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Of computer mice and men

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Abstract. *Many everyday skills, such as using a computer mouse and driving an automobile, require that people adjust to some type of transformation between motor activity and visual feedback. Historically, different approaches have been used to understand successful performance, including prism adaptation, S-R compatibility, and mirror tracing. In this study, more modern technology was exploited by using a digitizing tablet and computer screen, which work similarly to a computer mouse. Are X and Y dimensions of space independent of one another? In Experiment 1, students were given one of two mappings; either the range on the tablet was reduced with respect to the screen for both X and Y equally, or the reduction occurred for the X direction only. The study found that the two dimensions are linked. Experiment 2 suggested that X and Y are not symmetric. Overall, the findings indicated that the two dimensions are dependent. Consequently, a two-dimensional metric is required for understanding visual-motor tasks, and one based on geometric properties is advanced. Related issues including "telepresence", identity, and conscious awareness are discussed.*

Key words: Perceptual learning, geometry, transformations, visual-motor coordination, space, mappings.

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Virtual Reality is the latest in a series of inventions where we perform physical actions with intent to influence something other than our standard world. Virtual Reality is an extreme example where our actions affect a world that does not exist. An artificial world may be created through the simulation of visual information that would result from self-produced motor activity in that environment, such as walking, turning, or reaching. You reach with a scalpel to operate on a heart, and the heart changes accordingly with your scalpel's movements. Except there is no scalpel and no patient.

A less extreme example of atypical consequences of motor action involves using a computer mouse. You manually move a mouse-sized object on a table, but the intent is to produce an effect *not on the table* but on a computer screen in a different location. You might be drawing a figure on the screen, but by moving your hand on the mouse rather than on the screen. Similarly, playing home video games can involve slaying monsters on a television screen through the convenience and safety of a joystick located across the room. Operating an automobile or an aircraft requires manipulating a control with the intent of affecting the vehicle and not the control. Even flying a kite, or catching a fish, can be viewed as performing a motor activity with consequences that extend beyond the limits of the hand. You pull a string near your body, and it moves an object up in the clouds or down beneath the water. In general, the purpose of all these inventions is to go beyond the capabilities of our own sensory-motor capacities (e.g., Held & Durlach, 1989).

When will humans be successful at such tasks? All the tasks involve transforming the normal relation between motor activities and visual feedback in some way. For kite-flying, the visual consequences, seeing the kite move, is in a vastly different location than the motor activity, pulling at the string. Intuitively, some transformations seem harder to accommodate than others. For instance, the computer mouse would likely be less popular if the cursor on the screen moved to the right whenever you moved the mouse to the left. Mirror transformations require extensive practice as suggested both anecdotally by trying to perform simple activities while looking through a mirror,[1] and by the

1. The majority of research placed the mirror vertically in front of the subject, perpendicular both to the table surface and to the line of sight. This placement of the mirror actually produces a reversal of forward and backward (Woodworth, 1938; see also Ittelson, Mowafy, & Magid, 1991; Ittelson, 1993).

results of extensive research on tracing figures through a mirror (e.g., Woodworth, 1938). Different research programs have been relevant for understanding which transformations are easily accommodated. In addition to mirror tracing, research on "S-R compatibility" (e.g., Fitts & Seeger, 1953), and visual-motor adaptation to distortions produced by prisms and lenses (e.g., Welch, 1978) reflect two other approaches to the problem. Here, we exploit a different paradigm based on more modern technology, to ask novel questions concerning how visual-motor transformations are acquired.

The paradigm is similar to using a computer mouse, except that a pen substitutes for the mouse. The position of the pen on a tablet is read by the computer which displays a visual representation of the pen on the computer screen (see Figure 1). As the pen moves, the position of the cursor on the screen is updated without noticeable delay. For instance, moving the pen 2 inches to the left would cause the visual cursor to move 2 inches to the left. As with a mouse, motor activity in one location has an effect on a different location. Additional transformations can be created through software designed to alter how the tablet locations are mapped onto the screen. For instance, a mirror transformation would cause the cursor to move to the left when you moved your hand

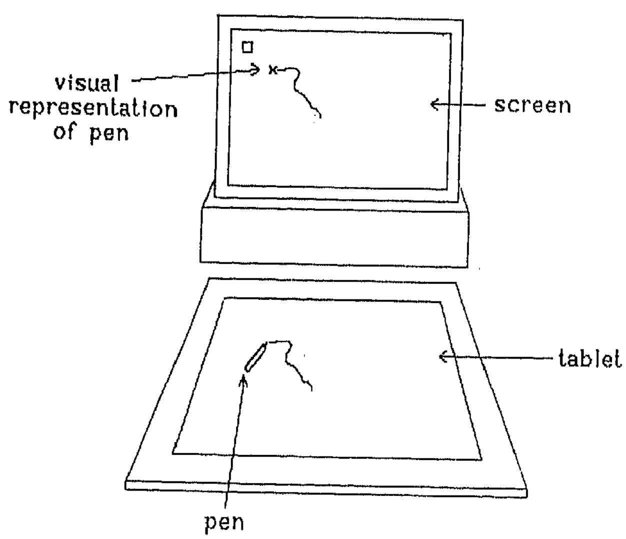

Figure 1. Sketch of computer screen and digitizing tablet.

to the right. Cunningham (1985) documented this relatively new paradigm as useful for the study of novel visual-motor mappings, and used it to study rotation transformations of one space with respect to another (1989; Cunningham & Welch, in press).

The question we asked was whether two dimensions of space are *independent* of one another. Many tasks require control over more than one spatial dimension. For instance, using a mouse controls a cursor both left to right and up and down. Flying an aircraft requires control over all three dimensions. When acquiring such skills, do orthogonal dimensions get processed separately from one another? If they do behave independently, then we should be able to learn about the properties of one dimension without regard for what is done to another.

