

In Search of Experimental Support for an Asymmetric Equilibria Solution in Symmetric Investment Games: A Reply

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Abstract

In one of their experimental studies, Rapoport and Amaldoss (2000) evaluate the behavior of subjects in a two-person investment game with symmetric players using the symmetric (completely) mixed-strategy equilibrium solution as the normative benchmark. Dechenaux et al. (2005) claim additional support for an alternating (asymmetric) equilibria solution. However, both aggregate and individual level analyses of our data soundly reject the asymmetric alternating equilibria solution.

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Rapoport and Amaldoss (R-A, 2000) conducted two experiments to study the behavior of subjects in an investment game. In Experiment 1 the players were symmetric, and we assessed the behavior of the subjects against the symmetric (completely) mixed strategy solution. In Experiment 2 players faced different budget constraints, and we compared the behavior of subjects against the asymmetric mixed-strategy equilibrium solution that involves iterative deletion of dominated strategies followed by randomization over a subset of the original strategies. The focus of the comment of Dechenaux et al. (DKL, 2005) and our present reply is confined to Experiment 1.

Experiment 1 consisted of 160 trials. Eighteen subjects divided into 9 pairs participated in one of two sessions (a total of 36 subjects). A random pairwise matching design was used to assign subjects to pairs in each trial. Furthermore, on any particular trial each subject was neither informed of the identity nor the past decisions of the other member in his/her pair. This design was implemented to minimize reputation effects. Each subject received an endowment of $e=5$ units of money on each of the 160 trials. With a strategy space $\{0, 1, \dots, 5\}$, each subject was asked to choose an investment level on each trial. The player who invested the greatest amount received a reward of r ($r=8$ for 80 trials in Game L and $r=20$ for 80 additional trials in Game H), whereas the other player received nothing. Ties were counted as losses for both players. Investments were forfeited. At the end of each trial, subjects were only informed of the outcome of their pair.

There is no pure-strategy equilibrium solution to this game. The symmetric (completely) mixed-strategy equilibrium solution is for each player to invest 0, 1, 2, 3, 4, and 5 with probabilities $1/r, 1/r, 1/r, 1/r, 1/r,$ and $(r-e)/r$, respectively. In the alternating equilibria solution (DKL, 2005), however, the two symmetric players behave asymmetrically, one of them invests 0, 2, and 4 with probabilities $(r-e+1)/r, 2/r,$ and $2/r$, respectively, and invests 1, 3, and 5 with probability zero; whereas the other player invests 1, 3, and 5 with respective probabilities $2/r, 2/r,$ and $(r-e+1)/r$, and invests 0, 2, and 4 with probability zero.

What are the implications of the alternating mixed-strategy equilibria for the analysis of our experimental data? After introducing the asymmetric mixed-strategy equilibrium solution, DKL write: “The existence of these alternative equilibria and the possibility that some subset of experimental subjects may sometimes play the corresponding equilibrium strategies leaves open the possibility that the experimental evidence examined in Rapoport and Amaldoss (2000) is more consistent with Nash equilibrium behavior than is evident from R-A analysis” (p. 3). They illustrate this claim by including in Tables 1 and 2 the range of equilibrium predictions and the range of expected payoffs, respectively, in an attempt to show that taking alternating equilibria into consideration extends the set of actions consistent with Nash equilibrium behavior. They bolster this very general claim by arguing that mean payoffs in the R-A study increased with the prize value, a property that is not consistent with the symmetric mixed-strategy equilibrium. They further argue that the frequency of zero investment in the R-A study is consistent with the asymmetric but not the symmetric equilibrium (p. 7).

What are we to make of the DKL claim? The answer depends on the assumptions we make in testing the implications of the equilibrium solutions conditional on the design of our experiment (multiple subjects, multiple trials, random assignment of subjects to pairs on each trial, and outcome information restricted to individual pairs, not the population). We consider below three possible models.

Model 1 assumes that *all* 18 subjects adhere to the *same* equilibrium strategy over *all* iterations of the stage game. This model allows us to competitively test the descriptive power of symmetric and asymmetric equilibrium strategies against each other.

Model 2 is more general. It asserts that different *subsets* of subjects adhere to the *same* equilibrium over *all* iterations of the stage game. The size of each subset is left unspecified.

Model 3 is also more general. It relaxes the assumption of Model 1 that players draw their decisions from the same equilibrium (distribution) on all trials. Rather, it assumes that *all* players play the *same* equilibrium on any given trial, but that the equilibrium strategy may *vary* from one trial to another in some unspecified way. This interpretation was suggested to us by an anonymous reviewer who writes: “Nothing in the specification of Nash behavior requires that the players play according to the same equilibrium in each period. In fact, included in (subgame perfect) Nash behavior over a finite repetition of the game (even with varying players who never meet more than once and no information about past play) is *any pattern* (emphasis is ours) of joint one-shot equilibrium play across periods.”

