Characterization of the micro-scale surface roughness effect on immiscible fluids and interfacial areas in porous media using the measurements of interfacial partitioning tracer tests

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\begin{abstract}
This study presents a model-based methodology to characterize the surface roughness effect on immiscible fluids in porous media using the measurements obtained with the gas-phase interfacial partitioning tracer test (IPTT). The characterization approach captures how adsorbed wetting film configuration on grain surfaces influences fluid-fluid interfaces in unsaturated porous media. The method establishes a novel representation of surface and interface roughness that delineates the micro-scale fractal nature of grain surfaces and the fluid-surface interactions at these scales. The method was tested using reported experimental data for several soils. The results showed that the methodology was effective for natural porous media comprising a range of physical and geochemical properties. Comparisons between characterized parameters of different media revealed that micro-scale surface roughness was only partially correlated to soil texture properties. Images of the test media obtained with scanning electron microscopy (SEM) illustrate the complexity of micro-scale surface roughness, and its variability among different media. Tests with an organic liquid–water system validated the generalness of surface roughness properties generated by the model. The proposed methodology is anticipated to provide a means to characterize and quantify the effects of surface roughness on fluid-solid interaction and fluid-fluid interfacial area, which are critical to various environmental disciplines.
\end{abstract}

1. Introduction

The interactions between immiscible fluids and rough solid surfaces play a significant role in various soil and hydraulic properties and in mediating mass and energy transfer. It has been shown that surface roughness has significant impacts on the wettability of solid surfaces (e.g., Wenzel, 1936; Cassie and Baxter, 1944; Hirasaki, 1991; Oliver et al., 1980), which is closely associated with the configuration of fluids in soil. It has also been demonstrated that surface roughness can influence the resistance of flow over fractured surfaces in unsaturated media (e.g., Brown, 1987; Lampurlanés and Cantero-Martínez, 2006; Or and Tuller, 2000; Thompson and Brown, 1991; Tokunaga and Wan, 1997), which is a critical factor for the hydraulic conductivity of a medium. Wetting-fluid configuration on rough surfaces can also influence both film interfacial area and film thickness (e.g., Kim et al., 2012; Tokunaga, 2011; Zheng et al., 2015; Jiang et al., 2020). Thus, potential relationships between surface roughness and other fluid and flow properties are of interest for numerous applications.

The complex nature of surface roughness poses challenges to develop a universal fluid-surface function for the great variety of natural porous media. Typically, surface roughness can be defined from three different viewpoints (e.g., Ghanbarian et al., 2016; Wenzel, 1936), each of which represents a specific principle for the interactions with fluids. The three approaches are conceptualized in the left column of Fig. 1.

The traditional definition of surface roughness is based on the average value of measured vertical coordinates compared to a reference level (relative surface height), e.g., arithmetic roughness ($R_a$) or root-mean-square roughness ($R_{rms}$). However, the relative surface height definition was found insufficient in practice due to the lack of consideration of fractal topology (Brown, 1987, 1989; Ghanbarian et al., 2016). The second definition of surface roughness originates from the basic concept of fractal geometry (Mandelbrot, 1983), in which either the pore surfaces (2D) or the entire porous medium (3D) are modified with corrections of fractal dimension, resulting in higher dimensions for surfaces (Brown, 1987; Ghanbarian et al., 2016) or lower dimensions for entire media (Crawford, 1994; Giménez et al., 1997; Toledo et al., 1990). It is...
noted that a related characterization is sometimes employed, the Hurst roughness exponent. This term is related to the fractal dimension, and ranges from 0 to 1. The third definition was proposed by Wenzel (1936), in which surface roughness was defined as an area factor, e.g., the ratio of “actual surface area” over “geometric smooth-surface area”. Some studies have used this definition to modify flow equations or evaluate fluid-fluid interfacial area (e.g., Philip, 1978; Liu et al., 2014; Zheng et al., 2015; Jiang et al., 2020).

Another challenge for an explicit fluid-surface function is the method of characterization of surface roughness, especially for the inner pore surfaces of natural porous media. Most reports of grain surface roughness based on relative surface height are obtained by direct measurements, such as laser profilometry and atomic force microscopy (AFM) (e.g., Tokunaga et al., 2003; Kim et al., 2012), or photogrammetric reconstruction of images obtained from scanning electron microscopy (SEM) (e.g., Kibby, 2013). However, these methods have very small fields of view (e.g., less than 100 μm in one dimension in AFM), which brings about uncertainties at larger scales. An alternative approach is to characterize surface roughness from soil hydraulic measurements, thereby capturing larger-scale effects. For instance, fractal-dimension definitions (Ghanbarian et al., 2016; Pachepsky et al., 1995; Perfect, 1999) and area-factor definitions (Or and Tuller, 2000; Zheng et al., 2015) of surface roughness have been reported in modeling works that were extracted from measurements of soil-water characteristic (SWC) curves or unsaturated hydraulic conductivity. A limitation to this approach is that these SWC and unsaturated hydraulic conductivity data may not be highly sensitive to roughness and its influence on wetting-fluid configuration.

