Parallel Hopfield Networks
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Abstract
We introduce a novel type of neural network, termed the Parallel Hopfield Network, that uses precise spike timing in order to simultaneously affect the dynamics of many different, independent Hopfield networks in parallel in the same piece of neural hardware. We analyze the network theoretically using an approximate set of self-consistent mean field equations which allows us to generate phase diagrams for the network. Each Hopfield sub-network is found to have finite memory capacity as the number of neurons goes to infinity. Simulations with finite numbers of neurons confirm the predictions of the theory.

Activation rule
Different spatio-temporal ‘memories’ (μ) hold different Hopfield-like sub-networks. Hopfield patterns (p) are generated randomly with the activity of the ith neuron in subnetwork μ and pattern p given by

\[ q_{i}^{μp} = \begin{cases} 1 & \text{with probability } b \\ 0 & \text{otherwise} \end{cases} \]

The weights are then given by the Hebb rule...

\[ W_{ij}^{μ} = \frac{1}{N} \sum_{p=1}^{P} (q_{i}^{μp} - b)(q_{j}^{μp} - b) \]

if \( i \neq j \)

\[ 0 \]

if \( i = j \)

The update rule is outlined in the figure below.

Simple demonstration
This figure shows the results of a simulation of a PHN made up of 200 neurons, containing 9 different memories, each one loaded with 10 different Hopfield patterns. A shows 100 time steps of the simulation; the black dots correspond to neural spikes and the red boxes denote the spatio-temporal mask for one of the memories. B shows the activity at the mask points for this memory and compares it with the activity profile of the pattern being recalled.

Order Parameters
Using the method of self-consistent signal to noise analysis [1], it is possible to derive approximate mean field equations for the networks which are then described in terms of four order parameters:

- f - the frequency of spiking in the network which is further split into \( f^{gen} \) - the rate of genuine spiking, and \( f^{sp} \) - the frequency of spurious spiking
- m - the overlap of the subnetworks with the memories being recalled
- q - the Edwards-Anderson order parameter describing the level of spin glass ordering in the subnetworks
- r - which describes the level of random activation in the subnetworks.

References

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