Clearly, Models 2 and 3 generalize Model 1; Model 2 reduces to Model 1 if there is only a single subset, and Model 3 reduces to Model 1 if there is no variation in the choice of equilibrium distributions (strategies) across trials. Both these models are consistent with the general claim (see above) made by DKL. In what follows, we discuss the properties of the alternating equilibria solution and examine the extent to which our data are consistent with this solution. Starting first with Model 1, we show that our data reject the alternating equilibria solution on the aggregate as well as the individual level.

Model 1

Property 1: According to the alternating equilibria solution, on the aggregate the probabilities of investing $c=0$ and $c=e$ ($e=5$) are the *same*, namely, $(r-e+1)/(2r)$. Thus, as per the alternating equilibria, on average subjects in the low-reward condition should invest $c=0$ and $c=5$ on 25% of the trials, whereas those in the high-reward condition should invest $c=0$ and $c=5$ on 40% of the trials (see DKL, Table 1). However, as reported in the R-A paper, in the low-reward condition (Game L) subjects invested $c=0$ on 16.9% of the trials and $c=5$ on 41.8% of the trials. Similarly, in the high-reward condition (Game H) on the aggregate subjects invested $c=0$ on 14.1%

of the trials but invested $c=5$ in 62.8% of the trials. We observe similar differences at the level of the individual groups. We can overwhelmingly reject the null hypothesis that the probabilities of investing $c=0$ and $c=e$ are equal to $(r-e+1)/(2r)$ (*reward=20, $c=0$: $t=9.3, p<0.001$; $c=5$: $t=5.9, p<0.001$; $c=5$: $t=5.9, p<0.001$; $c=0$: $t=2.7, p<0.01$; $c=5$: $t=4.3, p<0.001$). Rather, the observed behavior is consistent with the symmetric mixed-strategy solution, which predicts that the probability of investing $c=0$ will be smaller than the probability of investing the entire endowment $c=e$. Furthermore, we note that the difference in the frequencies of choosing $c=0$ and $c=5$ increases with the reward. That is, when $r=8$ the observed difference is 24.8% and it grows to 48.7% when $r=20$. Such an investment behavior is consistent with the symmetric mixed-strategy equilibrium but contrary to the predictions of the alternating equilibria.*

Property 2: In the alternating equilibria equilibrium, one player randomizes placing positive probability at only investment levels $c_i=0, 2, 4, \dots, e-1$, and the other player randomizes placing positive probability at only investment levels $c_j=1, 3, \dots, e$. The equilibrium derives its name from this alternating structure. Zero probabilities are placed on the other investment levels. On examining the strategies chosen by each subject, we find that NONE of the 72 subjects across the two treatments chose only the even elements in the strategy space, namely $c_i=0, 2, 4$. Then, allowing for 10% decision errors, we examined the subjects who used even elements in the strategy space on at least 90% of the trials. We find that only in 1 out of 72 subjects (36 subjects times 2 reward levels) chose even elements on more than 90% of the trials. Weakening the criterion for model support even further, a very weak prediction of the alternating equilibria is that half of the subjects will place greater mass on the even elements in the strategy space. That is, $p(c=0)+p(c=2)+p(c=4) > p(c=1)+p(c=3)+p(c=5)$. This very weak criterion allows substantial scope for potential decision errors. In the high-reward condition, only 5 of the 36 subjects chose even elements more often than odd elements. On further examination, we find that 3 subjects in Group 1

chose even investment levels on 63.75%, 71.25%, and 55% of the trials. Thus, the subjects who chose even elements more often also chose odd elements for a substantial proportion of their choices. In Group 2, 2 subjects chose even strategies on 52.5% and 83.75% of the trials. In the low-reward condition, 5 of the 36 subjects chose even strategies more often. Specifically, in Group 1 we find that 2 subjects chose even strategies on 52.75% and 92.5% of the trials. In Group 2, 3 subjects chose even strategies on 65%, 60% and 67.5% of the trials out of the total 80 trials. Now, it is easy to see that the existence of a segment of subjects who choose only or predominantly even elements seems moot.

Property 3: The alternating equilibria solution predicts that player i who invests $c_i=0, 2, 4, \dots, e-1$ should earn a payoff equal to her endowment e , whereas player j who invests $c_j=1, 3, \dots, e$ should earn a payoff equal to the reward r . In the low-reward condition, none of the 18 subjects received an average payoff equal to their reward ($r=8$; $p<0.001$). Similarly, in the high-reward condition not a single subject's average payoff was equal to the reward ($r=20$, $p<0.001$). Thus, we find no support for the alternating equilibria solution in the payoff space. The observed behavior is more consistent with the symmetric equilibrium as discussed in R-A. Thus, the mean payoffs of the two groups in the low-reward condition (Game L) are not significantly different from e . In the high-reward condition (Game H), mean payoff is 6.845 which is significantly higher than $e=5$. This deviation from the symmetric equilibrium prediction is largely attributed to only a single group.