Measurements from gas-phase interfacial partitioning tracer tests (IPTT) have shown that total (film-meniscus) air-water interfacial areas ($A_{nw}$) in natural porous media increase exponentially at lower wetting-fluid saturations ($S_w$) (Kim et al., 1999; Costanza-Robinson and Brusseau, 2002; Peng and Brusseau, 2005; Brusseau et al., 2006). Comparison between the measurements from gas-phase IPTT, aqueous-phase IPTT, and X-ray microtomography revealed that the exponential increase is due to the significant involvement of surface roughness to film-associated interfaces at lower saturations (Brusseau et al., 2007). This is supported by the results of pore-scale modeling conducted using models that either implicitly (Or and Tuller, 1999) or explicitly (Jiang et al., 2020) account for surface roughness and its impacts on interfacial area.

The experimental and modeling results demonstrate that the $A_{nw} - S_w$ function is highly sensitive to surface roughness, particularly at lower water saturations. Based on this behavior, Jiang et al. (2020) proposed using measured $A_{nw} - S_w$ functions as a “wetting-film characteristic curve” to provide robust characterization of surface roughness and its interaction with wetting-fluid films. With this approach, the gas-phase IPTT method can be used to conduct measurements of total $A_{nw}$ for a specific soil, and these data can be used to characterize fluid-surface interactions specific to that soil. It is anticipated that this approach would have multiple advantages, including greater sensitivity compared to other macroscale approaches, the ability to support both fractal-dimension and area-factor definitions, and provide characterization of surface roughness across a broader range of scales.

In the present study, we develop a general approach to characterize fluid-surface interactions and the associated model parameters from gas-phase IPTT measurements. This effort is based on the pore-scale model reported in our prior paper, which employs a triangular-pore-space bundle-of-cylindrical-capillaries (BCC) representation of the porous medium to characterize the impact of surface roughness on fluid-fluid interfacial area (Jiang et al. 2020). The basic principles of characterization are universal, which can be implemented via most of the current pore-scale models (i.e., bundle-of-cylindrical-capillaries, pore-network, pore-morphology, Lattice-Boltzmann). We use published IPTT measurements reported for four soils, and generate the characteristic parameters for each sample. We also test the validity of the characterization approach for different conditions. In addition, we have conducted
SEM imaging at different magnifications for each soil to visualize and verify the representation of surface roughness parameters. The present approach can be used to quantify fluid-solid and fluid-fluid interfacial interactions for a wide range of environmental problems.

2. Model

The present work is based on the previous study of Jiang et al. (2020). Essential concepts of the Jiang et al. (2020) model are briefly discussed herein. Variables and equations that are not specifically given in the following sections are listed in Appendix A. More comprehensive explanations of the basis and limitations of the model can be found in the original study of Jiang et al. (2020), wherein the physical, mechanical basis for the approach is established.

2.1. General theory

Experimental and simulation evidence has indicated that wetting films on grain surfaces are held by a combination of surface adsorption (DLVO) and capillary forces (e.g., Kim et al., 2012; Tokunaga, 2011). The dominant force depends on matric potentials, and thus wetting-phase saturations, as well as the properties of the surface. Wetting films are only a few molecules thick when surface adsorption completely dominates, in which case the film surface area (and concomitantly the fluid-fluid interfacial area) is equivalent to the solid surface area obtained from gas adsorption experiments (e.g., nitrogen-BET). When capillarity dominates, film thickness increases such that the film may partially or fully cover the solid surface roughness. In the extreme case of full coverage, the wetting film can be regarded as smooth. Jiang et al. (2020) presented the concept of minimum and maximum theoretical \( A_{\text{min}} - S_w \) functions, with the actual behavior observed dependent upon a number of factors. This concept is illustrated in Fig. 2a by a theoretical “Fluid-Surface Triangle”, which illustrates the function of surface adsorption and capillarity with respect to fluid-fluid interfacial area.

The fluid-surface triangle is anchored by a zero interfacial area at \( S_w = 1 \) and two points at \( S_w = 0 \): a "smooth" specific solid surface area \( (A_s) \) and a "rough" specific solid surface area \( (A_r) \), which follows the assumption that the maximum interfacial area is equal to the solid surface area (Or and Tuller, 1999; Peng and Brusseau, 2005; Jiang et al., 2020). The lower and upper curves correspond to two imaginary cases of "smooth-surface" and "maximum-roughness", respectively. For the smooth-surface curve, it is assumed that no surface roughness exists in grain surfaces, or equivalently that surface roughness is completely masked by thick wetting films at all saturations. For the maximum-roughness curve, it is hypothesized that only surface adsorption participates in the configuration of wetting film, so that the area of wetting film is always identical to the rough solid surface covered by the wetting film.

The two limits of the surface-fluid triangle are determined by the inherent properties of the specific porous medium. The smooth-surface curve is controlled simply by the pore structure and pore-size distribution of the medium. Conversely, the maximum-roughness curve indicates the greatest degree to which surface roughness can affect wetting film area. The extent can be derived from the two anchor points in the \( A_{\text{min}} - S_w \) axis. For a rough porous medium, the surface roughness factor, \( X \), is defined by the ratio of its rough specific surface area, \( A_r \), to the smooth specific surface area, \( A_s \), such that:

\[
X = \frac{A_r}{A_s}
\]

Typically, \( A_r \) can be obtained from the nitrogen-BET measurement of specific solid surface area. Multiple methods are available to determine \( A_s \), including calculation using a smooth-sphere assumption and mean particle diameter, estimation using SWC data, or measurement by X-ray microtomography.