EXPERIMENT 1

Consider two mappings, both of which create exactly the same transformation of one dimension, left/right on the screen ("X"), but manipulate an orthogonal dimension, up/down on the screen ("Y"), differently. Both mappings shrink motor space with respect to visual space to half its former size in the horizontal direction. For instance, moving the pen left to right by 2 inches on the tablet causes the cursor to move 4 inches left to right on the screen. One of the mappings also shrinks Y by the same amount as X, to half its former size. The second mapping leaves Y unchanged from normal. If the two spatial dimensions are independent, then the novel reduction of the tablet ranges in the X direction should be learned equally effectively in both situations, because X is changed equally in both situations. If the dimensions are dependent, then learning about one dimension will depend not just on the complexity of its own transformation, but on the characteristics of the other dimension's transformation as well. Will transforming Y make it harder to learn about the transformation of X? Easier?

Two mappings were presented to different subjects. For one mapping, both X and Y tablet range were reduced to .5 the original size. For the other mapping, only X was reduced to .5 the original range.

Subjects

The subjects were 16 undergraduates at the University of Arizona. Students received either experiment credits towards an Introductory Psychology requirement, or were paid.

Procedure

Students were first familiarized with using the pen, tablet, and computer. A small square appeared on the screen and students were required to get the character "*X*", which also appeared on the screen, to fit into the square. The *X* was moved by moving the pen on the tablet. The visual representation of the pen (the *X*) was visible throughout the movement, and both the *X* and square vanished when subjects succeeded in placing the character inside the square. The square could appear in one of 9 locations. The 9 locations together would make up a large square if all 9 were visible at the same time: three positions on the top row, three in the middle, and three on the bottom. Practice was given only once at each of the 9 locations. The familiarization trials served, therefore, primarily to orient the subject in the new environment, rather than to teach anything new about visual-motor coordination. The mapping from tablet to screen during familiarization was the natural mapping where a given distance on the tablet corresponded to the same distance on the screen.

Next, subjects were tested to determine performance before training any new visual-motor relation. Testing occurred by withholding the visual consequences of their movements. That is, they were instructed to move the pen to the position on the tablet that *would* make the *X* fit inside the square, if the *X* was visible. Consequently, they had to use their own internal sense of the correct mapping, because they could not rely on visual feedback to guide them to the desired destination. Each trial was terminated by the subject, who pushed down on the pen (which served as a button) when satisfied. Two repetitions at each of the 9 locations in random order completed the testing.

At this point, the mapping was further changed through software[2] and students given practice with a novel mapping. Students were not informed of the nature of the new relation, or told that anything had been changed. Half the subjects were assigned to the Minify X and Y group and the other to the Minify X only group. Practice was made possible through returning the *X* character to the screen. Subjects again attempted to place the cursor within the square. Here, they were given 4 seconds

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2. The digitizing paradigm already imposes a novel transformation before software manipulations because the screen is in a different location than the tablet. For convenience, the pre-software mapping will be referred to as the "identity" or "normal" mapping, unless specified otherwise.

to complete the task, at which point the display vanished. Under the Minify X only mapping, moving the pen horizontally from left to right a given distance caused the *X* cursor to move *twice as far* left to right on the screen. In the orthogonal direction, a given distance on the tablet corresponded to the same distance on the screen. Under the Minify X and Y mapping, moving the pen a given distance either left to right, or bottom to top caused the cursor to move twice as far in the appropriate direction on the screen. Subjects practiced with 10 repetitions for each of the 9 positions, with a short rest break halfway through. They were allowed 4 seconds for each trial, for a total of 6 minutes of training. Subjects were then tested, given 3 additional minutes of training, and tested again.

For testing after practice, the same procedure was used as in testing before practice. Thus, subjects must again make use of their internal sense of how vision and motor activity are related, perhaps this time a different internal sense modified through practice. For convenience of data collection, the mapping was returned to normal during testing. From the subjects' point of view, this doesn't matter because the *X* cursor is invisible; consequently, there isn't any information on what the mapping is. The difference between where subjects place the invisible *X* (i.e., position of pen on tablet) before and after practice measures the effects of training.

Two further procedural details are relevant. During the entire experiment, subjects' sight of the *actual* pen and hand was occluded. Thus the visual consequences of pen movement was restricted to the activity on the screen. Second, after each testing and training trial, students were requested to "scribble around on the pad" between trials. The purpose was to minimize any stereotyped responding that might occur with a small number of positions, as well as to keep separate trials as independent as possible. The position of the pen on the tablet after the scribble served as the starting position for the next trial.

Results

The most notable outcome occurred for the mapping which specified a reduced tablet range for the X direction only. Not only did the X direction shrink, as specified by the mapping, but so did the Y direction. Despite practice that the up and down direction was unaltered, that tablet range also became smaller with respect to the visual range. The mean position of the pen on the tablet for each of the 9 target locations, both before and after training, is shown in Figure 2. For both mappings, subjects essentially learned that a square in visual space maps onto a smaller square in motor space, even though that relation is correct for one of the mappings provided, but not correct for the other.

