)$ is the price and $w(s)$ is the labor cost at time s , and

$$R(k_j, L(s), p(s), w(s)) = p(s)Ak_j^\alpha L^\beta(s) - w(s)L(s)$$

where $\alpha > 0, \beta > 0, \alpha + \beta < 1$.

Let $\log p(s) = \xi(s), \log w(s) = \eta(s)$.

Assumptions. (i) The bivariate stochastic process $\{(\xi(s), \eta(s)), s \geq 0\}$ is a two-dimensional Brownian motion (B.M.) with linear drift (π_p, π_w) per unit time and covariance matrix $\begin{bmatrix} \sigma_p^2 & \rho\sigma_p\sigma_w \\ \rho\sigma_p\sigma_w & \sigma_w^2 \end{bmatrix}$ per unit time, starting at $(\xi(0), \eta(0))$.

(ii) $b = \pi_p + \frac{1}{2}\sigma_p^2 - r - \delta < 0$.

The labor function $L(s), 0 \leq s \leq T$ and the capital outlays $\{k_j\}$ are control variables which are non-negative, non-anticipative functions of price and labor cost, i.e., $L(s) \geq 0$ depends only on $\{(\xi(t), \eta(t)), t \leq s\}$ and $k_j \geq 0$ depends only on $\{(\xi(t), \eta(t)), t \leq jh\}$. For a given choice of $\{k_j\}$ and L_j , the expected present discounted value of the firm over the period $[0, T]$ is:

$$V(0; \{k_j\}, L_j) = E \sum_{j=0}^{n-1} e^{-jr} F_j(k_j, L).$$

We are interested in the maximum value

$$V^*(0) = V(0; \{k_j^*\}, L_j^*) = \max_{\{k_j\}, L} V(0; \{k_j\}, L)$$

where $\{k_j^*\}$ and L_j^* are the optimal choices for the control variables.

Since by assumption (i) the behavior of the stochastic processes $p(s)$ and $w(s)$ are better understood in terms of $\xi(s) = \log p(s)$ and $\eta(s) = \log w(s)$, it will be convenient to work with logarithmic transforms of the control variables, viz.,

$$\phi_j = \log k_j \quad \text{and} \quad \psi(s) = \log L(s),$$

and rewrite $F_j(k_j, L_j)$ as

$$(1) \quad F_j(\phi_j, \psi) = -\exp[\phi_j + \xi(jh)] + \exp[-(r + \delta)h + \phi_j + \xi((j+1)h)] + \int_{jh}^{(j+1)h} \exp[-r(s-jh)] \{A \exp[\alpha\phi_j + \beta\psi(s) + \xi(s)] - \exp[\psi(s) + \eta(s)]\} ds,$$

and $V(0; \{k_j\}, L)$ and $V^*(0)$ as

$$(2) \quad V(0; \{\phi_j\}, \psi) = E \sum_{j=0}^{n-1} e^{-jr} F_j(\phi_j, \psi)$$

and

$$(3) \quad V^*(0) = V(0; \{\phi_j^*\}, \psi^*) = \max_{\{\phi_j\}, \psi} F_j(\phi_j, \psi).$$

For the maximization in (3), first fix $\{\phi_j\}$ and maximize $V(0; \{\phi_j\}, \psi)$ given by (2), with respect to ψ . Since $\psi(s)$ is allowed to depend on the observed prices and labor costs up to time s , the optimum $\psi^*(s, \{\phi_j\})$ for arbitrary $\{\phi_j\}$ is obtained by maximizing the integrand in the integral appearing in (1) at each s and for each observed $(\xi(s), \eta(s))$. In other words, $\psi^*(s, \{\phi_j\})$ is obtained by maximizing

$$A \exp [\alpha \phi_j + \beta \psi(s) + \xi(s)] - \exp [\psi(s) + \eta(s)]$$

with respect to $\psi(s)$. Thus

$$(4) \quad \exp [\psi^*(s; \{\phi_j\})] = (A\beta)^{\frac{1}{1-\beta}} \exp \left\{ \left(\frac{\alpha}{1-\beta} \right) \phi_j + \frac{1}{1-\beta} [\xi(s) - \eta(s)] \right\}.$$