Results of IPTT, especially the gas-phase tests at lower water saturations (< 0.4), have shown that actual \( A_{\text{min}} \) curves reside within the fluid-surface triangle as shown in Fig. 2b. The inset in Fig. 2b illustrates the possible shape of wetting film in an actual drainage process. When capillarity forces are dominant at higher saturations, the film thickness is greater than the magnitude of surface roughness, which in effect masks the impact of surface roughness. In this case, the actual \( A_{\text{min}} \) curve will be similar to the smooth-surface curve. As wetting films become thinner and surface adsorption becomes increasingly significant at lower saturations, the contribution of surface roughness to film interfacial area increases, resulting in the exponential increase in interfacial area shown in Fig. 2b. Finally, the \( A_{\text{min}} \) curve ascends towards the roughness-based specific solid surface area, and merges with the maximum-roughness curve. For a specific porous medium, the degree to which surface roughness is masked can vary depending on its features of fractal-scale roughness, which should be expressed via fractal dimension or area factor. Thus, it is reasonable to introduce an interfacial area factor \( X \), which is always

![Fig. 2. Illustration of (a) the “Fluid-Surface Triangle” for the \( A_{\text{min}} - S_w \) relationship, and (b) the location of an actual \( A_{\text{min}} - S_w \) curve. Model parameters corresponding to each curve are marked. \( A_{\text{min}} \) has units of cm\(^2\)/cm\(^3\) or cm\(^{-1}\).](image-url)
lower than the solid surface roughness factor $X$, such that:

$$X_a = A_{nw}/A_0$$

(2)

It is worthwhile to note that $X$ in Eq. (1) only depends on the properties of the soil, but $X_a$ in Eq. (2) is a function of saturation, and ultimately, matric potential. Therefore, for a universal approach to characterize surface roughness from sample-scale measurements of soils, the methodology requires two steps: (i) determination of the value of $X$; (ii) determination of $X_a$ as a function of saturation or matric potential.

2.2. Characterization model

Following the approach developed in Jiang et al. (2020), the bundle of cylindrical capillaries (BCC) method is selected to establish the characterization model. The major advantage of BCC over other modeling approaches (e.g., pore-network) is the specification of pore-size distribution for a specific porous medium, which facilitates incorporation of the surface roughness representation of $X$ and $X_a$ in accordance with experimental data. In addition, the simplification from 3D to 2D pore geometry in the BCC model has been validated in various studies on porous media (Dahle et al., 2005; Diamantopoulos et al., 2016; Helland and Skjaeveland, 2007; Or and Tuller, 1999). The justification for using this method is fully described in Jiang et al. (2020).

In the present BCC model, a porous medium is approximated as a bundle of equilateral triangular pores, whose side length, $L$, follows a lognormal distribution, $f(L)$:

$$f(l) = \frac{1}{L_0 \sqrt{2\pi} \sigma} \exp \left( -\frac{\ln(l/L_0)^2}{2\sigma^2} \right)$$

(3)

The two parameters $L_0$ and $\sigma$ are the mean side length and logarithm standard deviation for lognormal distribution, respectively. For an unsaturated smooth porous medium, both specific solid surface area ($A_s$) and interfacial area ($A_{nw}$) are only controlled by the pore-size distribution parameters $l_0$ and $\sigma$ (see Fig. 2a). For a rough porous medium, first, it is assumed that every pore has a uniform surface roughness factor, $X$, defined by Eq. (1); then, under a specific matric potential, $\mu$, the interfacial area factor, $X_a$, is specified on each pore surface in the form of wetting film (Fig. 1b). According to Jiang et al. (2020), $X_a$ is assumed to follow a logistic function, which is given by:

$$X_a = X + \frac{\rho_0}{\rho} \left[ \frac{\ln(x - \rho)}{\ln(x + \rho)} \right] \left( \frac{\ln(x - \rho)}{\ln(x + \rho)} \right)$$

(4)

where the parameters $k$ and $h_m$ correspond to the change of wetting film in drainage under the impact of micro-scale fractal roughness. Eq. (4) is based on the experimental profiles between interfacial area (related to $X_a$) and matric potential (related to $h_m$) (e.g., Brusseau et al., 2006; Kim et al., 1997; Porter et al., 2009). The rationale and support for using this equation is described fully in Jiang et al. (2020). Parameter $k$ (unit of reciprocal length) controls the growth rate of film area in drainage, and $h_m$ (unit of length) indicates the thickness of a critical sub-layer of roughness, which triggers the exponential increase in the $A_{nw}$ curve. The argument variable $h_m$ in Eq. (4) is a function of matric potential $\mu$ (see Appendix for the equation). With the constraints in Eq. (4), $X_a$ is only controlled by one variable, $\mu$, and is always limited in the range between 1 and $X$.

