

Negative Propagating Light

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Abstract

New materials and techniques have presently enabled scientists to significantly alter the speed of light. Recently light has been recorded traveling faster than the speed of light, at so called superluminal velocities. It has also been drastically slowed down to a fraction of its original speed. Under extreme circumstances, light can also be seen to propagate backward as time elapses. Robert Boyd is one of the leading researchers of backward propagating light.

Optics Overview

There are at least two velocities associated with the propagation of light, the group velocity and the phase velocity. Imagine waves emanating outward from a rock thrown in a lake. The collection of waves as a whole represents the group velocity (v_g). The peaks of individual waves represent the phase velocity. Each wave appears to be moving faster than group. The group velocity is very important because it “defines the speed with which energy or information is propagated¹”. Under average circumstances both the group and phase velocities of light will be positive and propagate in the forward direction. However, new materials and techniques have made it possible to have the phase and group velocities travel in opposite directions. The latest science is now able to increase the group velocity to greater than the speed of light ($v_g > c$) and to make light travel backwards.

Backward Propagating Light

Light behaves as an electromagnetic wave that oscillates as it propagates forward. The wave motion of light implies that it has a frequency and wavelength associated with it that contributes to its speed. There are two types of frequency, linear (f) and angular (ω). Linear and angular frequency vary by a constant factor of 2π such that $\omega = 2\pi f$. In a vacuum, the speed of light (c) is equal to the frequency (f) multiplied by the wavelength

(λ) so that $c = f\lambda$. The phase velocity (v) equals $\frac{c}{n}$, where n is the frequency dependant refractive index. Therefore distinct frequencies will be refracted differently according to Snell's Law² $n_1\sin(\theta_1) = n_2\sin(\theta_2)$. This phenomenon is known as dispersion. As illustrated by Graph 1, normal dispersion occurs when the refractive index of the material decreases as the wavelength increases. Anomalous dispersion occurs when the refractive index increases with wavelength or mathematically when $\frac{dn}{d\lambda} > 0$. There is abnormally high absorption in regions of anomalous dispersion as illustrated by Figure 1.

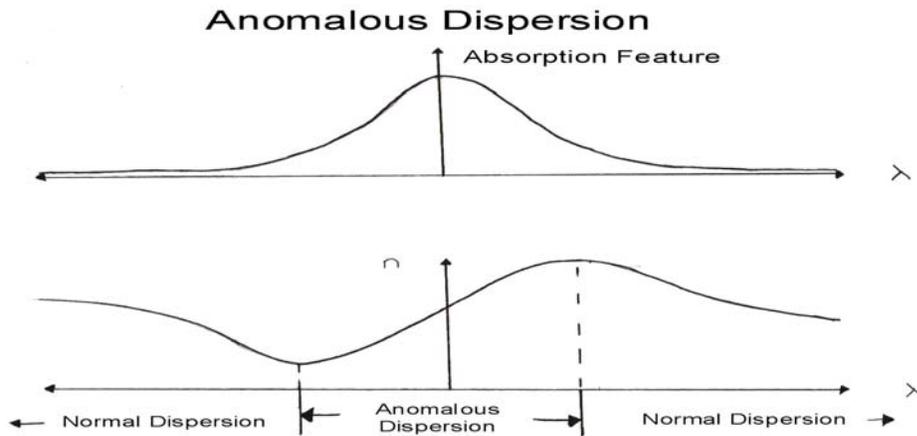


Figure 1: Top graph: Strength of absorption versus wavelength.

Bottom graph: Refractive index versus wavelength with the regions of anomalous and normal dispersion labeled.

Anomalous dispersion is a fundamental part of achieving backward propagating light. Anomalous dispersion can be used to produce group and phase speeds that are in different directions³. The group velocity⁴ is defined by

$$v_g = \frac{c}{n + \omega \frac{dn}{d\omega}} \quad (1)$$

Materials that exhibit large anomalous dispersion allow the group velocity of the light to exceed c and/or become negative⁵. This is because in the anomalous region $\frac{dn}{d\omega}$ is negative and will therefore contribute a negative value to the velocity of the group.

Equation 1 predicts backward propagating light for $\omega \frac{dn}{d\omega}$ values larger than n because the denominator will be negative. When $\omega \frac{dn}{d\omega}$ is greater than n and $n - \omega \frac{dn}{d\omega}$ is less than 1 the denominator will become negative and less than one. This will result in a negative group velocity at superluminal speeds. Anomalous dispersion happens in areas of rapid spectral variation with respect to the refractive index. Therefore, negative values of the group velocity will occur in these areas.