To obtain $\{\phi_j^*\}$ and $\psi^* = \psi^*(\{\phi_j^*\})$, the expression

$$(5) \quad V(0; \phi) = E \sum_{j=0}^{n-1} e^{-jrh} F_j(\phi_j, \psi^*(\{\phi_j\})) = E \sum_{j=0}^{n-1} e^{-jrh} E_j F_j(\phi_j, \psi^*(\{\phi_j\}))$$

has to be maximized with respect to $\{\phi_j\}$, where the value of the function $\psi^*(\{\phi_j\})$ at s is given by (4) and

$$(6) \quad E_j F_j = E[F_j | (\xi(t), \eta(t)), t \leq jh].$$

For this, we have to first compute the conditional expectations given by (6). The following lemmas will be useful for this purpose. These are well-known properties of normally distributed random variables and their proofs are omitted. We shall use the notation $X \stackrel{d}{=} Y$ to mean that X and Y are random variables with the same distribution.

LEMMA 1. $(\xi(s), \eta(s))$ can be expressed as

$$\begin{aligned} \xi(s) &= \xi(0) + \pi_p s + \sigma_p Z_1(s) \\ \eta(s) &= \eta(0) + \pi_w s + \rho \sigma_w Z_1(s) + \sqrt{1 - \rho^2} \cdot \sigma_w Z_2(s) \end{aligned}$$

where $\{Z_1(s), s \geq 0\}$ and $\{Z_2(s), s \geq 0\}$ are independent standard B.M.'s.

LEMMA 2. If W is a standard normal random variable and a is a constant, then $E \exp [aW] \equiv \exp \left[\frac{1}{2} a^2 \right]$.

LEMMA 3. If $\{Z_1(s)\}$ and $\{Z_2(s)\}$ are independent B.M.'s and W is a standard normal random variable, then $a_1 Z_1(s) + a_2 Z_2(s) \stackrel{d}{=} \sqrt{(a_1^2 + a_2^2)} s W$.

The conditional expectation given by (6) can now be expressed in terms of $\{Z_1(s)\}$ and $\{Z_2(s)\}$ as

$$(7) \quad E_j F_j = E[F_j(\phi_j, \psi^*(\{\phi_j\})) | Z_1(t), Z_2(t), t \leq jh].$$

For the purpose of taking expectations, F_j may be replaced by another random

variable with the same distribution. We make such a replacement using the Markov property of B.M.'s in conjunction with Lemma 3 to get

$$\begin{aligned}
 (8) \quad F_j(\phi_j, \psi^*(\{\phi_j\})) &\stackrel{d}{=} -\exp[\phi_j + \xi(0) + \pi_p jh + \sigma_p Z_1(jh)] \cdot \\
 &\{1 - \exp[(\pi_p - r - \delta)h + \sigma_p \sqrt{h} W_1]\} + (A\beta^\beta)^{\frac{1}{1-\beta}} \cdot \\
 &(1 - \beta) \exp\left[\left(\frac{\alpha}{1-\beta}\right)\phi_j + \frac{\xi(0) - \beta\eta(0)}{1-\beta} + \left(\frac{\pi_p - \beta\pi_w}{1-\beta}\right)jh\right. \\
 &+ \left(\frac{\sigma_p - \rho\beta\sigma_w}{1-\beta}\right)Z_1(jh) + \sqrt{1-\rho^2} \frac{\beta\sigma_w}{1-\beta} Z_2(jh)\bigg] \times \int_0^h \exp[-rs \\
 &+ \left(\frac{\pi_p - \beta\pi_w}{1-\beta}\right)s + \frac{\sqrt{(\sigma_p^2 + \beta^2\sigma_w^2 - 2\beta\rho\sigma_p\sigma_w)s}}{1-\beta} \cdot W_2] ds
 \end{aligned}$$

where W_1 and W_2 are standard normal random variables independent of $\{Z_1(s), Z_2(s), s \leq jh\}$.