The surface roughness of drainage has been parameterized into five explicit parameters: $l_0$, $\sigma$, $X$, $k$, and $h_m$. Each of them can be determined from experimental data sets, e.g., SWC ($P_\sigma$, $S_p$), solid surface area (nitrogen-BET), and IPTT ($A_{nw}$, $S_p$) measurements. SWC data are used to determine the pore-size distribution parameters $l_0$ and $\sigma$, and subsequently, the overall roughness factor $X$ can be calculated as the ratio of the measured nitrogen-BET specific surface area $A_s$, and the smooth surface area obtained from $l_0$ and $\sigma$. Upon obtaining $l_0$, $\sigma$, and $X$, the remaining parameters $k$ and $h_m$ can be obtained by fitting the model to the $A_{nw}$ data set. Determining the values of $k$ and $h_m$ for a specific porous medium is the critical step in the characterization procedure.

It is emphasized that all five parameters are not obtained from one single data set. Rather, three distinct and independent data sets are used to parameterize the model, SWC data for $l_0$ and $\sigma$, nitrogen-BET for $X$, and IPTT data for the remaining two. The degrees of freedom in parameter determination are considerably constrained by the use of these three independent data sets, which limits the uncertainty in model output. Furthermore, the use of SWC data to characterize pore-size distributions of porous media is a widely used and well-established approach. The use of nitrogen-BET analysis to determine solid surface areas is the gold standard for characterization of surface areas. The use of IPTT data to characterize interfacial areas has been established in multiple works. Hence, use of each of the three data sets for parameter determination is well established; the combination and integration of the three into a unified approach to characterize fluid-solid interactions is the unique aspect of the present study.

The detailed procedure of optimization requires a classification of the wetting-fluid distribution in pores, which can range from fully-filled to partially-drained to fully-drained pores. In partially-drained pores, wetting fluid is distributed along the pore side walls, and form different configurations as film fluids or meniscus fluids, corresponding to film and meniscus interfacial area. Under a given drainage potential, there is a critical size of pore where the nonwetting phase begins to invade into a filled pore (onset of drainage). This critical pore size, $L_1$, corresponds to a geometrical constraint given by Tuller et al. (1999):

$$L_1 = \frac{L}{\rho \mu}$$

(5)

where $\gamma$ and $\rho$ are the surface tension and density of the wetting fluid, respectively, and $C_3$ is a constant regarding to triangular pore geometry (see Appendix). Pores larger than that size are either partially or fully drained, with the same meniscus radius, $r$, controlled by the Young-Laplace law (the wetting phase is assumed to be completely wetting to the solid surface):

$$r = \frac{\gamma}{\rho \mu}$$

(6)

Knowing the relationship between matric potentials and critical pore sizes, the equations for capillary pressure (matric potential), saturation, and interfacial area can all be derived from pore geometry. The geometric relationships between liquid films and menisci within a triangular pore (partially-filled) are presented in Fig. 3.

Typical equations for both $S_{nw}$ and $A_{nw}$ in the literature are given with matric potentials as the argument variable. To conduct the surface roughness characterization, it is assumed that the three variables coalesce into an integrated $P_\sigma - S_p - A_{nw}$ relationship (Hassanizadeh and Gray, 1993), so as to make it convenient to implement the model optimization on $A_{nw}$, $S_{nw}$ data sets. The equations for $S_{nw}$ and $A_{nw}$ with the terms of all types of fluid configuration (filled-pore fluid, film fluid, and capillary-meniscus fluid) are given as (Jiang et al., 2020):

$$S_{nw} = S_{ad}(\mu) + S_{af}(\mu) + S_{ac}(\mu)$$

(7)

$$S_{ad}(\mu) = \int_{L_{min}}^{L_{max}} f(L)dL$$

(8)

$$S_{af}(\mu) = \int_{L_{min}}^{L_{max}} 3h(\mu)[1 - 2(\mu) \cot(\pi/6)] f(L)dL$$

(9)

$$S_{ac}(\mu) = \int_{L_{min}}^{L_{max}} 3F_2 L L^2 f(L)dL$$

(10)

$$A_{nw} = A_{nw}(\mu) + A_{nw}(\mu)$$

(11)

$$A_{nw}(\mu) = \int_{L_{min}}^{L_{max}} 3X_a L L^2 f(L)dL$$

(12)
The development of these equations are discussed in detail in Jiang et al. (2020).

Equations (7~10) and equations (11~13) indicate that total saturation is the sum of filled-pore (duct) fluid ($S_{np}$), film fluid ($S_{lf}$), and capillary-meniscus fluid ($A_{nw}$), while total interfacial area is only determined by film ($A_{nf}$) and capillary-meniscus ($A_{nw}$) fluid in partially-filled pores. Definitions of $h(\mu)$, $A_2$, $F_3$, $L_{min}$, and $L_{max}$ are given in the Appendix. The magnitudes of these components have been investigated in previous modeling studies (Or and Tuller, 1999; Zheng et al., 2015; Jiang et al., 2020), amongst which filled-pore fluid is dominant in saturation, and film fluid is dominant in interfacial area. It is then logical to minimize focus on the uncertainties in the less significant components during the characterization process, e.g., film fluid in fitting $P_c - S_n$ curves, and meniscus interface in fitting $A_{nw} - S_n$ curves. It is also noted that the maximum $A_{nw}$ in Eq. (11) as $\mu \to \infty$ is equal to the rough specific surface area $A_r$, which represents the assumption of Fig. 2b.