Property 4: On averaging the predictions of the alternating equilibria solution for the two players, on the aggregate players should choose 0, 1, 2, 3, 4, and 5 with probabilities $(r-e+1)/(2r)$, $1/r$, $1/r$, $1/r$, and $(r-e+1)/(2r)$, respectively. Using the aggregate distribution of strategies across the 18 subjects in each trial as the unit of observation, we can reject the null hypothesis that the empirical distribution of strategies and the predictions of the alternating equilibria are the same in the high-reward condition across the two groups covered in the study (K-S test: $D_{(160)}=0.258$,

$p < 0.01$). We obtain similar results at the level of individual groups (Group 1: $D_{(80)} = 0.265$, $p < 0.01$; Group 2: $D_{(80)} = 0.258$, $p < 0.01$). In the low-reward condition across the two groups, we again reject the null hypothesis ($D_{(160)} = 0.168$, $p < 0.01$). We obtain similar results in Group 1 ($D_{(80)} = 0.244$, $p < 0.01$), but not in Group 2 ($D_{(80)} = 0.093$, $p > 0.1$). As discussed in R-A, the aggregate behavior of subjects is more consistent with the symmetric mixed-strategy equilibrium.

Next, we discuss why the empirical support provided by DKL for the alternating equilibria solution is weak. First, they argue that subjects tended to invest zero more often than the symmetric mixed-strategy solution because of the alternating equilibria. In light of the violations of the properties of the alternating equilibria on both the aggregate and individual level, it seems that alternating equilibria is not the correct explanation for the observed departure from the prediction. Second, they argue that in the high-reward condition subjects earned more than the predicted payoff. Although the subjects earned, on average, 6.845 in the high-reward condition, which is significantly more than the symmetric mixed-strategy solution predicted payoff (i.e., $e = 5$), it is no way near the payoff predicted by the alternating equilibria (namely, $r = 20$). As discussed earlier, the observed payoffs are consistent with the symmetric mixed-strategy solution in the case of low-reward condition. Third, they suggest (see their Tables 1 and 2) that because the alternating equilibria increase the *range* of investment behavior and payoffs, they help to better explain the observed behavior. Under Model 1, in games with multiple equilibria, the correct procedure for the analyst is to identify the properties of the different equilibria and then search for empirical evidence for each equilibrium *separately* rather than seeking support for a *range* that encompasses the predictions of all equilibria (see DKL, Table 1). In this way, the analyst can determine the equilibria that best account for the experimental results. Such a focused search for experimental evidence in support of each individual equilibrium has been extensively used to *select* among the

multiple equilibria in coordination games (see, e.g., Camerer 2003 for a recent review of the experimental literature on coordination games).

Models 2 and 3

We make two arguments in this section, the first is formal and the second substantive. DKL claim that the “existence of these alternating equilibria ... leaves open the possibility that the experimental evidence examined in Rapoport and Amaldoss (2000) is *more consistent* (emphasis is ours) with Nash equilibrium behavior than is evident from the R-A analysis.” Under Models 2 and 3, this claim is *devoid* of any empirical content. Because Models 2 and 3 subsume Model 1 as a special case, any evidence in support of Model 1 also supports these two models. Therefore, no experimental evidence can refute the claim of DKL under these two alternative interpretations.

The substantive argument proceeds as follows. Consider the requirements for tacit coordination that are implied by Model 3. Recall that the strategies of any two members of a pair are in equilibrium, if either 1) both members of the pair choose the symmetric equilibrium strategy (Eq. 1 in DKL), or 2) one member of the pair chooses the mixed strategy in Eq. (2) and the other member the mixed strategy in Eq. (3). Under Model 3, this should hold for all nine pairs of subjects. Recall, too, that because pair membership varies randomly from trial to trial, there are no reputation effects. Moreover, there is no information at the end of each trial about the outcomes of all 18 players that might conceivably help in coordination. We do not see how the choice of strategies under Model 3 is experimentally possible in the absence of an exogenous coordination device that instructs all pairs which mixed strategy to play on any particular trial (e.g., play the symmetric equilibrium on odd trials and the asymmetric equilibrium—with a specification of which member of the pair plays which of the two asymmetric equilibria—on even trials). A similar argument holds even more forcibly in the case of Model 2. We argue that Models 2 and 3 are logically possible, but under our experimental design they are highly implausible.

Conclusion. Under the assumption that our symmetric players adhere to the same equilibrium strategy across all iterations of the stage game, our data strongly reject the asymmetric (alternating) equilibrium solution. The observed behavior on both the individual and aggregate levels is more closely aligned with the symmetric (completely) mixed-strategy equilibrium. If, however, one persists on interpreting the claim of DKL to say that any arbitrary pattern of joint one-shot equilibrium play is allowed, implausible as this hypothesis may be under the experimental design of R-A, then including alternating equilibria (along with the symmetric equilibrium) cannot shrink the set of actions and distributions consistent with Nash equilibrium behavior.

References

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