The conditional expectation of the future cash flows (7) is now computed for the expression given by (8) by using Lemma 2. Thus

$$\begin{aligned}
 (9) \quad E_j F_j &= -\exp[\phi_j + \xi(0) + \pi_p jh + \sigma_p Z_1(jh)] \cdot (1 - e^{bh}) \\
 &+ (A\beta^\beta)^{\frac{1}{1-\beta}} (1 - \beta) \times \exp\left[\left(\frac{\alpha}{1-\beta}\right)\phi_j + \frac{\xi(0) - \beta\eta(0)}{1-\beta}\right. \\
 &+ \left(\frac{\pi_p - \beta\pi_w}{1-\beta}\right)jh + \left(\frac{\sigma_p - \rho\beta\sigma_w}{1-\beta}\right)Z_1(jh) \\
 &+ \left.\sqrt{1-\rho^2} \frac{\beta\sigma_w}{1-\beta} Z_2(jh)\right] \times \exp[(g - r)h - 1]/(g - r)
 \end{aligned}$$

where

$$(10) \quad g = \frac{\pi_p - \beta\pi_w}{1-\beta} + \frac{1}{2} \cdot \frac{\sigma_p^2 - 2\beta\rho\sigma_p\sigma_w + \beta^2\sigma_w^2}{(1-\beta)^2}$$

which may also be written (after some manipulation)

$$\begin{aligned}
 (10') \quad g &= \frac{1}{1-\beta} \left(\pi_p + \frac{1}{2}\sigma_p^2\right) - \frac{\beta}{1-\beta} \left(\pi_w + \frac{1}{2}\sigma_w^2\right) \\
 &+ \frac{\beta}{(1-\beta)^2} (\sigma_p^2 - 2\rho\sigma_p\sigma_w + \sigma_w^2).
 \end{aligned}$$

Writing (9) as $E_j F_j = -C_1 \exp[\phi_j] + C_2 \exp\left[\left(\frac{\alpha}{1-\beta}\right)\phi_j\right]$, the optimal ϕ_j is seen to be

$$(11) \quad \exp[\phi_j^*] = \left[\left(\frac{\alpha}{1-\beta}\right) \cdot \frac{C_2}{C_1}\right]^{\lambda(1-\beta)} \quad \text{where } \lambda = \frac{1}{1-\alpha-\beta}.$$

Putting this back in (9), we have

$$\begin{aligned}
 E_j F_j(\phi_j^*, \psi^*(\phi_j^*)) &= -C_1 \exp[\phi_j^*] + C_2 \exp\left[\left(\frac{\alpha}{1-\beta}\right)\phi_j^*\right] \\
 &= \frac{1}{\lambda} \left[\frac{\alpha^\alpha}{(1-\beta)^{1-\beta}} C_1^{-\alpha} C_2^{1-\beta} \right]^\lambda = \frac{1}{\lambda} \left[\frac{A\alpha^\alpha \beta^\beta}{(1-e^{bh})^\alpha} \exp[(1-\alpha)\xi(0)] \right. \\
 &\quad \left. - \beta\eta(0) + \{(1-\alpha)\pi_p - \beta\pi_w\}jh\right] \cdot \left(\frac{e^{(g-r)h} - 1}{g-r}\right)^{1-\beta} \\
 &\quad \times \exp\left[\{(1-\alpha)\sigma_p - \rho\beta\sigma_w\}Z_1(jh) + \sqrt{1-\rho^2}\beta\sigma_w Z_2(jh)\right]^\lambda \\
 &\stackrel{d}{=} \frac{1}{\lambda} \left[\frac{A\alpha^\alpha \beta^\beta}{(1-e^{bh})^\alpha} \left(\frac{e^{(g-r)h} - 1}{g-r}\right)^{1-\beta} \right]^\lambda \exp[\lambda\{(1-\alpha)\xi(0) - \beta\eta(0)\}] \\
 &\quad \times \exp[\lambda\{(1-\alpha)\pi_p - \beta\pi_w\}jh] \\
 &\quad + \lambda\sqrt{\{(1-\alpha)^2\sigma_p^2 + \beta^2\sigma_w^2 - 2\rho\sigma_p\sigma_w\}jhW}
 \end{aligned}$$