### 3. Materials

Peng & Brusseau (2005, 2012) measured the air-water interfacial areas for various types of porous media via gas-phase IPTT. In addition, they reported the data of specific solid surface areas and soil-water characteristic curves for four media: Accusand, Granusil, Vinton, and Hayhook. Among the four porous media, Accusand and Granusil are well-sorted natural sands; Vinton is a sandy soil; Hayhook is a coarse loamy soil. The four selected porous media comprise a range of physical and geochemical properties, which are useful for the test of surface characterization methods on varying natural media. Basic soil texture properties for the studied porous media are summarized in Table 1.

Gas-phase IPTTs are limited to lower water saturations. Available $A_{nw}$ data for higher water saturations obtained via aqueous-phase IPTT (Brusseau et al., 2007, 2015) are combined with the gas-phase IPTT data. These data are available for Accusand and Vinton. To test the generalizability of the approach, aqueous IPTT data for a PCE-water system with Accusand from Zhong et al. (2016) are also chosen. Another sandy medium with interfacial area measurements by Kim et al. (1997, 1999) is also listed in Table 1 for comparisons in the following sections.

### 4. Results and discussion

#### 4.1. Parameterization of surface roughness effect

Experimental evidence has shown that wetting films on natural grain surfaces are sensitive to scale. Surface adsorption can only support thin films at the magnitude of 1~10 nm (Resurreccion et al., 2011; Tokunaga, 2011), while capillary forces tend to be enhanced on rougher surfaces, resulting in thicker films at the magnitude of 0.1~1 $\mu$m (Kim et al., 2012; Tokunaga, 2011; Tokunaga et al., 2003). Among the three types of surface roughness representations in Fig. 1, the relative surface height (Type I) is limited by its primary scale for the selected range of coordinates. The fractal dimension (Type II) has been used in the simulation of soil hydraulic properties such as hydraulic conductivity and relative permeability (Ghanbarian et al., 2016; Thompson and Brown, 1991). Fractal dimension is appropriate for multi-dimensional problems with a constant $D$ value (e.g., the perimeter and cross-section area are related by fractal dimension in tube flow), but is inconvenient for situations where $D$ is changing, which occurs in the change of wetting film controlled by matric potentials. Conversely, the area factor (Type III) approach used in this study, especially the use of interfacial
area factor $X_r$, provides a convenient cross-scale methodology to parameterize the scale-sensitive interaction between fluids and solid surfaces. At the lowest metric potentials, the wetting-film interfaces will tend to be smooth, which is the lowest $D$. As surface adsorption dominates at higher metric potentials, the films become sufficiently thin such that their surface topology mimics the finest structures of the solid surface, representing the highest fractal dimension, which is also the $D$ of rough solid surfaces. Therefore, the interfacial area factor $X_r$ contains information of the fractal-scale properties of wetting films. In addition, the measurement scale for the area factor is controlled by experiment-based variables, such as nitrogen-BET measured solid $A_m$, metric potentials, wetting-fluid saturations, and interfacial areas, which improves the accuracy of fractal representation.

Thus, it is elucidated that the “fluid-surface triangle” defined in Fig. 2 delineates the free fractal space of wetting film interfacial area, wherein the upper (maximum-roughness) and lower (smooth-surface) curves represent the maximum and minimum fractal dimensions of the film interface, respectively. Meanwhile, the two limiting conditions determine the range for $X_r$, within $X_r = 1$ (smooth) and $X_r = X$ (roughness factor of solid phase). With the interfacial area factor $X_r(\mu)$ determined from IPTT experiments, the surface-fluid interaction in fractal space can be quantified and parameterized.

The logistic Eq. (4) provides a simple approach for quantifying the roughness characteristic, with only two parameters: $k$ and $h_m$. The effects of $k$ and $h_m$ are illustrated in Fig. 4. In general, both parameters define the degree to which capillary forces mask the solid surface roughness beneath a wetting film, as shown by the phenomenon that some portions of the $A_{mw}$ -- $S_w$ curves are coincident with the smooth-surface curves in both plots. Inspection of Fig. 4 indicates that either increasing $k$ or decreasing $h_m$ can enhance the degree of masking. In addition, the trends of changes are different for $k$ and $h_m$: in an $A_{mw}$ -- $S_w$ plot, $k$ controls more of the horizontal region of $A_{mw}$ curves (the higher saturation region), while $h_m$ has a stronger influence on the vertical region (lower saturations).

Based on the mathematical features of the logistic function and fractal geometry, we are able to develop the physical representations for parameters $k$ and $h_m$. First, as a variable with the dimension of length, $h_m$ corresponds to a critical fractal metric, which is normalized as “critical adsorptive thickness” in the logistic function, such that wetting film thickness approaches the magnitude of the critical manifestation of fractal roughness. It is noted that the actual wetting film thickness at this stage is not equal to $h_m$, as the involvement of capillary forces in wetting film is still significant. The equation to calculate this film thickness is given in the Appendix. Larger values of $h_m$ cause the exponential stage of $A_{mw}$ -- $S_w$ curves to start earlier, i.e. at higher saturations.

