

Departure Times in Y-Shaped Traffic Networks with Multiple Bottlenecks

ABSTRACT

We study, theoretically and experimentally, the departure time behavior of commuters in a Y-shaped network with two bottlenecks and costs attached to early and late arrivals as well as time spent in the bottlenecks. A mixed-strategy equilibrium solution is constructed which implies that, for certain parameter values, *expanding* one bottleneck, while keeping the capacity of the other fixed, may induce a shift in the endogenously-determined departure times so as to *increase* total travel costs. Our experimental results are strongly supportive of this paradoxical prediction and the equilibrium solution that gives rise to it. (JEL C72, C92, R49)

Keywords: Bottleneck paradox, traffic congestion, queuing, coordination, group decision making.

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The economic analysis of congested traffic networks has a venerable history. First gaining prominence in the classic debate over social costs between A.C. Pigou and Frank Knight (Frank Knight (1924) and later in the seminal article in this journal by William Vickrey (1969), the problem continues to attract academic and public interest. Indeed, increasing costs of congestion in traffic networks may signal the beginning of “a golden age of congestion pricing”¹. The experimental and theoretical research in our paper follows directly from Vickrey’s model. He studied the decisions commuters make when deciding when to depart from home and travel to a common destination along a single road during the morning rush hour. In Vickrey’s model, there is a single bottleneck on the road with a fixed and known capacity. If the arrival rate at the bottleneck exceeds this capacity a queue is formed behind it. If the capacity is binding so that not all commuters can experience no waiting in queue (and, therefore, cannot arrive at their common destination at the same time), then each commuter faces a trade-off between departure time and schedule delay. A commuter may choose to depart early and pay the penalty of arriving at her destination too early (with respect to an exogenously determined desired arrival time), depart late and pay the penalty of arriving too late, or depart at the peak hour when travel time and cost are high but the expected probability of arriving very early or very late is low. In determining when to leave home to the airport for a scheduled flight and negotiate the queues that may form at the baggage checking counter and later on at the security control, airline commuters are fully aware of this tradeoff. So are people who flock to watch a popular movie and have to wait in line to purchase a ticket (if they do not have one) and subsequently get checked for security at the

¹ As proclaimed in a New York Times article (February 11, 2007) discussing recent federal budget initiatives to stimulate the management of congestion on American roadways.

entrance. In Vickrey's model, each commuter independently chooses her departure time to minimize her travel costs, which are assumed to be linear in the travel time and schedule delay. Departure times are in equilibrium when no commuter has an incentive to unilaterally alter her departure time.

Paradoxes in Traffic Networks. Alternative formulations and extensions of Vickrey's model have been reported by, among others, Chris T. Hendrickson and George Kocur (1981), Mike J. Smith (1983), Carlos F. Daganzo (1985), Masao Kuwahara (1990), and Arnott, De Palma and Lindsey (1990, 1993a, 1993b, 1999). Of particular relevance to our paper is the model by Arnott, De Palma and Lindsey (1993a, hereafter ADL), who considered a Y-shaped traffic network comprised of *two* upstream bottlenecks with service capacities s_1 and s_2 , and a single downstream bottleneck with service capacity s_d . In their model, two groups of commuters travel along the Y-shaped corridor from home to work, one entering each arm and passing through the corresponding upstream bottleneck, as well as the downstream bottleneck that is common to both groups. Arnot et al. then showed a remarkable and counterintuitive result, namely, that for certain combinations of bottleneck capacities, schedule delay costs, and travel costs, *expanding* one of the upstream bottlenecks may induce the commuters to alter their departure times in such a way as to *increase* the sum of total queueing and schedule delay costs². Our study reports the results of an experiment primarily designed to study the implications of this theoretical result in the controlled environment of the laboratory with a finite and relatively large number of network users.