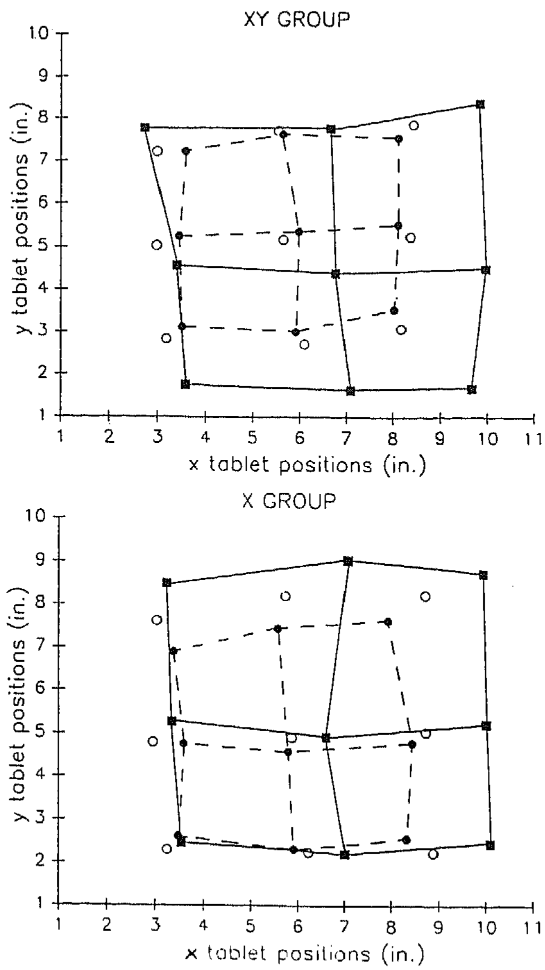

Figure 2. Mean tablet positions for Experiment 1 averaged across subjects. Connected squares show positions before training, unconnected (unfilled) circles after 6 minutes of training and connected (filled) circles after 9 minutes. Lower left corner of tablet is located at (0,0). "XY Group" is the mapping which shrinks both X and Y. "X Group" is the mapping which shrinks X only.

The slopes for the two groups before training, after 6 minutes, and after 9 minutes of training are shown in Figure 3. Slopes were calculated by fitting general linear equations describing the positions of the pen on the tablet as a function of the positions of the targets on the screen.[3] The decrease in the x and y slopes shown reflect the uniform

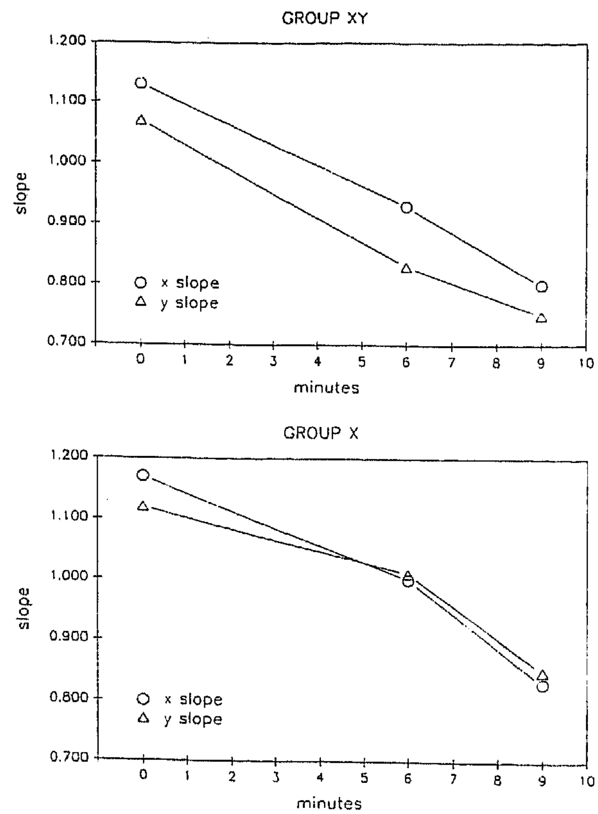

Figure 3. Mean slopes for Experiment 1 (see text and footnote 2).

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3. The general linear equations were of the form $T_x = aV_x + bV_y + c$ and $T_y = dV_x + eV_y + f$, where T refers to tablet positions and V to visual positions. The slopes shown in the graphs correspond to the parameters *a* ("x slope") and *e* ("y slope"). Perfect performance before training would have values for parameters a, b, c, d, e, and f, of 1, 0, 0, 0, 1, 0, respectively. Perfect performance after training for group Minify X and Y would have values .5, 0, 0, 0, .5, 0, and for group Minify X only, .5, 0, 0, 0, 1, 0.

reduction of the tablet range with respect to the visual range that occurred in both the horizontal and vertical directions for both groups. After 9 minutes of training, the change in x slope and in y slope for the Minify X and Y group were significant [two tailed - $t(7) = 3.45$, $p < .05$; $t(7) = 3.88$, $p < .01$], as were the changes in both x slope and y slope for the Minify X only group [two tailed - $t(7) = 6.01$, $p < .001$, $t(7) = 2.83$, $p < .05$].