by Lemma 3, where W is a standard normal random variable. By an application of Lemma 2, we now have

$$\begin{aligned}
 (12) \quad EE_j F_j(\phi_j^*, \psi^*(\phi_j^*)) &= \frac{1}{\lambda} \left[\frac{A\alpha^\alpha \beta^\beta}{(1-e^{bh})^\alpha} \cdot \left(\frac{e^{(g-r)h} - 1}{g-r}\right)^{1-\beta} \right]^\lambda \\
 &\quad \exp[\lambda\{(1-\alpha)\xi(0) - \beta\eta(0)\}] \cdot \exp[g'jh],
 \end{aligned}$$

where

$$(13) \quad g' = \lambda\{(1-\alpha)\pi_p - \beta\pi_w\} + \frac{1}{2} \lambda^2\{(1-\alpha)^2\sigma_p^2 + \beta^2\sigma_w^2 - 2\rho\sigma_p\sigma_w\},$$

which may also be written (again after some manipulation):

$$\begin{aligned}
 (13') \quad g' &= (\lambda\beta + 1)\left(\pi_p + \frac{1}{2} \sigma_p^2\right) - \lambda\beta\left(\pi_w + \frac{1}{2} \sigma_w^2\right) \\
 &\quad + \frac{1}{2} (\lambda\beta + 1)(\lambda\beta)(\sigma_p^2 - 2\rho\sigma_p\sigma_w + \sigma_w^2).
 \end{aligned}$$

From (3), (5) and (12), we now have

$$\begin{aligned}
 (14) \quad V^*(0) = V(0; \phi^*) &= E \sum_{j=0}^{n-1} e^{-jrh} E_j F_j(\phi_j^*, \psi^*(\phi_j^*)) \\
 &= \frac{1}{\lambda} \left[\frac{A\alpha^\alpha \beta^\beta}{(1-e^{bk})^\alpha} \left(\frac{e^{(g-r)h} - 1}{g-r}\right)^{1-\beta} \right]^\lambda e^{\lambda[(1-\alpha)\xi(0) - \beta\eta(0)]} \sum_{j=0}^{n-1} e^{(g'-r)jh} \\
 &= \frac{p(0)}{\lambda} \left(\frac{p(0)}{w(0)}\right)^{\lambda\beta} \left[\frac{A\alpha^\alpha \beta^\beta}{(1-e^{bk})^\alpha} \left(\frac{e^{(g-r)h} - 1}{g-r}\right)^{1-\beta} \right]^\lambda \frac{e^{(g'-r)T} - 1}{e^{(g'-r)h} - 1}.
 \end{aligned}$$

The following theorem summarizes the above derivation. In this theorem, $k_j^* = \exp[\phi_j^*]$ comes from (11), $L^*(s) = \exp[\psi^*(s; \{\phi_j^*\})]$ comes from (4) and

$V^*(0)$ comes from (14) by converting the ζ and η processes back to the p and w processes and writing $\hat{w}(s) = w(s)/p(s)$ for the real wage process.