The result reported in ADL seems paradoxical, as the usual remedy for combating congestion is to expand the road system (increase service capacity). Transportation researchers have

² This result could be used as the theoretical justification for the practice of regulating the flow of traffic at freeway on-ramps ('metering').

identified additional paradoxes in which this remedy fails (Richard Arnott and Kenneth Small, 1994). Perhaps the most famous, the *Pigou-Knight-Downs* (PKD) paradox, concerns a simple network with a common origin and a common destination connected by two roads, a direct road containing a narrow bridge and a wider, more circuitous, road. For certain parameter combinations (number of commuters, bridge capacity), expanding the bridge's capacity has no effect on travel time. A second, more perverse, paradox is the *Downs-Thompson* paradox. It is like the previous paradox except that the alternative to taking the congested road with the narrow bridge is replaced by a privately operated train line. Both paradoxes are illustrated numerically by Arnott and Small (1994). The third paradox, which is due to Dietrich Braess (1968), consists of showing that adding one or more links to a traffic network, thereby allowing for more routes between a common origin and a common destination, may cause total travel time to increase. In all of these paradoxes, the commuters depart from the origin of the network simultaneously. In the *Bottleneck Paradox* of ADL, which is the subject of this paper, the number of network users, origin, destination, and travel routes are all *fixed*, but departure times are endogenous.

Previous Empirical and Experimental Research. All of these paradoxes occur in abstract networks with alternative routes or modes of transportation, on at least one of which unpriced congestion occurs. Similarly to the Prisoner's Dilemma game, the equilibrium solution (called *user equilibrium* in the context of traffic networks) is inefficient because network users make their decisions selfishly without taking into account the negative externalities they impose on others. After reviewing and numerically illustrating some of these transportation paradoxes, Arnott and Small (1994, p. 451) raised the question: "Are these paradoxes more than intellectual curiosities?" Empirical data that either reject or support the paradox are very hard to come by, as the assumptions underlying these models cannot be exactly met. As early as 1970, John D.

Murchland noted with regard to the Braess paradox: “It seems to me that the importance of the Braess’s paradox for practical networks will only become apparent when sufficiently accurate congested traffic assignment calculations become available and the phenomenon emerges, or fails to emerge, during systematic searches for optimal link additions” (1970, p. 393). Few of these systematic searches have been forthcoming. Perhaps the most frequently cited example is the one by W. Knödel (1969), who remarked that major road improvement in the center of Stuttgart failed to yield the benefits expected. They were only obtained when a cross street was subsequently withdrawn from traffic use (see Murchland, 1970). Intriguing as it may be, this is only anecdotal evidence that does not meet the requirement for “systematic searches for optimal link additions.”

An alternative approach, that has recently emerged in experimental economics, is to test the implications of the equilibrium analyses of various traffic networks and compare user equilibrium with system equilibrium experimentally. This approach consists of simulating simple traffic networks in the controlled environment of the laboratory, having users choose routes in these simulated traffic networks before and after they are subjected to changes in the cost function or structure of the network, and searching for the emergence of systematic and replicable patterns of behavior. If they emerge, then it is of significant interest to determine whether these behavioral patterns support equilibrium play and the paradoxical results it may imply. Experimental studies that subscribe to this approach have recently been reported by Erel Avineri (2006), Yasunori Iida, Takamasa Akiyama, and Takashi Uchida (1992), Laurent Denant-Boement and Romain Petiot (1999), Dirk Helbing, Martin Schönhof, and Daniel Kern (2002), Helbing (2004), Yannick Gabuthy, Matthieu Neveu, and Denant-Boement (2004), Reinhard Selten et al. (2004, 2007), Kerstin Schneider and Joachim Weimann (2004), Avineri and Joseph

N. Prashker (2005), Amnon Rapoport et al. (in press), John Morgan, Hendrik Orzen, and Martin Sefton (2006), Rapoport, Vincent Mak, and Rami Zwick (2006), and Ziegelmeyer et al. (in press).