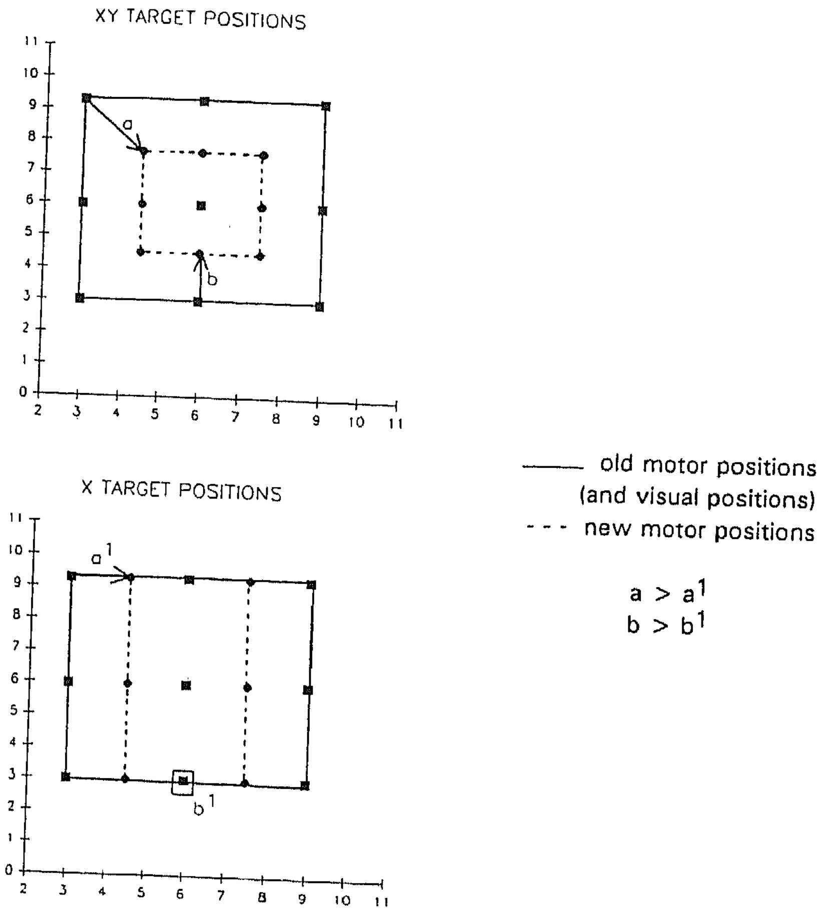

Figure 4. The change in the locations of nine positions on the tablet for the mapping which minifies both X and Y (top) and the mapping which minifies only X (bottom). The nine positions are also the 9 targets used in Experiments 1 and 2.

Discussion

The orthogonal dimensions of space appear to be dependent, but note that the relation manifested itself in a way other than anticipated. In the mapping where X alone was reduced, learning about X was not impaired by Y, but instead caused Y to change inappropriately. Nonetheless, the results suggest that the two dimensions are dependent. Moreover, it appears easier to rescale both dimensions equally than just the Y dimension alone. A practical consequence is that any visual-motor task which imposes this asymmetry between X and Y may not only be difficult to acquire, but may be "learned" incorrectly.

At a theoretical level, consider first a simple metric that *cannot* account for the data. A simple view of which mappings are easiest could be that the more a novel mapping requires that the position of the hand change from the normal identity mapping, the harder that mapping is to acquire.[4] In this view, a mapping which shrinks only X should be *easier* to acquire than one which shrinks both X and Y. On average, a hand position on the tablet would need to change less from its usual position if X alone is altered then if Y is altered as well. Figure 4 shows the change in position required by the mappings for several locations, including one located in the upper left of the tablet, and another located in the middle bottom region. The data from Experiment 1 suggest that such a "minimization principle" based on how much the position of the hand changes is not useful.[5]

An alternative account is that it is not the *overall amount of change in hand position* which determines the difficulty of a mapping, but instead it is *the number of geometric properties* altered by the mapping

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4. This metric of simplicity is based on a minimization of distance principle discussed for paths of apparent motion (e.g., Proffitt, Gilden, & Kaiser, 1988). In apparent motion, there are an infinite number of paths along which a stimulus can appear to travel to get from one location to a second location. One solution is to interpolate a path such that, on average, each point moves as little as possible. In the present paradigm, the analogy to minimizing the distance travelled from the initial position is minimizing the distance on the tablet from the visual positions (or the old tablet positions under the normal mapping).

5. The preference is also unlikely to be caused by prior experience with a mouse, which causes a transformation more like the shrink XY transformation than the shrink just X transformation. The data from several subjects who reported never using a mouse does not appear to differ from those who had.

which is critical. A mapping which shrinks both X and Y equally preserves shape and changes only the overall size of space, whereas a mapping which shrinks only X alters properties of shape as well as size. Shrinking both spatial dimensions equally turns a square into a small square and shrinking only one turns a square into a rectangle (refer to Figure 4). A square and a small square have more geometric properties in common than a square and a rectangle.

This view can account for the data where a mapping which shrinks both X and Y equally was easier to acquire accurately than one which shrinks only X. In general, in this view, the more geometric properties altered by a mapping, the more difficult that mapping will be to acquire. This geometric approach, which derives from work of mathematician Felix Klein, will be expanded in the general discussion.

However, there is an alternative account of the outcome which must be dealt with first. Perhaps subjects have a tendency to reduce the range they use on the tablet simply because of fatigue. Was it actually the reduction of X which pulled Y with it, or would the change happen anyway?

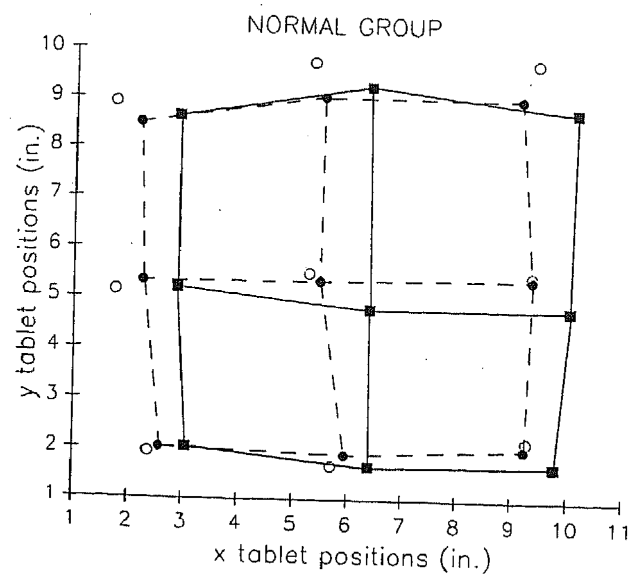
EXPERIMENT 2

A new set of subjects were given a mapping which shrunk *neither* X nor Y. That is, they were allowed to practice on the normal or identity mapping where distances in motor space and visual space are identical. If the tablet range shrinks in the Y direction as a result of fatigue, we would expect to observe the change for this mapping as well. A second group was run which received the remaining condition of shrinking Y but not X. This mapping tests whether the results will be symmetrical for both dimensions.

