Astrophysics and Cosmology Special Interest Group
"Galaxy Simulations"

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October 19, 2003

Abstract

Galaxy simulations using powerful computers or groups of computers have been imperious models of galactic and cosmological evolution. I present not only the theoretical side of galaxy simulations, but I also delineate the historically prominent galactic observations.

Observational Galactic Astronomy

Since no science can exist without observation, we will discuss first the leading observations of galaxies in radio and optical light performed in the twentieth century.

Shapley-Curtis Debate

During the 1920s, Harlow Shapley and Heber Curtis debated whether observed fuzzy "spiral nebulae," which are today known as galaxies, exist either in our Milky Way Galaxy or beyond it. Shapley took the stance that these "spiral nebulae" observed in visible light do indeed exist in our galaxy while the more conservative Curtis believed in the converse. While both sides of this debate seemed to either make too many suppositions or supported their claims with flawed data, the main points of both stances made the debate fairly close at the time. [6]

Shapley's Argument

Shapley argued that these "spiral nebulae" are actually within our galaxy's halo. He presented faulty data which seemed to confirm that M101, a large angular diameter galaxy, changed angular size noticeably. We know now that for M101 to change angular size appreciably (0.02 arcsecond / year) it would have to recede from us at warp speeds based upon our current knowledge of its diameter and distance. To account for all the galaxies' observed recession from earth, not knowing of the universe's expansion, Shapley invented a special repulsion force to explain this strange phenomenon. He was not cogently able to explain why galaxies observed from earth are less densely distributed along the galactic equator, though. [6]

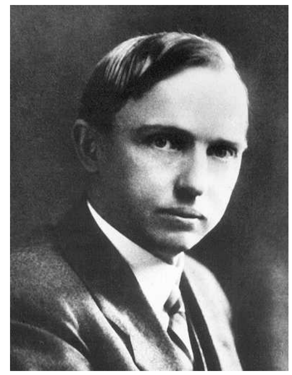

Figure 1: Harlow Shapley was born in 1885 in Missouri and died in 1972. [4]

Curtis' Argument

Curtis remained more conservative in his argument by not introducing outlandish suppositions like the special repulsion forces of Shapley. Curtis, although not able to explain galaxies' redshifts well either, made a convincing claim for the distribution of galaxies in the sky. Because observed galaxies seen edge-on often contain opaque gas lanes and if our galaxy is similar to those observed galaxies then we should not see extragalactic objects near our equator because of our own opaque dust lane. Hubble's observation of Cepheid variables in the Andromeda Galaxy confirmed the distance to a galaxy: millions of light years, not thousands! Curtis was obviously correct. [6]

Radio Astronomy

Oort assigned one of his students, van de Hulst, to find an interesting, useful radio frequency for observing HI, or unionized atomic hydrogen. (HII is singly ionized atomic hydrogen, and H_2 is molecular hydrogen.)

Figure 2: Heber Curtis was born in 1872 in Michigan and died in 1942. [3]

He then discovered the 21 cm hydrogen band which is predominately used today in radio astronomy. Caused by the energy released from magnetic alignment of the atomic hydrogen, this useful band has observed much about the kinetics of galaxies.

By looking at different portions of the galactic equator, astronomers like Lindblad and Oort have determined our galaxy differentially rotates, faster towards the center. By using high resolution radio telescopes like today's Very Large Array (VLA), individual galaxies' rotational velocity dispersions have been determined. Our Milky Way Galaxy also contains spiral arms. Our galaxy's center was determined by the geometric center of Shapley's observations of globular cluster positions using optical means; however, without radio astronomy we would not have determined the need for dark matter, a hotly debated topic today. [6]

Theoretical Galactic Astronomy

Models of galaxies on computers enable accurate predictions of galactic evolution. Almost all of them are based on Newtonian mechanics where force on an object, defined as

$$\vec{F} = m\vec{a}, \tag{1}$$

is proportional to a resultant acceleration $\vec{a}$ by the mass of the object m ($\vec{F} \propto \vec{a}$, where m is a proportionality constant).