With the exception of the experiments by Schneider and Weimann and by Ziegelmeyer et al., all of these experiments have focused on *route choice*, not on *departure time*. Schneider and Weimann tested a road-pricing model formulated by Arnott, De Palma and Lindsey (1990). In their model, a fixed number of commuters have to travel from a common origin, O , to a common destination, D , on a single road and pass through a single bottleneck with a fixed capacity s . The Bottleneck Paradox of ADL, which is the focus of the present study, does not arise in such a simple network. Ziegelmeyer et al. reported two laboratory experiments designed to study the impact of public information about past departure rates on travel costs and congestion levels. Their focus is on population size, relative cost of delay, and outcome information about previous congestion rates, not on the paradoxical results of ADL.

The rest of the paper is organized as follows. Section I presents the model. Our model is similar to the models of Kuwahara (1990) and ADL with these major exceptions. The exceptions are that 1) we allow for late arrivals (at high cost) to the common destination with respect to a common desired arrival time (as does Kuwahara) and, to render our model testable in the laboratory, 2) we reformulate the model for a finite number of agents (the ADL and Kuwahara models assume a continuum of infinitesimally small agents). Section II presents, illustrates, and discusses the Nash equilibrium solution for the case under experimental investigation. Section III presents the experimental method and results. Presentation of the equilibrium is relegated to an appendix. Our contribution is threefold. First, and most importantly, we show experimentally that the Bottleneck Paradox is not just a theoretical curiosity. Using a between-subject design, we

show that an increase in the capacity of the upstream bottleneck causes a significant shift in the departure time distribution and, consequently, a significant increase in the commuters' total travel cost. Second, although the observed departure times are in qualitative agreement with the equilibrium solution, our results point to a small but significant departure from the equilibrium departure time distributions: network users who only have to pass through the downstream bottleneck depart from home later than predicted. Third, we show that experience in traversing the network matters. As the network users who have to pass through two bottlenecks gain more experience with the network and learn about the decisions and outcomes of all users, their mean payoff increases and the between-user variability in payoff decreases. Section IV concludes with a discussion regarding the significance and generality of our results.

I. The Model

We consider a Y-shaped traffic network that comprises two upstream bottlenecks with maximum, non-stochastic, capacities s_1 and s_2 , and a single downstream bottleneck with maximum capacity s_d . Capacity is measured by the number of network users per unit time. There are two groups of network users with exogenously determined routes. There are n_1 users in group 1 who depart from one origin and have to first pass through bottleneck 1, and n_2 users in group 2 who depart from a different origin and begin by first traversing bottleneck 2. The routes of the two groups then intersect to flow through the downstream bottleneck that is common to both. Following ADL and using their notation, we assume that the capacity s_1 is sufficiently large that it is not binding. Given this assumption, the network reduces to a corridor with one upstream bottleneck with capacity s_2 (that is only encountered by group 2) and one downstream bottleneck with capacity s_d (that is encountered by both groups). See Fig. 1. If the arrival rate of commuters at either of the two bottlenecks exceeds its maximum capacity, a queue forms behind this

bottleneck. This is the only source of congestion on the network³. The only decision variables in the model are the departure times of the users, and these are endogenously determined. In the next section, we will construct a Nash equilibrium distribution of departure times, $f_g(t)$, for each group g ($g = 1, 2$) of users. Let τ_g be the set of times for which $f_g(t)$ is greater than 0 (the set of times at which users in group g depart).

--Insert Fig. 1 about here--

There are two sources of cost that are common to the network users of both groups. There is the cost per unit of *travel time*, which we denote by α . Without loss of generality, we assume that travel time is zero except for delay experienced in the two bottlenecks. There are also the costs of arriving either too early or too late with respect to a common, exogenously determined, desired arrival time, t^* . The parameter β denotes the cost per unit time of *early arrival*, and the parameter γ the cost per unit time of *late arrival*. Like ADL, we assume $\gamma > \alpha > \beta$. This inequality generally holds in practice. Each commuter is, thus, presented with a departure time choice problem that involves trading off delays in bottlenecks, which are clearly a function of the departure times of the other commuters, with desired arrival time at the common destination.