The procedure was identical to the first Experiment, except that practice was given with these "Normal" and "Y" mappings instead.

Results and discussion

Figure 5 shows that for the normal mapping, the Y tablet range does not shrink. In fact, after 6 minutes of training there was a tendency to inappropriately *expand* the range used on the tablet. This may be due to the fact that the tablet is physically larger than the computer screen. Subjects might find it natural to map the whole surface of the tablet onto the whole surface of the screen, rather than to equate absolute distances.



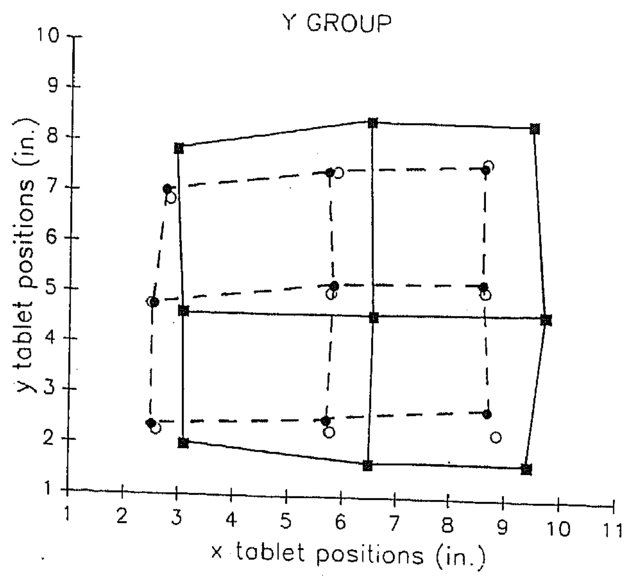


Figure 5. Mean tablet positions for Experiment 2. Connected squares show positions before training, unconnected (unfilled) circles after 6 minutes of training and connected (filled) circles after 9 minutes. Lower left corner of tablet is located at (0,0). "Normal Group" is the mapping where neither X nor Y is changed. "Y group" is the mapping which shrinks Y only.

Consistent with this is the pretest biases in all 4 manipulations of the study. On average, subjects start out using a larger range of the tablet rather than matching distances equally (reflected by slopes > 1 before training). If true, this is in and of itself an interesting phenomenon for further investigation. Overall, the results suggest that Y did not shrink in the previous experiment simply because of fatigue, or any other aspect of the paradigm in general.

The results of the remaining group were surprising. When the mapping specified that Y alone was to be reduced, subjects learned that

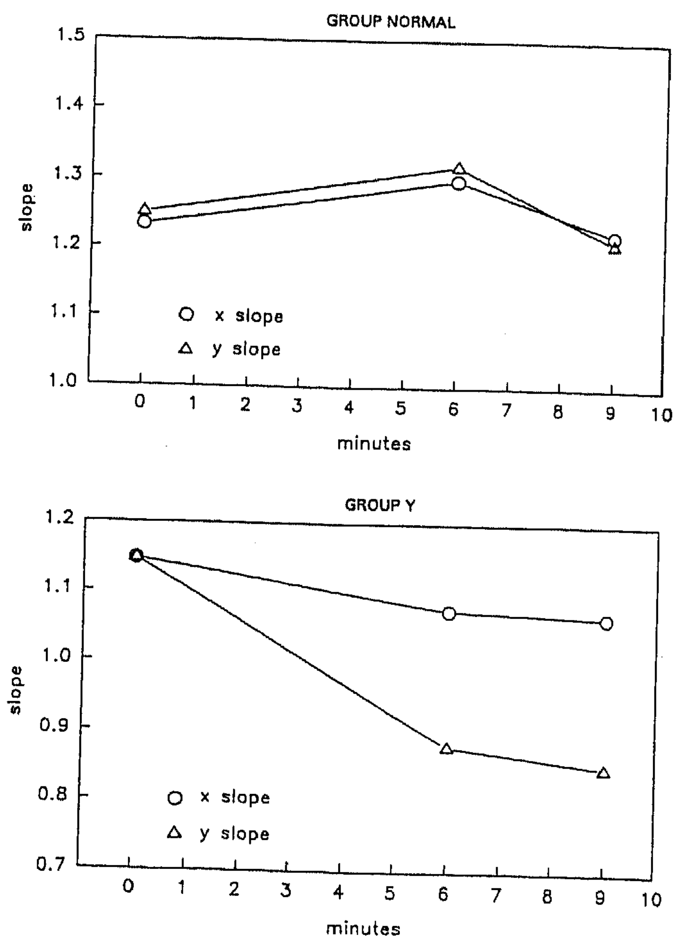


Figure 6. Mean slopes for Experiment 2.

Y was reduced; however, Y did not pull X with it (Figures 5 and 6). Although the X range did not expand as in the control group, it also did not clearly shrink. This outcome suggests an asymmetry between the two dimensions: A novel X relation affects Y, but not vice-versa.

The asymmetrical influence may be due to the asymmetry imposed by the paradigm. The X dimension maps straightforwardly from the tablet to the screen, but the Y dimension does not. The paradigm imposes 3 spatial transformations of its own, even if the normal mapping is used: The tablet is lower down than the screen (a y displacement), it's closer to the observer (a z displacement), and it's flat on a table whereas the screen is upright (rotation around an x axis). Moving the hand from left to right on the tablet causes the cursor to move from left to right on the screen. Moving the hand in a forward direction on the tablet (close to far), which lies flat on the table, causes the cursor to move from the bottom to the top of the screen. Consequently there may be less certainty about the correspondence between visual space and motor space in the latter situation. When confronted with novel transformations, it may be sensible for the more certain dimension to guide the less certain dimension, but not the other way around. Practically, many computer based tasks have the asymmetry of the current paradigm – mice and video games for instance. Yet it is also possible the asymmetry is inherent in the dimensions themselves. Left/right and up/down differ in a number of ways. Objects tend to have left right symmetry but not up down (or front back) symmetry. Gravity imposes a different sort of asymmetry on up vs. down, but not left vs. right. The exact conditions under which X and Y are dependent or independent and under which X and Y have symmetrical or asymmetrical effects on each other need to be investigated further.

