Dynamic Cournot Oligopoly with Output Adjustment Cost

by
Chrystie T. Burr

under
Dr. Ferenc Szidarovszky

Agenda

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2. Intro – Variant: Output Adjustment Cost
3. Best Responses
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Classical Cournot Oligopoly

1 of 4

- Researches into the Mathematical Principles of Wealth (1838)
- Several firms produce a homogeneous product
- Outputs are chosen simultaneously by firms
- The goal is to choose the output quantity that will maximize its own profit

Let:
- Firm 1 produces output $x_1$ and $C_1(x_1)$ is the cost
- Firm 2 produces output $x_2$ and $C_2(x_2)$ is the cost
- Price is $P(x_1 + x_2)$
- Profit of firm 1 is $P(x_1 + x_2)x_1 - C_1(x_1)$
- Profit of firm 2 is $P(x_1 + x_2)x_2 - C_2(x_2)$
Classical Cournot Duopoly

- Cournot equilibrium = Nash equilibrium

\[ \Pi_1(x_1^*, x_2^*) \geq \Pi_1(x_1, x_2^*), \forall x_1 \in X \]
\[ \Pi_2(x_1^*, x_2^*) \geq \Pi_1(x_1^*, x_2), \forall x_2 \in X \]

Classical Cournot Oligopoly

- Finding Nash equilibrium: use best response functions

![Diagram 1](image1.png)
![Diagram 2](image2.png)
Classical Cournot Oligopoly

Finding Nash equilibrium: use best response functions

Variant: With Output Adjustment Cost

- When the optimal output increases from one time period to the next, additional cost is required.
- This is called output adjustment cost
- Denoted by $K$
Output Adjustment Cost

- Two components: marginal cost and fixed cost

\[ K = \omega(x^t - x^{t-1}) + \tau \]

- Case 1: Continuous OAC \((\tau = 0)\)
  
  \[ \text{Profit} = x(A - Bs) - \alpha - \beta x - \omega(x^t - x^{t-1}) \]

- Case 2: Discontinuous OAC \((\tau > 0)\)
  
  \[ \text{Profit} = x(A - Bs) - \alpha - \beta x - \omega(x^t - x^{t-1}) - \tau \]

Best Response

with Continuous Output Adjustment Cost

\[ \Pi_k = -Bx_k^2 + (A - Bs_k - \beta_k)x_k - \alpha_k \]

\[ \Pi_k = -Bx_k^2 + (A - Bs_k - \beta_k - \omega_k)x_k - \alpha_k - \omega x_k^{t-1} \]
Best Response
with Continuous Output Adjustment Cost

\[ \Pi_k = -Bx_k^2 + (A - B_{sk} - \beta_k)x_k - \alpha_k \]

Best Response
with Discontinuous Output Adjustment Cost

\[ \Pi_k = -Bx_k^2 + (A - B_{sk} - \beta_k - \omega_k)x_k - \alpha_k - \omega_kx_{k-1} - \tau_k \]
Best Response
with Discontinuous Output Adjustment Cost

Case 1: \( L_k - x_{k+1} \frac{\tau_k}{V} > \frac{\tau_k^2}{B} \)

Case 2: \( L_k - x_{k+1} < \frac{\tau_k^2}{B} \)
Equilibrium Condition
with Continuous Output Adjustment Cost

Equilibrium is reached iff for all \( k \), \( x_k^* \) is the best response of firm \( k \), that is

\[
\frac{\partial \Pi_k}{\partial x_k} \bigg|_{x_k=x_k^*+0} \leq 0 \leq \frac{\partial \Pi_k}{\partial x_k} \bigg|_{x_k=x_k^*-0}
\]

Ex. Given \( A = 20, B = 1, L_1 = L_2 = 10, \beta_1 = \beta_2 = \omega_1 = \omega_2 = 5 \), the equilibrium condition becomes

\[
10 - (x_1 + x_2) \leq x_{1,2} \leq 15 - (x_1 + x_2)
\]
Equilibrium Condition
with Discontinuous Output Adjustment Cost

- Very difficult to study the equilibrium cond.-
  1. Infinitely many equilibria
  2. The discontinuity exemplified by the jump in the production level – profit function.

- Therefore the use of computer simulation is needed.

Simulation

- Windows deployable program
- Based on Matt Dabkowski’s *Duopoly Basin Calculator*
  - Iterate all possible initial production levels to find the corresponding equilibria
  - The number of steps taken to reach equilibrium is then categorized by colors.
Algorithm

Initialize
for \( y_0 = y_{\text{min}}; y_0 \leq y_{\text{max}}; y_0 = y_0 + \text{stepsize} \) {
    for \( x_0 = x_{\text{min}}; x_0 \leq x_{\text{max}}; x_0 = x_0 + \text{stepsize} \) {
        Then Determine whether firm x changes its production level in the next time period by comparing the profits generated by the vertices of the two parabolas.
        If converged, stop.
        If not, update. Count ++
    }
}

Simulation Results
Simulation Results

$A = 20, B = 5, L_1 = L_2 = 10, \beta_1 = \beta_2 = \omega_1 = \omega_2 = 1, \tau_1 = \tau_2 = 1$

Sensitivity Analysis

- **Stage 1**: No jump is assumed while the parameters $A, B, \omega_x = \omega_y$ and $\beta_x = \beta_y$ are systematically varied.

- **Stage 2**: Discontinuity in the additional cost function is applied. Only $\tau$ varies.
Sensitivity Analysis (Stage 1)

A = 20

Sensitivity Analysis (Stage 1)

A = 25
Sensitivity Analysis (Stage 1)

$B = 0.3$

Sensitivity Analysis (Stage 1)

$B = 0.5$
Sensitivity Analysis (Stage 1)

$B = 0.7$

Sensitivity Analysis (Stage 1)

$B = 0.9$
Sensitivity Analysis (Stage 1)

$\beta = 3$

Sensitivity Analysis (Stage 1)

$\beta = 5$
Sensitivity Analysis (Stage 1)

$\beta = 7$

Sensitivity Analysis (Stage 1)

$\beta = 9$
Sensitivity Analysis (Stage 1)

\[ L = 4 \]

Sensitivity Analysis (Stage 1)

\[ L = 6 \]
Sensitivity Analysis (Stage 1)

\[ L = 8 \]

\[ \omega = 0.00001 \]
Sensitivity Analysis (Stage 1)

$\omega = 0.001$

Sensitivity Analysis (Stage 1)

$\omega = 1$
Sensitivity Analysis (Stage 1)

$\omega = 1$

Sensitivity Analysis (Stage 1)

$\omega = 5$
Sensitivity Analysis (Stage 2)

τ = 1.5

Sensitivity Analysis (Stage 2)

τ = 1.9
Summary

- $N$-firm single product oligopolies with production adjustment cost were examined.
- The best response function is always decreasing, not necessarily continuous, and might have two different values.
- This leads to infinitely many equilibria
- Simulation study shows the sensitivity and convergence rate of the equilibrium set.

Q & A