Denote by $Q_i(t)$ the time required to traverse bottleneck i ($i = 2, d$) at time t . Then, the duration of the trip for a member of group g , who departs home at time t , is given by

$$(1) \quad T_g(t) = Q_g(t) + Q_d(t + Q_g(t)), \quad g = 1, 2.$$

The expression in Eq. (1) assumes different forms for the two groups. For group 2, we have

$$T_2(t) = Q_2(t) + Q_d(t + Q_2(t)).$$

whereas for group 1 (that does not encounter an upstream bottleneck), we have

³ The two bottlenecks are assumed to be sufficiently separated so that the physical length of a queue behind the downstream bottleneck does not interfere with the upstream bottleneck.

$$T_1(t) = Q_d(t).$$

In general, $Q_i(t)$ will depend on the congestion at queue i caused by all departures prior to time t . This will be discussed further in the next section of the paper. The travel cost of a member of group g , who departs home at time t , can then be written as

$$(2) \quad C_g(t) = \alpha T_g(t) + \beta[t^* - (t + T_g(t))] + \gamma[(t + T_g(t)) - t^*].$$

The distributions of departure times of the two groups, $f_1(t), f_2(t)$, constitute an equilibrium if and only if the costs of departing at any time in the sets τ_1 and τ_2 are constant for each group,

$$(3) \quad C_g(t) \begin{cases} = \bar{C}_g & \text{for } t \in \tau_g \\ \geq \bar{C}_g & \text{for } t \notin \tau_g \end{cases} \quad g=1,2.$$

Because group 1 users face no upstream bottleneck, $\bar{C}_1 \leq \bar{C}_2$. The next section of the paper presents results for the case in which $\bar{C}_1 < \bar{C}_2$ and $s_2 > s_d$. This is one of the cases identified in ADL (Case A1) in which the Bottleneck Paradox emerges (Case 3 in Kuwahara).

II. The Equilibrium Solution

In this section, we present the Nash equilibrium distribution of departure times for the model used by ADL with one significant change, namely, the cost of late arrivals, γ , is no longer infinitely large and hence it may be optimal for some agents to arrive after the desired time. The ADL model assumes that the number of agents in each group is infinite and that the time each agent requires to traverse a bottleneck is infinitesimally small. Later in the section we discuss the impact on these results of having a finite set of agents in each group (as required in the experiments) who require finite increments of time to traverse the bottlenecks.

A graph of the cumulative distribution of equilibrium departure times of the two groups of network users is exhibited in Fig. 2, for a particular set of model parameters⁴ that satisfy $\bar{C}_1 < \bar{C}_2$ and $s_2 > s_d$. Derivation of the results shown in Fig. 2 and presented below are contained in Appendix A. These results have strong similarities to those of ADL (Case A1) with important and interesting differences due to the use of a finite late cost, γ .

--Insert Fig. 2 about here--

The first thing to observe in Fig. 2 (see upstream and downstream output curves) is that, in equilibrium, some network users arrive late (after t^*) and that they are all from group 2. Let t_g^o and t_g^e represent, respectively, the departure times of the first and last members of each group, g . The time of the last arrival at the network destination, t_{max} (see vertical line on the right-hand side of Fig. 2), can be shown (see Appendix A) to be

$$(4) \quad t_{max} = t_2^e = t^* + \beta(n_1 + n_2)/s_d(\beta + \gamma).$$

The first user to depart is also from group 2, and her departure time, t_2^o , is

$$(5) \quad t_2^o = t_{max} - (n_1 + n_2)/s_d.$$

Between t_2^o and t_{max} , group 2 users depart at *three* different rates⁵. Initially, they leave at a rate

$$(6) \quad d_2(t_2^o) = d_2^o = \frac{\alpha}{\alpha - \beta} s_d$$

that is greater than the upstream bottleneck capacity, s_2 , and a queue builds at this bottleneck.