GENERAL DISCUSSION

The experiments found that vertical and horizontal dimensions of space are not always independent. Consequently, any metric concerning how new visual-motor mappings are acquired must involve at least two dimensions of space. In addition, the experiments found that when the two dimensions are dependent, rescaling two dimensions equally is easier than adjusting just one (x) alone. One promising approach towards these issues is based on an application of geometry to visual and motor space.

In what can be called the *geometric view*, I suggest that the more geometric properties a new mapping alters, the more difficult that mapping will be to learn. Geometric properties are those we intuitively think of as defining shape, such as the *angle* formed by two intersecting lines, the *parallelism* or non-parallelism of two lines, the *order* of points along a line, and so forth. To apply formal studies of geometry to visual-motor tasks, visual space can be imagined as a form and the different transformations as different ways of pulling and pushing on the form to turn it into a different form. The resulting product is motor space. A mapping specifies a correspondence between points before (visual space) and after (motor space) a transformation. Under the typical mapping, the visual form and the motor form are identical; point for point, they superimpose perfectly. New mappings can be characterized by the number of geometric properties they alter. Such an approach based on geometry may be more useful than other possible ways to order the complexity of mappings, such as amount of overall change in hand position (discussed earlier) or of polynomials of increasing degree (e.g., Carroll, 1966). Space can readily be thought of as a large form, which makes geometry a natural candidate; other mathematical frameworks don't have as clear a linkage.

How specifically can mappings be ordered? In the transformation approach to geometry, Klein (1893/1957) laid out a hierarchy of groups of transformations to a form and showed how each transformation group was the basis of a different sized geometry. Considering the transformations themselves, there are five levels, each level a larger set permitting greater changes to the form. They are: isometric, similarity, affine, projective, and topological. Figure 7 shows the increasing changes permitted to a square under the different groups of transformations.

The two levels relevant to the present situation are the second and third levels: *similarity* and *affine* transformations. In the present view, mapping a square visual space into a smaller square in motor space was easier to acquire than mapping a square visual space into a rectangle in motor space because the later transformation (affine) altered more geometric properties than the former transformation (similarity). Similarity transformations allow uniform expansions and contractions to both dimensions equally. Consequently, they can turn a square into a larger or smaller square, or a rectangle into a larger or smaller rectangle with unchanged width-to-height ratio, and so forth. These transformations alter properties of location and "size", but all remaining shape properties are unaltered, such as collinearity ("straightness"), parallelism, angle, and ratio of all lengths. Similarity transformations are a subset of affine transformations, which permit the uniform expansions and contractions to be applied separately to two orthogonal dimensions. Consequently, affine transformations can not only turn squares into different size squares, but also into rectangles, or into rhombi. That is, they can alter the properties of angle and ratio of lengths along with those of location and size, but still leave other shape properties intact such as parallelism, collinearity and some ratio of lengths. All similarity transformations are affine transformations, but not all affine transformations are similarity transformations. In general, affine transformations destroy more geometric properties than similarity transformations, and consequently will be more difficult to acquire.

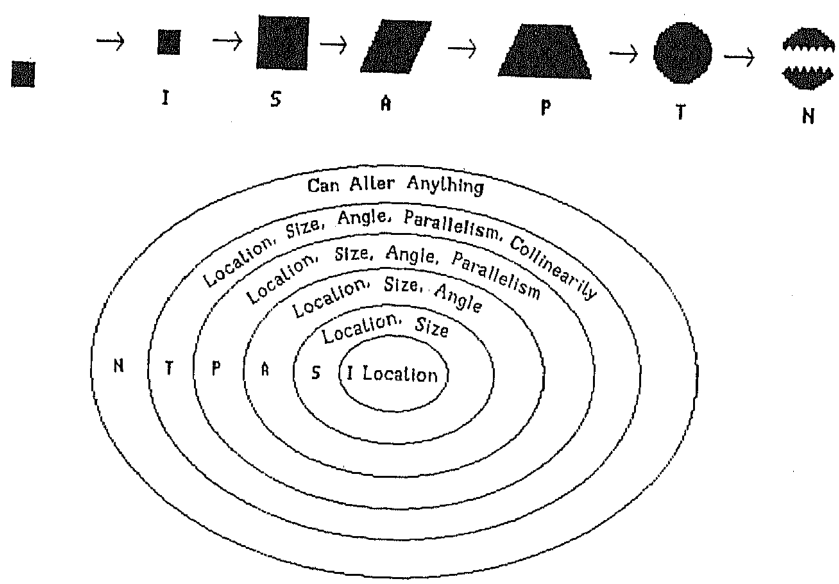


Figure 7. a) Increasing transformations of a square permitted by the different levels of Klein's hierarchy on the transformation approach to geometry. The original square is the first form on the left. I = isometric, S = similarity, A = affine, P = projective, T = topological, NT = non-topological. b) Properties altered by the different groups of transformations. The figure also depicts the characteristic that each level is a subset of, or nested within, all subsequent levels.

Note that the computer mouse can be viewed as causing a similarity transformation. A small range of the mouse corresponds to a larger range on the screen, and the ratio is equal for both X and Y directions. *At a low velocity*, the mouse on my computer for a particular software package has a scale chance of 1 to 4, such that one inch of mouse movement equals 4 inches on the screen. (Computer mice actually map velocity of mouse movement onto distance on the screen.) The reduction of x and y ranges equally may contribute to the relatively easy acceptance of the mouse.