Newton's Gravity Law

Newton observed that the gravitational force of one object with mass m_1 on another object with mass m_2 separated by a distance r obeys the law

$$\vec{F} = \frac{Gm_1m_2}{\vec{r}^2} \tag{2}$$

Figure 3: Swede Bertil Lindblad was born in 1895 and died in 1965. He began working on measuring our galaxy's differential rotation while director of the Stockholm Observatory. [4]

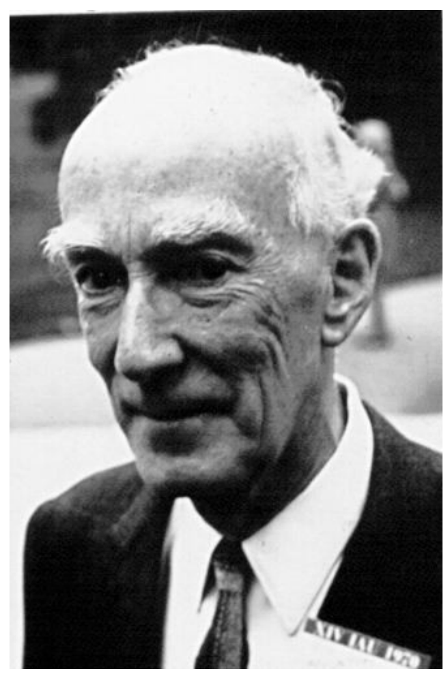

Figure 4: Jan Hendrik Oort lived from 1900 to 1992 and worked at the University of Leiden. Oort finished formulating Lindblad's mathematical description of the dynamics of the Milky Way. [4]

Figure 5: Discoverer of the 21 cm hydrogen emission line, Hendrik Christoffel van de Hulst lived between 1918 and 2000. [4]

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where G is the experimentally determined *gravitational constant*. This is an *inverse square* law because as r increases the graviational F decreases as $F \propto 1/r^2$.

Numerical Simulation Methods

Particle-Particle and Barnes-Hut are two fairly popular algorithms for simulating the position evolution of many objects like stars in galaxies or globular clusters. They both either trade accuracy for speed or vice versa. They are all numerical because they approximate to certain degrees of accuracy the positions of the stars at certain discrete time intervals. [2]

Particle-Particle $\mathcal{O}(n^2)$

This is the simplest of the simulation methods. We will denote $\vec{F}_{a,b}$ as the gravitational force $\vec{F}$ on a from b. To find for a particular object the net gravitational force, all one has to do is sum the forces exerted on that object by the rest of the objects. So for an object a

$$\vec{F}_{net} = \sum_{i=1}^{n} \vec{F}_{a,i} \tag{3}$$

where n is the number of objects in the simulation. From (2)

$$\begin{aligned} \vec{F}_{a,i} &= \frac{G m_a m_i}{\vec{r}_{a,i}^{\,2}} \\ &= m_a \vec{a}, \end{aligned}$$

with m_a a's mass, m_b b's mass, and $\vec{r}_{a,i}$ the distance from a to i; therefore, acceleration caused by i

$$\vec{a} = \frac{G m_i}{\vec{r}^{\,2}}. \tag{4}$$

Relation (3) rewritten determines

$$\vec{a}_{net} = \sum_{i=1}^{n} \frac{G m_i}{\vec{r}_{a,i}^{\,2}}. \tag{5}$$

This algorithm then determines $\vec{a}_{net}$ for each star, integrates over a discrete time step to find position, updates each star's position, and repeats this procedure an arbitrary number of times.

Usually G is normalized to 1 to reduce the overhead of unnecessary multiplication operations.