THEOREM 1. Under assumptions (i) and (ii),

$$(a) \quad k_j^* = \left[A\alpha^{1-\beta}\beta^\beta \left(\frac{1 - e^{(g-r)h}}{r-g} \right)^{1-\beta} \left(\frac{1}{1 - e^{bh}} \right)^{1-\beta} \left(\frac{1}{\hat{w}(jh)} \right)^\beta \right]$$

$$(b) \quad L^*(s) = [A\beta k_j^* / \hat{w}(s)]^{\frac{1}{1-\beta}}$$

$$(c) \quad V^*(0) = \frac{p(0)(1 - e^{bh})}{\lambda\alpha} \cdot k_j^*(0) \cdot \left[\frac{1 - e^{(g'-r)T}}{1 - e^{(g'-r)h}} \right]$$

where $b = \pi_p + \frac{1}{2}\sigma_p^2 - r - \delta$, $\lambda = 1/(1 - \alpha - \beta)$ and g and g' are given by (10) and (13), respectively.

The expression for the optimal capital acquisition decision for each future decision point (jh) is a function of all the parameters of the wage and price process (as are the labor demand and value of the firm) which appear in the g and b terms (and g' for the value). Reference to the definition of those terms demonstrates that g (and g') is the expected rate of growth of the real wage raised to a power (determined by the production function parameters) times the price. In the optimal capital quantity, this growth rate is shown minus the discount rate. Given the lognormality of these processes, variances appear in the expected growth rate of each process; thus the first two terms in the definition (10') for g and (13') for g' are the expected rate of price and wage appreciation, respectively. The term b represents expected price appreciation net of the discount rate and depreciation.

The capital decision can be seen to be influenced in two ways by the stochastic processes. The term in g in the expression for $k^*(jh)$ is seen as the future value at the end of period of growth h of a dollar at $g - r$ percent. This represents the expected increase in the profits from production over the interval h due to changes in wages and prices. We call this the "production effect" on the optimal capital stock. The b term, on the other hand, represents the rate of price increase on capital goods net of the rate of depreciation and the discount rate, and represents the "capital ownership effect" on the optimal decision.

Investment in capital goods at discrete intervals is affected by two separate considerations: its effect on production over the decision period and the expected capital gain derived from title to production assets. Capital is qualitatively differentiated from labor in that the ownership of capital introduces an additional source of speculative gain (or loss) which affects the optimal capital decision. This speculative activity of the firm is easiest to see if k is considered to be inventories, but is nonetheless present whatever the nature of the "fixed" asset.

The nominal market value of the firm given by Theorem 1 is seen to represent the value of the opportunity for entrepreneurs to acquire the optimal capital stock

$((\lambda\alpha)^{-1}$ is the share of residual rents as a proportion of capital's share of income) multiplied by the current dollar value of the capital and the term in g' . The g' term is the ratio of the present values of two streams of income of length T (in the numerator) and h (in the denominator), discounted at the rate $g' - r$. This term can be interpreted as a multiplier reflecting the value in terms of rents to capital ownership from future investment opportunities at time intervals of length h for T periods into the future.

3. PROPERTIES OF OPTIMAL CAPITAL INVESTMENT

Two central issues are of interest to us in discussing the demand for capital. The first is the way price and wage variability affect the demand for capital goods. The second issue is the way in which the length of the decision-making interval, h , and the terminal time, T , interact with wage and price variability to influence capital spending decisions.

In discussing the role of variability in the wage and price process, it is necessary to appreciate that, as mentioned above, the variances of the lognormally distributed future wage and price levels affect the expected future values of those variables. For this reason, equations (10') and (13') factor out the role of the variance of prices and wages on expected future levels of those variables and include that influence in the first two terms of those expressions. The third term of g and g' represents the effect of the covariability of the wage and price process on the prospects for the firm. In the case of perfect covariability of wages and prices ($\rho = +1$), the last term of g and g' drop out.

In general, imperfect correlation between wages and prices ($\rho < 1$) means that the real wage will vary with the stochastic wage and price changes. Since management may adjust the use of labor instantaneously, it can soften the blow of adverse real wage investments. Conversely, it may exploit any deterioration of the real wage. Because of the convexity of the profit function (due to the rents associated with decreasing returns to scale), variability in the real wage raises the expected growth rate of the firm's rents. This has the effect of increasing the firm's demand for capital with wage and price uncertainty.