As a result of the Kramers-Kronig relationships⁶ these regions of rapid spectral variation are also in the vicinity of “a narrow absorption feature or a narrow dip in a gain feature⁷”. Near the absorption peaks, the refractive index of a material takes on a complex form:

$$n' = n - ik \quad (2)$$

In Equation 2 k represents the extinction coefficient and n is the real part of the refractive index⁸. The imaginary part of the refractive index, $\text{Im}(n)$, represents the absorption strength of the medium. Kramers-Kronig relations are important because they allow the real and imaginary parts of the refractive index to be evaluated. They also allow the refractive index to be determined by measuring the absorption of a material. The Kramers-Kronig relationships for the case of backward propagating light are⁶:

$$\text{Re}n(\omega) + 1 = \frac{2P}{\pi} \int_0^{\infty} \frac{\omega' \text{Im}n(\omega')}{\omega'^2 - \omega^2} d\omega' \quad (3)$$

$$\text{Im}n(\omega) = \frac{-2P\omega}{\pi} \int_0^{\infty} \frac{\text{Re}n(\omega') + 1}{\omega'^2 - \omega^2} d\omega' \quad (4)$$

Professor Boyd’s team decided to create a decline in the gain curve rather than look at a narrow absorption area. The gain (g) is related to the refractive index and the group index through a series of equations⁷.

$$g(\delta) = \frac{g_o}{1 + I_o} \left[1 - \frac{I_o(1 + I_o)}{(T_1\delta)^2 + (1 + I_o)^2} \right] \quad (5)$$

Where g_o = initial gain coefficient; δ = the difference between the frequency (ω) of the measured gain value and the frequency (ω_o) of a strong wave with intensity (I); $I_o = I/I_{\text{sat}}$; T_1 = relaxation time of the population inversion⁷. The population inversion

refers to an optical fiber with more electrons in an excited state than the ground state due to externally applied energy. The relaxation time is then the time for the electrons to be restored to a normal ground state. The refractive index will then vary spectrally by:

$$n(\delta) = n_{host} - \frac{g_o c T_1 I_o}{2\omega_1 (1 + I_o)} \left[\frac{\delta}{(T_1 \delta)^2 + (1 + I_o)^2} \right] \quad (6)$$

For n_{host} = refractive index of the fiber in use. The index of the group can be simplified by evaluating it at $\delta = 0$ so that group refractive index is given by ⁷.

$$n_g = n_{host} - \frac{g_o c T_1 I_o}{2 (1 + I_o)^3} \quad (7)$$

Using the set-up from Figure 2, the direction that light is traveling can be determined. The first part of the experiment determines a reference value for the output waveform of the 1550 nm pulsed laser. The 980 nm laser is used to establish gain in the E.D.O.F. (erbium doped optical fiber) which amplifies the signal of the system. The length of the E.D.O.F is repeatedly reduced and measurements are taken. The direction of propagation of the wave can then be determined by analyzing the output waveform.

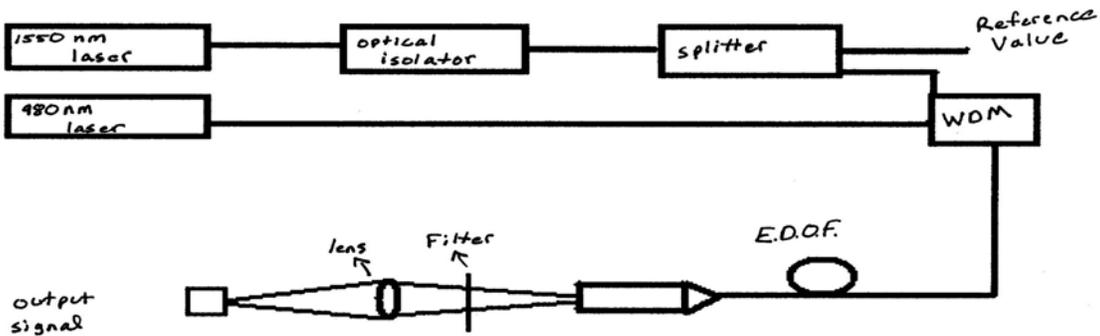


Figure 2: An illustration of a possible setup for monitoring the propagation of light in a fiber.