The remainder of the hierarchy makes a number of additional predictions. Isometric transformations should be even easier to accommodate than similarity transformations. Isometric transformations only permit the form to be displaced and/or rotated in its entirety to a new location. From a geometric standpoint, this group is the most benign of all possible groups of transformations. Only absolute location is changed; all other characteristics of the initial form remain intact such as the distance between all pairs of points, the angles, etc. Perhaps the ease with which people use the digitizing tablet paradigm with minimal exposure reflects the ease with which just location changes are handled. These devices change only location (in 3 dimensional space, 2 displacements and 1 rotation)[6] but nothing else. In this view, rotations of motor space with respect to visual space should also be easier to acquire than similarity transformations. The fourth level, which should be harder to acquire than affine transformations, are projective transformations. The transformations permit the parallelism of lines to be altered, as well as location, size, and angle. Finally, the most radical transformations from a geometric standpoint are topological. Here straight lines can even be turned into curved lines thus destroying most of the properties we intuitively think of characterizing the form of a stimulus. The present study is consistent with the predicted ordering of the second and third levels. We have begun directly testing whether all levels are accommodated such that topological are harder than projective, which is turn harder than affine, and so forth. The data look promising and a preliminary report is given elsewhere (Bedford, 1993b).

There are two concepts related to successful visual-motor performance that may have similar solutions based on geometry. One concept

6. Klein's hierarchy can be extended to three dimensions of space, which adds a number of additional levels.

is *telepresence* (e.g., Akin, Minsky, Thiel, & Kurtzman, 1983; Held & Durlach, 1989; Kubovy, 1986, who calls it movable "egocenter"), a rarely studied construct in perception whose importance has recently been recognized. Telepresence refers to the experience of moving where we usually *feel ourselves to be located* to a different location. The location where we feel ourselves to be seeing from, and more generally where we localize ourselves in space, does not appear to be completely determined by the physical boundaries of our bodies (Dennett, 1980; Kubovy, 1986). Instead it appears flexible and movable, and in part determined by what we are seeing and manipulating. For instance, perspective in a painting can make an observer have a felt point of view which differs from the actual point of view (Kubovy, 1986). When watching a movie, we can feel ourselves adopting the camera's point of view rather than our own (see Cutting, 1987).

Considering some tasks with transformed visual-motor space, video games seem to transport the player to the screen although the joystick is located elsewhere. Operating a robot arm in a distant location is easier if you have the experience of being where the robot's arm is. Alteration of a sense of "presence" may also include changes in felt size and shape in addition to location. Does a player feel smaller when his/her alter ego, which is slaying monsters on a computer screen, is only 3 inches tall? Telepresence is difficult to study because it involves an experience that is difficult to quantify and may even sound mystical. Nonetheless, there appears to be a high correlation between success at different transformations and anecdotal reports of being transported or transformed, although the origins of this relation are not well understood. It has been suggested that the feeling of telepresence is governed by the degree of similarity between the motor activity and the visual feedback. This sense of similarity may also be quantifiable by exactly the same taxonomy as determines similarity between the visual and motor space. In the geometric view, the sense of telepresence would be more likely to break down the more geometric properties are changed when mapping from one space to the other.

The second concept related to successful visual-motor performance is *identity*. Success at different transformations depends, in part, on whether the visual information and the motor information are judged to derive from the same object (Bedford, 1993a, 1993b, 1992; Held & Durlach, 1989; Welch, 1978; Welch & Warren, 1980). For instance, if you move the pen with your hand on the digitizing tablet, then the visual information on the screen must be judged to refer to the same pen and hand. Intuitively speaking, if visual and motor experiences are too dissimilar, then the sense that your motor actions are causing the visual feedback may disappear, even though they occur simultaneously. Under these circumstances, learning the new relation is impaired. The hierarchy of transformations provides a metric for determining when two sources of information will be judged to derive from the same object. In this view, if the two forms (i.e., shape of the space) differ only in location then they are very likely to refer to the same object, if they differ in size as well, then they are slightly less likely, if different in angle then even less likely and so forth. Note that in this view, the judgement of identity is a graded or probabilistic decision, rather than an all or none decision.

The present study raises a number of new questions. At a more molecular level, what types of computations are used to acquire a new mapping? It is interesting to note that the preference between mappings found by the present study is easier to model in a system of *polar coordinates* then in one of *Cartesian coordinates.* Cartesian or rectangular coordinates represent the location of any point in coordinates of x (left to right) and y (up to down) [x,y]. In contrast, polar coordinates use angle and distance parameters to locate a point in space [o,d]. In Cartesian coordinates, a mapping which shrinks only X, but leaves Y unchanged requires only one parameter to code the change (e.g., $X_{new} = .5X_{old}$) whereas a mapping which shrinks both X and Y requires two parameters (e.g., $X_{new} = .5X_{old}$; $Y_{new} = .5X_{old}$). In polar coordinates, the ease of representation is reversed. A mapping which shrinks both X and Y equally affects only one parameter (distance), whereas now it is a mapping which shrinks only X which requires two parameters (distance and angle). Note that not all the levels of the geometric hierarchy would order themselves neatly with polar coordinates. For instance, some isometric transformations (displacements) should be easier than similarity transformations within the geometric hierarchy, but harder to model in polar coordinates than similarity transformations. However, intuitively, polar coordinates can be viewed as integrating X and Y directions from the start of the computations, whereas Cartesian coordinates keep them apart. Given the general dependence of two dimensions of space, a useful starting point may be a polar coordinate model (see Cunningham, 1989, for one polar coordinate model).