We may contrast two inflationary economies with the same expected price and wage level growth rates but different variances. This would be the case where policy makers in the higher variance economy lower their instantaneous growth rate targets associated with monetary policy to account for the effect of their economy's higher variability on expected future price and wage level changes. The existence of the third term in g and g' would still assure a higher level of demand for capital in that economy, everything else being equal. Thus variability has a unique role in raising the demand for capital goods.

In the following analysis, we will consider the impact of increased variability represented by the third term in equations (10') and (13'), which we call $\bar{\sigma}^2$, on the demand for capital goods. We ignore the effects of higher variances on expected future price and wage levels, assuming that the increased variability's

effect on expected growth rates is offset by lower instantaneous growth rates of wages and prices. We do this in order to isolate the effects of wage and price variability, as opposed to inflation, on demands. (However, even when $\rho=1$, whenever $(\sigma_p - \beta\sigma_w)$ increases, g and g' increase.) If larger $\bar{\sigma}^2$ is associated with an increase in that expression, preserving the mean rate of wage and price inflation is not necessary to our results concerning the effect of variances on capital demands.

To analyze the effect of variability on capital demands, we compute the elasticity of the demand for capital with respect to $\bar{\sigma}^2$. This is given by:

$$(15) \quad E_{k_j^*} \bar{\sigma}^2 = \frac{\partial k_j^*}{\partial \bar{\sigma}^2} \frac{\bar{\sigma}^2}{k_j^*} = \frac{1}{2} \bar{\sigma}^2 \frac{\lambda\beta}{1-\beta} \left(\frac{1}{r-g} - \frac{h}{e^{(r-g)h} - 1} \right) > 0.$$

The implication of equation (15) is that the higher degree of variability represented by $\bar{\sigma}^2$, the larger the demand for capital. Furthermore, the greater $\bar{\sigma}^2$, the more sensitive (in percentage terms) is the demand for capital.

We are interested in the effect of the decision making interval, h , on the sensitivity of the capital demand to the variability in the wage and price process. The fact that capital can be adjusted from previous levels only after time intervals of length h is a key attribute of capital relative to labor. The fact that h is fixed can be interpreted as the fact that more frequent adjustments are prohibitively costly. The elasticity of the demand for capital with respect to variability is increased by larger h , since:

$$(16) \quad \frac{\partial E_{k_j^*} \bar{\sigma}^2}{\partial h} = \frac{1}{2} \frac{\lambda\beta}{1-\beta} \left[\frac{1 - e^{(r-g)h}(1-h(r-g))}{(e^{(r-g)h} - 1)^2} \right] > 0.$$

This means that the longer the interval between capital acquisition decisions, the more sensitive is the demand for capital to the variability in the system. Thus, while it can be shown that h has an ambiguous effect on the optimum capital decision (and $V^*(0)$), economies with larger variability and longer decision making intervals will be characterized by larger demands for capital.

At the maximum, h can be equal to T . In the case where the firm has an infinite horizon, but can only make an initial capital decision (i.e., $h=T=\infty$), the expressions in Theorem 1 become particularly simple. For example, k^* becomes:

$$(17) \quad k_j^* = \left[A\alpha^{1-\beta}\beta^\beta \left(\frac{1}{r-g} \right)^{1-\beta} \left(\frac{1}{\hat{w}(0)} \right)^\beta \right]^\lambda, \quad h = T = \infty$$

which can hold only if $r-g>0$. This is the case reported in Dietrich and Heckerman [1980]. The g' and b terms do not appear in this solution, since capital is never sold and never adjusted.

In summary, the effect of variability is to increase the demand for capital. Furthermore, the effect on this demand of variability is greater, the longer the period over which each capital acquisition decision obtains.