Then, at a later point in time when departing group 2 users will subsequently exit the upstream

⁴ For comparability, we follow for the most part the choice of parameters selected in ADL. They provide arguments regarding the appropriateness of the cost parameters in the context of highway traffic commuting. Note that the model is actually a flow model with each user considered to be an infinitesimally small element of the population of users. As a result, n_1 and n_2 are normalized to 0.5 and 1.0 and the bottleneck capacities are likewise set at values such as 0.75/hr. Model results would not change if, for example, $n_1 = 5,000$, $n_2 = 10,000$ and $s_d = 7,500$ /hr.

⁵ The probability of a departure from group g in time increment Δt is $n_g f_g(t) \Delta t$ and, in keeping with the flow model interpretation of ADL, $d_g(t) = n_g f_g(t)$ will be referred to as the *rate* of departures from the group at time t .

queue at the same time as the first of the group 1 users depart, t_1^o , the departure rate for group 2 falls to a rate that is equal to the capacity of the upstream queue, s_2 .

The departure time of the last group 2 user to subsequently arrive at the destination on time (on or before t^*) can be shown to be

$$(7) \quad t^f = \frac{\alpha + \gamma}{\alpha} t^* - \frac{\gamma}{\alpha} t_{\max}.$$

After t^f , group 2 users depart at a third, much lower (assuming large γ), rate of

$$(8) \quad d_2^f = \frac{\alpha}{\alpha + \gamma} s_d.$$

There are $s_d(t_{\max} - t^*)$ users in this last set of group 2 users and they arrive at the destination at times between t^* and t_{\max} . This completes the description of the three departure rates of group 2.

Group 1 users depart at times in the mid-range of the departure times for group 2 users and, as is shown in Appendix A, all group 1 departures must take place while there is a queue at the upstream bottleneck. In particular, the last user in group 1 departs at

$$(9) \quad t_1^e = t_2^o + \frac{n_2 - s_d(t_{\max} - t^*)}{s_2},$$

and the first departure in group 1 occurs at

$$(10) \quad t_1^o = t_1^e - \frac{n_1}{d_1^o},$$

where $d_1^o = \frac{\alpha}{\alpha - \beta} s_d - s_2$ is the constant departure rate of group 1 users over τ_1 .

As is shown in Appendix A, the departure times and rates in equations (4) through (10), that are illustrated in Fig. 2, result in equal costs to all group 1 users of

$$(11) \quad \bar{C}_1 = \alpha \left(\frac{\gamma(n_1 + n_2)}{s_d(\beta + \gamma)} - \frac{n_2}{s_2} + \frac{\beta(n_1 + n_2)}{s_d(\beta + \gamma)} \right),$$

and for all group 2 users of

$$(12) \quad \bar{C}_2 = \beta \left(\frac{\gamma(n_1 + n_2)}{s_d(\beta + \gamma)} \right).$$

The resulting total system cost for all users in both groups is, therefore,

$$(13) \quad TC = n_1 \bar{C}_1 + n_2 \bar{C}_2.$$

From the above expressions it is easy to verify that $\frac{\partial TC}{\partial s_2} > 0$ under the conditions of our model,

$\bar{C}_1 < \bar{C}_2$, and $s_2 > s_d$. This means that increasing the capacity of the upstream bottleneck results in an increase of total network costs. This is the Bottleneck Paradox discussed by ADL.

Figures 3(a) and 3(b) illustrate the equilibrium distributions of departure times for the two conditions that will be used in the subsequent experiment. In both cases, the following model parameters apply:

$$n_1 = 8, n_2 = 16, s_d = 12, t^* = 8:30, \alpha = \$6/hr, \beta = \$3.60/hr, \gamma = \$49.8/hr. \quad \text{The}$$

parameter that varies between the two cases is s_2 .