The exponential parameter $k$ determines the growth rate of the logistic curve, which is, from another perspective, a representation of the complexity of the fractal structure of the surface. This term can be conceptualized as “thickness per area”. Surfaces with smaller $k$ values have smaller degrees of surface-roughness masking for the $A_{mw}$ -- $S_w$ curves. Thus, the curves exhibit a more uniform rate of increase as saturation decreases. This indicates that surfaces with smaller $k$ values comprise a greater number of fractal iterations between smooth surface and maximum roughness, or between the varying ranges of fractal dimensions. It is important to note that the influence of surface charge, wettability, and other factors on the formation and disposition of wetting films is implicitly incorporated in the model through these two parameters (Jiang et al., 2020).

### 4.2. Characterisation results

Results of the fitted curves generated by the characterization model are illustrated in Fig. 5 in comparison with the measured data of $P_c$ -- $S_w$ and total $A_{mw}$ -- $S_w$. All simulated curves are generated by the optimization toolbox in MATLAB. It is shown that the model produces very good fits of the measured data. The lognormal pore size distribution parameters $L_m$ and $\sigma$ are derived from the $P_c$ -- $S_w$ data sets, the solid roughness factor $X$ is determined from the solid specific surface area measured via nitrogen-BET, and the logistic parameters $k$ and $h_m$ are derived from the total $A_{mw}$ -- $S_w$ data sets. Overall, the good matches of the measured data for all four media shows that the proposed parameters and mathematical assumptions are sufficient to address the characteristics of surface roughness for natural granular media. One possible source of uncertainty is the values of lognormal standard deviation, which is affected by some inconsistencies on the low saturation data of $P_c$ -- $S_w$ curves. However, as discussed by Jiang et al. (2020), the standard deviation has very little impact on the determination of interfacial area parameters.

Values of the fitted model parameters, along with some other parameters of soil texture, e.g., median particle diameter $d_{50}$ and uniformity coefficient $U$ calculated by $d_{50}/d_{10}$ are given in Table 2 accord-
Fig. 5. Model characterization results for the experimental data of $P_e - S_e$ and $A_{sw} - S_w$ measurements. The measured data are from Peng and Brusseau (2005).
Thus, the reported data (Peng and Brusseau, 2012). The results from another model test run on the literature data reported by Kim et al. (1997, 1999) are also listed in Table 2. The actual critical film thickness, \( h_c \), which corresponds to the value of \( h_m \), is calculated via the empirical film thickness equation given in Zheng et al. (2015) (see Appendix), showing the combined contributions of surface adsorption and capillarity to film thickness. In the view of surface roughness characterization, the primary concern is whether there are quantitative correlations between the surface roughness-related parameters and the sample-scale features, such as particle and pore-size distribution.

Characterization results in Table 2 show that the traditional sample-scale parameters, e.g., uniformity coefficient \( U \), lognormal average pore size length \( L_m \), specific surface area \( A_s \), and the solid roughness factor \( X \), all show some degree of correlations with soil texture, especially between sands and soils. The one exception for the solid surface roughness factor is that of Hayhook, which is smaller than that of Vinton even though the former has a 70% larger specific solid surface area and a larger uniformity coefficient.

To test potential correlations between micro-scale roughness properties and the sample-scale soil properties, results between \( k \) vs \( X \) and \( h_c \) vs \( X \) are plotted in Fig. 6. The logistic parameter \( k \) is poorly correlated to surface roughness factor \( X \) for the five media used in this study. Thus, it would be difficult to predict the fractal surface properties of natural porous media from soil texture. For instance, Vinton has far more specific surface area than Granulis, but it also has a larger value of \( k \) as well as a smaller \( h_m \). Consequently, the \( A_{\text{max}} - S_w \) curve of Vinton, in terms of Fig. 4a, could be closer to the smooth-surface curve at some saturations, meaning that Vinton has stronger surface roughness masking than Granulis. As noted above, \( k \) represents the complexity of the fractal structure of the surface, whereas \( X \) represents the overall magnitude of surface roughness (roughness capacity). The fractal complexity of natural solid surfaces is expected to vary greatly among different media, as is illustrated in the following section. Thus, it is reasonable to anticipate that different media that have similar overall magnitudes of surface roughness (i.e., similar \( X \) values), may have different fractal structuring (different \( k \) values). Such behavior would explain the poor correlation between the two variables. A larger data base would be needed to further test this relationship.

Conversely, the critical thickness \( h_c \) presents some degree of correlation with \( X \) (\( R^2 = 0.84 \)), which implies that rougher grain surfaces can induce the exponential increase of interfacial area at larger measurement scales (larger saturations).

The quantitative results in Table 2 and Fig. 6 demonstrate the complexity of fluid-surface interactions. The interplay between surface adsorption and capillarity forces for wetting film is assumed to occur at scales much smaller than the size of soil particles, and even smaller than the magnitude of relative surface height. Fluid-surface interaction on such micro-scale rough structures is controlled by the complicated and chaotic nature of fractal growth. Furthermore, the meso-scale (between single particle and finer structures) surface heterogeneity, such as pits and fractures on single grain surfaces (Aratú and Brusseau, 2019), would also increase the complexity of fractal properties for micro-scale roughness. This illustrates the importance of surface roughness characterization for improved characterization of wetting film and its subsequent hydraulic properties.