At a more molar level, what is the role of conscious awareness? For example, are squares more similar to smaller squares than they are to rectangles at the conscious level? If so, was it this limitation which determines the v-m mappings most readily learned? One of the most interesting reports from subjects following the experiment involves subjects who received the mapping where only X was shrunk but not Y. Several subjects incorrectly reported that *both* X and Y dimensions seemed to be smaller on the tablet. Where did the awareness of false information come from? Did the false awareness cause the mapping to be acquired incorrectly, or was it the acquisition of the wrong mapping which caused the false awareness?

Finally, when will the dimensions behave independently? The present study also found that when Y alone is shrunk, X does not clearly change with it. The asymmetry may be due to the asymmetry in the paradigm, or in the spatial directions themselves. Other factors which could influence the independence/dependence include the paths along which the pen travels (diagonal paths vs. vertical and horizontal paths only), the starting point on the tablet (middle vs. a corner) and the type of practice task (e.g., travelling to reach a small target vs. tracing the contours of a large target).

To conclude, how can people learn to use devices such as kites, automobiles, airplanes, robotic arms, video games, computer mice, and even virtual reality, all of which alter the normal visual-motor relation? One possibility is that we make use of general all-purpose cognitive machinery. Humans excel at flexibility in response to seemingly arbitrary new tasks. An alternative, which I prefer, is that adjusting to these transformations taps into the more specialized domain of *perceptual learning* (see Bedford, 1993b, especially pp. 40-43). Visual-motor transformations occur naturally with physical growth in childhood and drift of complex systems in adulthood (Held, 1965; Howard, 1980). Processes which deal with these situations operate according to highly structured rules and constraints to keep internal perceptual systems functioning optimally (e.g., Bedford, 1989, 1993a, 1993b, 1994, in press; Bedford & Reinke, 1993). Future research will determine the best framework in which to place these modern motor skills.

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REFERENCES

Akin, D. L., Minsky, M. L., Thiel, E. D., & Kurtzman, C. R. (1983). Space applications of automation, robotics, and machine intelligence systems, (ARAMIS)-Phase II, Vol. 3: Executive Summary, M.I.T., Contract NAS 8-34381, NASA Marshall Space Flight Center.

Bedford, F. L. (1989). Constraints on learning new mappings between perceptual dimensions. *Journal of Experimental Psychology: Human Perception and Performance, 15,* 232-248.

Bedford, F. L. (1993a). Perceptual and cognitive spatial learning. *Journal of Experimental Psychology: Human Perception and Performance, 19,* 517-530.

Bedford, F. L. (1993b). Perceptual learning. In D. Medin (Ed.), *Psychology of learning and motivation, 30,* 1-60.

Bedford, F. L. (1994). A pair of paradoxes and the perceptual pairing process. *Cahiers de Psychologie Cognitive/ Current Psychology of Cognition, 13,* 60-68.

Bedford, F. L. (in press). Constraints on perceptual learning: Objects and dimensions. *Cognition.*

Bedford, F. L. & Reinke, K. S. (1993). The McCollough effect: Dissociating retinal from spatial coordinates. *Perception and Psychophysics, 54,* 515-526.

Cheng, K., & Gallistel, C. R. (1984). Testing the geometric power of an animal's spatial representation. In H. Roitblat, T. G. Bever, & H. Terrace (Eds.), *Animal cognition* (pp. 409-423). Hillsdale, NJ: Erlbaum.

Cunningham, H. A. (1984). An Apple microcomputer-based laboratory for the study of visual-motor behavior. Behavior Research Methods, Instruments, and Computers, 17, 484-488. Cunningham, H. A. (1989). Aiming error under transformed spatial mappings suggests a structure for visual-motor maps. *Journal of Experimental Psychology: Human Perception and Performance, 15,* 493-506.

Cunningham, H. A., & Welch, R. B. (in press). Multiple concurrent visual-motor mappings: Implications for models of adaptation. *Journal of Experimental Psychology: Human Perception and Performance.*

Cutting, J. E. (1987). Rigidity in cinema seen firom the front row, side aisle. *Journal of Experimental Psychology: Human Perception and Performance, 13,* 323-334.

Dennett, D. (1980). *Brainstorms.* Cambridge, MA: MIT Press.

Fitts, P. M., & Seeger, C. M. (1953). S-R compatibility: Spatial characteristics of stimulus and response codes. *Journal of Experimental Psychology, 46,* 199-210.

Held, R., & Durlach, N. (1989). Telepresence, time delay, and adaptation. In S. R. Ellis & A. Grunwald (Eds.), *Spatial displays and spatial instruments.* NASA Conference Publication 10032.

Ittelson, W. I. (1993). Mirror reversals: Real and perceived. *Perception, 22,* 855-861.

Ittelson, W. I., Mowafy, L., & Magid, D. A. (1991). The perception of mirror-reflected objects. *Perception, 20,* 567-584.

Klein, F. (1957). *Vorlesungen über hohere Geometrie* (3rd ed.). New York: Chelsea. (Original work, Einleitung in hohere Geometrie I, II, published 1893).

Kubovy, M. (1986). *The psychology of perspective and Renaissance art.* Cambridge: Cambridge University Press.

Modenov, P. S., & Parkhomenko, A. S. (1965). *Geometric transformations* (Vol. I & II). New York: Academic Press.

Proffitt, D. R., Gilden, D. L., Kaiser, M. K., & Whelan, S. M. (1988). The effect of con-figural orientation on perceived trajectory in apparent motion. *Perception and Psychophysics, 43,* 465-474.

Welch, R. B. (1978). *Perceptual modification adapting to altered sensory environments.* New York, San Francisco, London: Academic Press.

Welch, R. B., & Warren, D. H. (1980). Immediate perceptual response to intersensory discrepancy. *Psychological Bulletin, 88,* 638-667.

Woodworth, R. S. (1938). *Experimental psychology.* New York: Henry Holt & Co.

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