Because the set of agents in each group is now finite and relatively small (in the experiment there will be 8 people in group 1 and 16 people in group 2), the equilibrium results derived to this point require some modification. The equilibrium distributions presented in Figures 3 (a) and (b) were obtained numerically. They reflect small adjustments in the ‘flow’ model presented above to account for the need for the finite set of agents to play mixed strategies in equilibrium and to account for the finite times needed for an agent to traverse each bottleneck.

In Fig. 3(a), s_2 is set very close to s_d at 12.0001. This results in a large advantage to users from group 1. \bar{C}_1 is just \$4.30 compared to \$7.68 for \bar{C}_2 . Total cost, TC, is \$157.37. On the other hand, in Fig. 3(b) the capacity of the upstream bottleneck, s_2 , is increased to 20. Now $\bar{C}_1 =$ \$6.87, $\bar{C}_2 =$ \$7.52 and TC = \$175.33. Increased capacity results in a degradation in system

efficiency (total travel cost increases by 11.4%) providing a specific instance of the Bottleneck Paradox.

--Insert Fig. 3 about here--

III. The Experiment

A. Method

Subjects. One hundred and forty-four subjects, in approximately equal proportions of men and women, participated in the experiment. The subjects were primarily business administration undergraduate students who volunteered to participate in a group decision-making experiment with payoff contingent on performance. They were recruited using class announcements and bulletin board postings. The participants were divided into six equal-size groups of 24 members each (three groups in each of two conditions). Each group participated in a single session consisting of 55 trials of the departure game (trials 1-5 were for practice only) that lasted about 95 minutes, including 25 minutes of organization and instruction time.

Procedure. The experiment used a between-subject design with two conditions and three groups in each condition (total of six sessions). Hereafter, we refer to these two treatments as Conditions V3 and V5. The experiment was conducted at the Economic Science Laboratory (ESL) at the University of Arizona. The ESL includes multiple networked PC terminals separated from one another by partitions. Upon arrival to the lab, each subject chose a poker chip from a bag containing 24 chips to determine his or her seating. Any form of communication between subjects was strictly forbidden. Once randomly seated in their cubicles, the subjects proceeded to read the instructions (see Appendix B for the instructions for Condition V5) at their own pace. Questions asked during this period were privately answered by the experimenter.

The parameters for the two experimental conditions were chosen to correspond to the two cases for which model results were presented in the end of the previous section. Three transformations of those values were necessary. First, the monetary values for travel costs were expressed in terms of ‘points’ when presented to the subjects (and then converted into money when the subjects were paid at the end of the experiment). Second, the subjects began each trial with an endowment of 1000 points so as to generally (but not always) allow them to make a profit on each trial. Finally, the group population sizes, n_g , and bottleneck capacities, s_i , were chosen to match the cases studied in the previous section of the paper. Thus, the parameters in Condition V3 assumed the values $n_1 = 8$, $n_2 = 16$, $s_d = 12/hr$ (5 min. per person), $s_2 = 20/hr$ (3 min. per user), $\alpha = 10$ pts/min, $\beta = 6$ pts/min, $\gamma = 83$ pts/min, and $t^* = 8:30$ am. Condition V5 used the same parameter values with the only exception that $s_2 = 12/hr$ (5 min. per user). In order to approximate continuous time, departure times for the participants were restricted to the nearest full second over the time interval 4:00 am to 8:30 am – a total of 16,200 possible strategies (see a shot of the decision screen in the instructions in Appendix B).

At the beginning of the session, each subject was provided with a participation fee of \$5. At the end of the session, the cumulative earnings across all the 50 trials were added to the initial \$5 fee to determine the payoff for the session (thus, payoff = cumulative earnings plus fee). Almost all the subjects in Conditions V3 and V5 completed the session with positive payoffs. Participants who ended up with low payoffs were paid a flat payment of \$10.00, although they had not been informed of this compensation method ahead of time as to not affect their behavior (thus, payment = max(\$10, payoff)). Mean payoff for all subjects was \$26.56.