### 4.3. Analysis of SEM images

The present modeling approach provides a different perspective to view surface roughness based on measured data for fluid-fluid interfacial areas. Introduction of inherent solid phase roughness parame-
ters can avoid the scaling bias of direct measurement such as SEM or AFM, whereby higher resolutions lead to smaller measurable regions. To demonstrate these phenomena and compare with the characterization results, SEM images of the four porous media (Accusand, Granusil, Vinton, and Hayhook) used in this study are obtained with a series of magnifications (500x, 2000x, 5000x, 20000x) (see Fig. 7). The minimum and maximum magnifications (500x and 20000x) represent a change in image domain from approximately 35,000 $\mu$m$^2$ to 40 $\mu$m$^2$ and resolutions from micrometer to nanometer scales. Images were obtained with a Hitachi S-4800 Field Emission SEM in the Kuiper Materials Imaging & Characterization Facility of the University of Arizona. SEM images are used to analyze visual associations between the real images of natural grain surfaces and the surface roughness parameters $X$, $k$, and $h_m$ ($h_c$).

First, some meso-scale features, such as fractures on grain surfaces, can be identified from the 500x SEM images, as highlighted with the red boxes for Granusil and Vinton. The sub-iterations of these heterogeneous structures are obviously different from the more homogeneous surface domains. It is also observed that the meso-scale surfaces of Vinton appear more divergent than other media, as is indicated by the circles of two surface regions with highly distinctive apparent roughness. This may partially explain the apparently anomalous value of $k$ for Vinton. To mitigate the uncertainties from heterogeneity, the following magnified regions are selected to the seemingly roughest sites in the 500x images. The 2000x and 5000x images correspond to the reported magnitudes (0.1–1 $\mu$m) of relative surface height representation (Adams et al., 2012; Kim et al., 2012; Tokunaga et al., 2003). All media show apparent uniform surface structures at these two levels. It could be assumed that wetting films at these scales are held primarily by capillary forces and remain sufficiently thick. However, the potential for additional roughness scales for the soil media (Vinton and Hayhook) is manifest in the 5000x images, while the sandy media (Accusand and Granusil) exhibit a larger proportion of smoothness in the images.

In the 20000x images, the thickness of roughness layers is in the range of tens of nanometers, close to the magnitudes of critical thickness ($h_c$) in Table 2. It is possible to recognize the iterated sub-layers of fractal roughness at this level. Inspection of the 20000x images provides a possible verification to the correlation between critical thickness and surface roughness factor exhibited in Fig. 6. At this level, the sandy media (Accusand and Granusil) only have a limited number of small and flat structures (as indicated in images) contributing to surface area, while the soil media (Vinton and Hayhook) present larger and deeper roughness formations that allow for further iterations of roughness scales. Such differences explain the differences in the nitrogen-BET surface areas between the sands and soils. However, it is also noticeable that surface information at this magnification has been considerably lost. It would be difficult to establish the explicit quantitative connections between the characterization results and the SEM images at this degree of magnification. Rather, it is further highlighted that the characterization approach based on sample-scale measurement has the unique capability of extracting the micro-scale information of fluid-solid interaction that is operationally relevant to flow and transport, which is difficult to ascertain from imaging methods.
The generalness of the roughness characterization method is based upon the premise that solid surface roughness remains constant regardless of any arbitrary flow conditions. Actual environmental problems can involve fluids with different properties, such as immiscible (nonaqueous phase) organic liquids (NAPL). Hence, a limited test is conducted in this section using published experimental data for the same Accusand but with a different nonwetting fluid (NAPL). The results are presented in Fig. 8. Air-water A_{nw} measurements are reported in Brusseau et al. (2015) and Peng & Brusseau (2005), and NAPL-water A_{nw} measurements using tetrachloroethylene (PCE) are reported in Zhong et al. (2016). To implement the test, the parameters Lm, σ, X, k, and h, determined for the air-water Accusand data are directly applied to the PCE-water data. The only adjustment is by substituting the surface tension γ for water with the interfacial tension between PCE and water. Values used in Fig. 8 are 0.0728 N/m for air-water and 0.0459 N/m for PCE-water.

As is shown in Fig. 8, the predicted PCE-water A_{nw} curve matches the measured data points. This suggests that there is generalness in the approach for characterizing wetting-film configuration. Greater roughness masking is observed for the NAPL-water system compared to the air-water system at lower wetting-fluid saturations. However, at higher saturations, interfacial areas for both fluid pairs are similar. Similar experimental evidence was reported by Schaefer et al. (2000a, 2000b), in which air-water and NAPL-water fluid-pairs exhibited similar interfacial areas for mass-distribution-based IPTT measurements over a large range of saturation. However, such measurements do not fully access the adsorption-dominant wetting film at very low saturations, whose properties are better captured by the gas-phase IPTT measurements.

A recent comparison of IPTT data for NAPL-water systems demonstrated that similar NAPL-water interfacial areas were measured for different fluid pairs for the same porous medium (Brusseau and Taghap, 2020). The data sets included PCE-water, decane-water, and hexadecane-water pairs. This provides some indication that fluid properties that affect multi-phase flow in porous media, such as density, viscosity, and spreading coefficient, may not have significant impact on the inherent interfacial area of the system. Thus, it is anticipated that the IPTT characterization method will provide robust determination of interfacial areas for application to different fluid pairs.

