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Discussion

DONALD E. MYERS

Department of Mathematics, University of Arizona, Tucson, AZ 85721

1. Introduction

Bayesian methods in general incorporate ''prior knowledge''. Most often this prior knowledge is given in the form of a *prior* distribution which is then used to obtain a *posteriori* distribution. Consider the problem of estimating/predicting a value at a non-data location *given* data at several locations. If data is given at only a finite number of locations then the problem is ill-posed and does not have a unique solution, with no knowledge of the phenomenon or of a functional relationship between the data and the value to be predicted or of a functional relationship between data values and location coordinates, any method for predicting is equally good or reliable. The key question then is which ''prior'' information will be incorporated and how it will be incorporated. The various kriging estimators all depend strongly on the use of a variogram or covariance which must be modeled or estimated from the data, the modeling of the variogram and the derivation of the systems of equations that determine the weights in the kriging estimator all depend on certain assumptions such as the use of a random function model and an appropriate form of stationarity, i.e., ''priors''. Of course these are not given in the form an *a priori distribution*. As pointed out by the authors, the kriging variance quantifies the uncertainty associated with the kriging estimator in a certain sense but it does not incorporate the uncertainty associated with the choice of the ''priors''. In particular any valid variogram model will result in a unique solution for the kriging equations irrespective of whether the sample variogram looks anything like the model used or even how the model was selected.

The authors, like several others, have used Bayesian methods to attempt to quantify or incorporate the uncertainty associated with modeling the variogram/covariance. However one should note that a multi-variate gaussian distribution is essential in the analysis and the analysis does not incorporate any of the uncertainty associated with this assumption, one is just substituting one form of uncertainty for another. Since in general these assumptions are non-testable, the choice of the assumptions may be simply what works and what seems reasonable.

2. Implementation

In contrast to more classical statistical tools, geostatistics developed almost at the same time as modern computing and it might be argued that without access to rapid, convenient computing it would have been only of abstract interest. As yet most commercial statistics packages do not incorporate geostatistical algorithms or do it rather poorly, commercial

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geostatistics packages such as ISATIS and its predecessor, BLUEPACK, are too expensive for ordinary users but beginning with basic codes included in Journel and Huijbrechts (1978), the USGS package STATPACK, the EPA public domain package GEOEAS and the library of FORTRAN codes in GSLIB as well as codes published in Computers and Geosciences, the use of geostatistics spread rapidly through the application fields. Most applications and most of the new developments have come from non-statisticians. Basic programs to compute sample variograms and to apply kriging require nothing more complicated than solving a system of linear equations and sorting routines (data handling is the most complicated part of such programs). The authors do not discuss the software used in their numerical application, if Bayesian methods such as given in this paper are to be widely used then implementation in easy to use software is essential, perhaps as a new module for GEOEAS, as an add-on for GRASS, a new addition to the GSLIB library, even a module in IDL or a proc in SAS.

Whether we like it or not, geostatistics is used primarily by non-statisticians. The inclusion of reasonable default choices in such software would be highly desirable, e.g., see the GEOEAS package. ''Help'' in the form of generic information useful in interpreting intermediate computations would also be desirable. For example, even if the user does not intend to use one of the variants of kriging, the ''range'' of a variogram is a readily interpretable parameter, i.e., the range of spatial dependence and can sometimes be related to other known properties of the phenomenon.

3. Some details

The authors note that polynomial functions of the position coordinates are commonly used to model the non-constant mean of the random function, as contrasted with the categorical variables used in the external drift in the numerical example. While there are analogies with the use of polynomials in Trend Surfaces, the real reason is the invariance under translation, see Matheron (1973). However, there is a more general reason, a sufficient condition for the invertibility of the kriging matrix (universal kriging or kriging with external drift) is that the structure function (variogram, covariance, generalized covariance) be conditionally negative definite with respect to the set of drift functions and that the drift functions be linearly independent, see Myers (1988, 1992). As a special case consider the completely monotonic functions of order k and their relationship to polynomials of a given order. The condition on the drift functions is more complicated in the case of space-time applications, see Dimitrakopoulos and Luo (1997).

4. Deterministic or stochastic?

It is well-known that the universal kriging estimator can be re-written in a deterministic form in which case the connection with interpolation with radial basis functions becomes apparent, see Myers (1987, 1992). The properties of the latter interpolators have been derived in a functional analysis context hence the use of probabilistic assumptions may be an artifice.

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