For a fiber with a relaxation time of 10 ms, $g_o = 1/m$ and $I_o = .003$ the group velocity will equal -4465.88. This value is very close to what was recorded in the laboratory making slightly different assumptions about the gain, relaxation time, and intensity. The negative group index value shows that, theoretically, light can propagate in the negative direction. These results have been duplicated in the laboratory⁷.

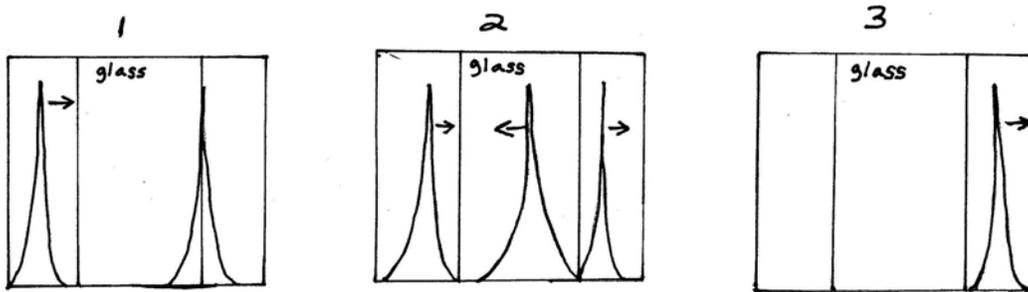


Figure 3: The motion of a wave as it approaches (1); propagates through (2); and is transmitted (3) by a piece of glass.

Figure 3 illustrates an observed example of backward propagating light. As the wave approaches a piece of glass a pulse forms at the opposite end of the glass (1). This new pulse splits into two different waves that travel in opposite directions (2). The backward traveling half races toward the incident light and cancels it out. The forward moving part continues propagating away from the medium as transmission (3). The group velocity of the wave in the piece of glass is negative. The peak of the transmitted pulse exits the medium prior to the peak of the incident light entering.

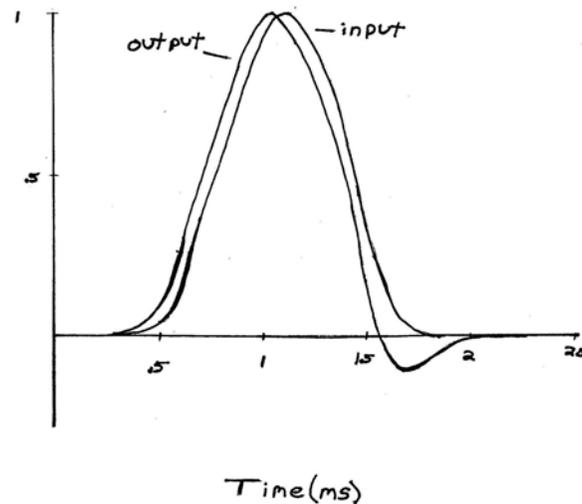


Figure 4: Normalized input and output waveform after propagation through an erbium-doped fiber.

Inspection of Graph 4 confirms the output wave is advanced in time with respect to the input wave. The output wave suffers from slight deformation around its leading edge. This is represented by the negative waveform values at its leading edge. Although the wave propagates backwards, the energy flow, represented by the Poynting Vector, is

positive for all combinations of phase and group velocities. The Poynting Vector points in the direction of energy flow⁹ and it always points in the forward direction³. Therefore information can only travel in the forward direction with respect to time.

Biography

Professor Boyd attended the Massachusetts Institute of Technology and in 1969 graduated with a bachelor's degree in Physics. He then attended the University of California at Berkeley and in 1977 received his Ph.D. in Physics for works involving the use of non-linear optical systems for infrared detection for astronomy. Later that year, he was hired by The Institute of Optics at the University of Rochester where he has worked ever since. In 1987 he was appointed to the Professor of Optics position. He has written two books and over 200 research papers in addition to being awarded five patents. He is a member of the Optical Society of America and the American Physical Society.

Conclusion

Robert Boyd's groundbreaking research in backward moving light is on the leading edge of technological discoveries. Light moving backwards in time is an outcome that has been predicted mathematically⁴ that has recently been observed in laboratory conditions. The negative group velocity of light can be calculated using already known equations for group velocity. Furthermore, experimentally determined values of the propagation of light agree with those predicted mathematically. Normal materials are not sufficient to produce such striking results. In order to achieve such abnormal effects materials with high gain are being used. Anomalous dispersion allows for a negative contribution to the group velocity. When this value becomes large enough it can dominate the equation for group velocity and make it negative. Despite the negative group velocity the flow of information is always in the forward direction.

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