5. Conclusions

A methodology is developed to employ measured A_{nw} – S_{w} curves to characterize the fluid-surface interaction controlled by micro-scale surface roughness of porous media solids. The method is based on a general theory of adsorbed wetting films on rough solid surfaces, where the films are held by some combination of surface adsorption and capillary forces. The characterization is realized by quantifying the change of film-associated fluid—fluid interfacial area between capillary-dominant states (smooth film) and adsorption-dominant states (rough film) controlled by matrix potentials. As the quantification of wetting-film formations employs use of sample-scale measurements of porous media (e.g., nitrogen-BET and IPTT), it provides an effective approach to conceptualize and simulate the multiple scales of roughness associated with the fractal geometry of natural grain surfaces. The wetting-film vs matric-potential approach incorporates the advantages of surface roughness representations using both fractal dimension and area factor, which expands the scope of grain surface roughness beyond the traditional approach of relative surface height. The use of IPTT measurements establishes a complete characterization of fluid-surface interaction across all grain surfaces within a porous medium, which is free of the spatial measurement-scale limitations influencing traditional surface metrological approaches such as SEM and AFM.

The model characterization and SEM imaging results for the selected porous media in this study highlight the complexity of natural grain surfaces. The surface heterogeneity of natural grain surfaces is present across multiple scales—from sample and particle scales, to the meso-scale on one grain surface, and then to the micro-scale surface roughness following fractal dimensions. Given the complicated nature of grain surfaces, the well-developed experimental measurement of fluid-fluid interfacial area, especially the low-S_{w} measurement using gas-phase interfacial tracers, provides a reliable and controllable approach to characterize the surface properties based on the formation of wetting film. The proposed characterization methodology is applicable to a wide range of environmental systems involving flow, transport, and mass transfer.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Hao Jiang: Conceptualization, Methodology, Investigation, Formal analysis, Data curation, Writing - original draft. Bo Guo: Data curation, Methodology, Supervision, Writing - review & editing. Mark L. Brusseau: Conceptualization, Methodology, Data curation, Writing - review & editing, Supervision.

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Appendix. Summary of Model Parameters and Derivations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nomenclature</th>
<th>Dimension</th>
<th>Annotation</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid properties</td>
<td>density of wetting fluid</td>
<td>ML⁻¹</td>
<td>Can be switched to different fluid pairs.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>surface or interfacial tension</td>
<td>ML⁻¹T⁻²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variables capable of direct measurement</td>
<td>matric potential</td>
<td>L²T⁻²</td>
<td>Equivalent to capillary pressure $P_c$ or pressure head $H$.</td>
<td>$\mu = gH$</td>
</tr>
<tr>
<td></td>
<td>wetting-fluid saturation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>nonwetting-wetting interfacial area</td>
<td>$L^{-1}$</td>
<td>Referred to the total interfacial area, but almost equal to film-associated area at most $S_w$. Measured by IPTT.</td>
<td></td>
</tr>
<tr>
<td>Specific solid surface area (rough surface area)</td>
<td>$A_s$</td>
<td></td>
<td>Measured by $N_d$/BET.</td>
<td></td>
</tr>
<tr>
<td>Parameters in wetting film model</td>
<td>overall surface roughness factor</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>interfacial area factor</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>adsorptive film thickness</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>actual film thickness</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>film thickness factor</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>hamaker constant</td>
<td>ML²T⁻²</td>
<td>Given in literature (Or and Tuller, 1998; Tokunaga, 2011).</td>
<td>$A_H = 6 \times 10^{-20}$ J</td>
</tr>
<tr>
<td></td>
<td>logistic growth factor</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>critical adsorptive film thickness</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parameters in sample upsizing</td>
<td>meniscus radius</td>
<td></td>
<td>Given by Young-Laplace equation.</td>
<td>$r(\mu) = \frac{L}{\mu}$</td>
</tr>
<tr>
<td></td>
<td>pore side length</td>
<td></td>
<td>Side length of an equilateral triangular pore.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>mean pore side length</td>
<td></td>
<td>Given by lognormal equation.</td>
<td>$f(l) = \frac{1}{\mu} \exp\left(-\frac{(ln(l/\mu))^2}{2\sigma^2}\right)$</td>
</tr>
<tr>
<td></td>
<td>lognormal standard deviation</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>critical pore size</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>minimum pore side length</td>
<td></td>
<td>Set as a very small number.</td>
<td>$L_{\text{min}} = 5 \times 10^{-9}$ m</td>
</tr>
<tr>
<td></td>
<td>maximum pore side length</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Air-entry potential</td>
<td></td>
<td></td>
<td>Can be derived from Brooks &amp; Corey method.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>corner area factor</td>
<td></td>
<td>Geometric factor for equilateral triangles.</td>
<td>$F_3 = \sqrt{3} - \pi$</td>
</tr>
<tr>
<td></td>
<td>pore area factor</td>
<td></td>
<td>Geometric factor for equilateral triangles.</td>
<td>$A_s = \sqrt{3}$</td>
</tr>
<tr>
<td>Drainage blob radius factor</td>
<td></td>
<td></td>
<td>Geometric factor for equilateral under drainage.</td>
<td>$C_1 = 2\sqrt{3} + \sqrt{8/\sqrt{3}}$</td>
</tr>
</tbody>
</table>

References