4. VARIABILITY AND THE DEMAND FOR CAPITAL IN THE CONTINUOUS CASE

To examine the impact of wage and price variability on the demand for capital when there are no limitations on adjustment, we take the limits of the optimum values for the discrete model given in Theorem 1 as h , the decision interval, goes to zero. We must then show that this limiting value is equivalent to the optimization of a continuous model. The first step is straightforward, where we can see, given the continuity of the wage and price functions, that:

$$\begin{aligned}
 (18) \quad (a) \quad & \lim_{h \rightarrow 0} k_t^* = A^\lambda \alpha^{\lambda(1-\beta)} \beta^{\lambda\beta} \left(\frac{1}{\hat{w}(t)} \right)^{\lambda\beta} \left(-\frac{1}{b} \right)^{\lambda(1-\beta)} \\
 (b) \quad & \lim_{h \rightarrow 0} L^*(t) = \left[A\beta \frac{k_t^*}{\hat{w}(t)} \right]^{\frac{1}{1-\beta}} \\
 (c) \quad & \lim_{h \rightarrow 0} V^*(0) = \frac{p(0)}{\lambda\alpha} k_0^* \frac{e^{(g'-r)T} - 1}{(-b)^{\lambda\alpha}(r-g')}
 \end{aligned}$$

where use of l'Hopital's rule has been made in evaluating the limits.

We must show that the limiting values of the optimal values from the discrete model represent the optimal values from a continuous time model. In continuous time, the market value of the firm can be written:

$$V(0) = E \left[\int_0^T e^{-rs} F(k(s), L(s), p(s), w(s)) ds \right]$$

and the problem is to determine the optimal paths $\{k^*(s), L^*(s)\}$ to maximize this expectation. Define a new non-anticipative capital function

$$k_n(s) = \bar{k}(jh) \geq 0, \quad jh \leq s < (j+1)h,$$

where the interval $[0, T]$ has been subdivided as in the discrete case and $k_n(s)$ converges to $k(s)$ as the interval length, $h = T/n$, goes to zero. Recall the discrete multiperiod model,

$$V_h(0) = E \left[\sum_{j=0}^{n-1} e^{-jrt} F_j(k_j, L_j) \right],$$

where the subscript h now connotes that capital acquisition decisions are made at discrete time intervals of length h . $V_h(0)$ can be rewritten (using the notation from the first section)

$$\begin{aligned}
 V_h(0) = E \left\{ \sum_{j=1}^{n-1} e^{-rh} p(jh) e^{-\delta jh} [e^{\delta jh} k_j - e^{\delta(j-1)h} k_{j-1}] \right. \\
 + \int_{0^+}^T e^{-\delta s} p(s) d[e^{\delta s} k(s)] - \int_0^T e^{-\delta s} p(s) d[e^{\delta s} k(s)] \\
 \left. + e^{-rT-\delta h} p(T)k(T) - p(0)k(0) + \int_0^T e^{-rs} R(p(s), w(s), k(s), L(s)) ds \right\}
 \end{aligned}$$

$$+ \sum_{j=0}^{n-1} \left[\int_{jh}^{(j+1)h} e^{-rs} R(p(s), w(s), k_n(s), L(s)) - R(p(s), w(s), k(s), L(s)) ds \right] .$$

Now, as $n \rightarrow \infty$, i.e., $h \rightarrow 0$, the first three terms of the above become:

$$- \int_0^T e^{-\delta s} p(s) d[e^{\delta s} k(s)] + e^{-rT} p(T) k(T) - p(0) k(0)$$

and the last two terms become:

$$\int_0^T e^{-rs} R(p(s), w(s), k(s), L(s)) ds .$$

Hence the continuous decision model is:

$$V(0) = E \left[- \int_0^T e^{-\delta s} p(s) d[e^{\delta s} k(s)] + \int_0^T e^{-rs} R(s) ds + e^{-rT} p(T) k(T) - p(0) k(0) \right] .$$