$\mathcal{O}(n^2)$ refers to the efficiency of this algorithm given n stars. The larger n the more time forces must be summed which is why this algorithm, despite being accurate, is fairly impractical for large n. n^2 is quite a large number for simulations of globular clusters, for example, with *millions* of stars. This algorithm is deterministic, though. The number of operations is asymptotically n^2, but the number of operations for acceleration summation at a discrete time step of the simulation is exactly $n(n-1)$. [5, 2]

Barnes-Hut $\mathcal{O}(n \log n)$

This more efficient, less accurate algorithm relies on the concept of center of mass. For a grouping of n objects with positions $\vec{r}_1, \vec{r}_2, \vec{r}_3, ...$ and masses $m_1, m_2, m_3, ...$, their center of mass

$$\vec{R}_{com} = \frac{1}{M} \sum_{i=1}^{n} m_i \vec{r}_i \tag{6}$$

where the total mass of the grouping

$$M = \sum_{i=1}^{n} m_i.$$

It is essentially a mass-weighted average of all the objects' positions.

To approximate sufficiently distant groupings of objects by their centers of mass, the Barnes-Hut method creates what is called a *quad-tree* for a two spatial dimensional or an *oct-tree* for a three spatial dimensional simulation. For a two dimensional case, the star field to be simulated might look like Figure 6. By splitting the stars into recursively smaller spatial compartments, Figure 7 helps create the quad-tree to determine, when summing forces as in the Particle-Particle

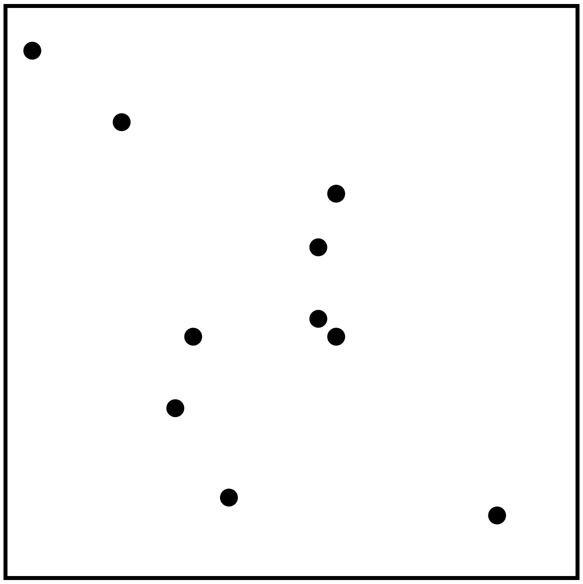

Figure 6: A 2D 10-body star field.

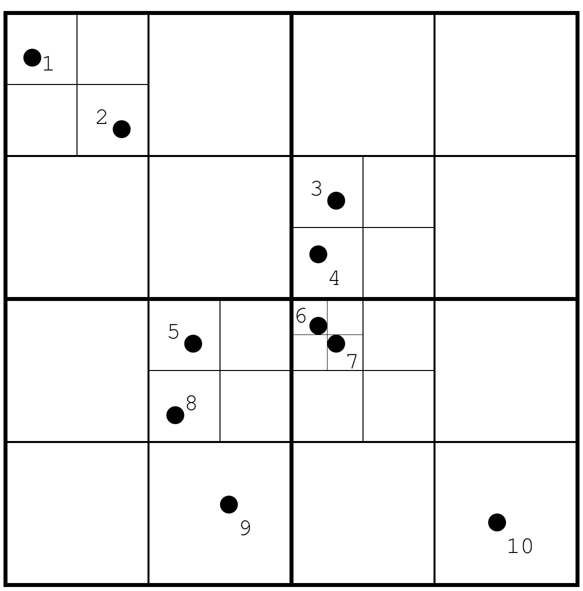

Figure 7: The same 10-body star field as Figure 6 but divided spatially recursively until there is only one or no star per compartment.

method, which groupings of objects can be approximated by their center of mass.

Once the algorithm generates the quad-tree, force, and thus acceleration, calculations proceed for every star. The tree is *traversed*, starting at the root node which is the system's entire center of mass. Traversing the tree repetitively calculates force on every particle stopping not once all the leaves have been visited during a particular traversal, but when it is sufficient to approximate a system whose descendants will not be traversed by its center of mass, a node in the tree. The algorithm then integrates the acceleration to find position, updates the particles' positions, generates a new quad-tree, traverses it repetitively to find net forces on each particle, and repeats this process arbitrarily for future time steps. [1]

References

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