Since $V(0)$ will reduce to $V_h(0)$ if $k(s)$ is replaced by $k_n(s)$, we can compare the optimized values of $V(0)$, $V_h(0)$ by comparing the sets of allowable controls, C . If $C_n = \{k, L\}$ is allowable for the multiperiod model with equal sized subintervals and $C = \{k, L\}$ is allowable for the continuous decision model, then $C_n \subset C$ for all n and $V_h^*(0) \leq V^*(0)$. Moreover, $C_n \subset C_{2n}$ for all n so that $V_h^*(0) \leq V_{n/2}^*(0)$. Now suppose that k^* and L^* are the choices of k and L which optimize $V(0)$. Then for each n there is a \hat{k}_n, \hat{L}_n such that $\hat{k}_n \rightarrow k^*, \hat{L}_n \rightarrow L^*$ and $\{\hat{k}_n, \hat{L}_n\} \in C_n$. Moreover, $V_h(0)$ evaluated using \hat{k}_n and \hat{L}_n will converge to $V^*(0)$. However, $V_h(0)$ evaluated using \hat{k}_n and \hat{L}_n is less than or equal to $V_h^*(0)$. Hence,

$$\lim_{\substack{n \rightarrow \infty \\ h \rightarrow 0}} V_h^*(0) = V^*(0) .$$

We have proven the following:

THEOREM 2. *The optimal values for capital, labor, and the market value for a firm under the assumptions of Theorem 1 in continuous time are given by equations (18a), (18b) and (18c).*

In contrasting the continuous and discrete period solutions, the most remarkable, if intuitively appealing, result is that the variability parameter, $\bar{\sigma}^2$, does not appear in the capital acquisition or labor demand functions. If the firm can make and implement investment plans instantaneously, expected stochastic changes in the real wage do not influence its decision making. The production effect on optimal capital is captured completely by the instantaneous real wage. A larger degree of variability in the wage and price process have no effect on the instantaneous demand for capital when the firm can adjust instantaneously.

The capital ownership effect remains in the continuous time model in the term in b . This term represents the typical net cost of capital term which appears

in capital demand models derived in continuous time, such as Jorgensen [1967]. In the continuous time case, however, the critical role played by the necessary capital holding period, namely h in the discrete time model, is not discernible. In contrasting the role of b in the discrete and continuous models, it can be seen that the ownership effect may be larger or smaller than the continuous case, depending on h . As stated above, the overall effect of h in the discrete case on k_j^* is not signable, however.

5. IMPLICATIONS

The role of variability on the demand for capital and the value of the firm has been shown to be due to the variability in the real wage, represented by $\bar{\sigma}^2$. The surprising conclusion is that variability in the real wage, under our assumptions that wages and prices follow a bivariate log Brownian motion, increases the demand for capital, essentially by increasing the expected growth rate of real and nominal profits. However, the degree to which this variability affects the capital acquisition decision is affected by the length of the decision making interval, and is not a factor in the continuous case.

Our results obtain for an economy where decision makers take the parameters of the stochastic wage and price processes as given. Comparisons between economies do not really provide implications for policy. Thus we are not saying that the government should seek to raise or lower the variability of the price and wage generating process. We only emphasize that variability enters the capital acquisition decision in a positive way, and thus uncertainty may work to raise the demand for investment capital.

Several interesting implications for further research are suggested by the above. First, one obvious question to be explored is how robust these results are to alternative assumptions concerning the stochastic wage and price process, the production function, and the cost of capital. A second question is how the optimal decisions change when the parameters of the stochastic process are not known with certainty, but are rather estimated by decision makers from observed realized values. In other words, how is the system modified by investors who are learning? Another obvious extension of the analysis is to make the decision making interval a variable to solve for explicitly by associating some cost with capital adjustments.

While this list does not exhaust the interesting extensions of this problem, it does highlight some limitations of investment analysis derived under the assumption of continuity. Capital is only differentiated from labor in a qualitative way in the discrete time case. In the discrete case, many factors are seen to affect the demand for capital which have heretofore not been considered.

University of California at Davis, U. S. A.
University of Southern California, U. S. A.
University of Arizona, U. S. A.
University of Arizona, U. S. A.

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