Each trial was structured in the same way. Once the trial commenced, each subject was asked to independently choose and register a departure time. Departure time decisions were made

anonymously with no time pressure. In this particular environment, in order not to provide any cues for coordinating behavior, the subject's station identification number--a number between 1 and 24--was never revealed. Once all the 24 group members typed in their departure time decisions, a central server calculated the delays, arrival times, and travel costs which were consequently presented to the subjects. In both Conditions V3 and V5, each subject was provided with the following information:

- the number of subjects ahead of her in each bottleneck that she encountered;
- the delays and costs incurred at each phase of their trip;
- the departure time, arrival time, delays at each bottleneck, and net profit for each of the 24 network users.

This was accomplished by presenting a "Results" screen that consisted of a 24×5 table with rows corresponding to the 24 players (but with no identification as to the station number of the subject) ordered in terms of their departure times and colored (blue = group 1; green = group 2) according to the player's group, and 5 columns designating the

- departure time,
- delay at the upstream bottleneck, called Delay A,
- delay at downstream bottleneck, called Delay B,
- arrival time,
- individual payoff for the trial

(refer to the Results Screen in Appendix B). The instructions also included detailed examples of the profit calculations resulting from hypothetical departure times and bottleneck conditions. The subjects played five practice trials for no payment to familiarize themselves with the mechanics of the game, and then played fifty additional trials with payment contingent on performance.

B. Results

The analyses presented in this section are organized as follows. First, we present means and standard deviations of the individual payoffs and use them to test the major implication of the Bottleneck Paradox.⁶ Next, we present the aggregate departure times, and for each group separately compare them to the equilibrium results. This is followed by an analysis of the dynamics of play that focuses on the switches in departure time between adjacent trials. Finally, individual differences are displayed and discussed in the last sub-section.

Payoffs. Table 1 (columns 3-6) presents the means and standard deviations (in dollars) of the payoffs across all 50 trials. The results are presented for group 1 (“Blue”, $n=8$) and group 2 (“Green”, $n=16$) by session. Within each group, they are shown separately for each of the two conditions. The right part of Table 1 (columns 7-10) displays the same statistics for the last ten trials, namely, trials 41-50. In equilibrium, expansion of the upstream bottleneck results in a substantial increase in the travel cost of group 1, but only a very small decrease in the travel cost of group 2. There is also a large increase in the total system cost as capacity is increased confirming the Bottleneck Paradox. As shown in the previous section, $\bar{C}_1=4.30$ in Condition V5 and $\bar{C}_1=6.87$ in Condition V3. When subtracted from the \$10 (1000 points) endowment for each trial, this results in equilibrium payoffs of \$5.70 and \$3.13 per trial for group 1 members in Conditions V5 and V3, respectively (bottom row of Table 1). In contrast, equilibrium payoffs for group 2, whose members encounter the upstream and downstream bottlenecks, are relatively small in both conditions, \$2.32 in condition V5 and \$2.48 in condition V3 (bottom row of Table 1).

⁶ The experimental units were “points”. To conform with the payoff structure in the model, we present the results below in dollars (100 points = \$1.00). We used a different conversion rate (see instructions) to determine the actual earnings at the end of the session.

--Insert Table 1 about here--

The mean payoffs in Table 1 provide strong evidence in support of the equilibrium predictions. A comparison of the mean payoffs in group 1 shows that the subjects in Condition V5 earned, on average, 64 percent *more* than the subjects in Condition V3 (compare 3.38 to 5.55). Using the session as the unit of analysis, the null hypothesis that the three sessions in Condition V3 and the three in Condition V5 are drawn from the same population was rejected by the Mann-Whitney U test ($p < 0.05$, one-tailed test). When mean payoffs are compared between conditions by a parametric test, using this time the subject (rather than the session) as the unit of analysis, the null hypothesis of equality of means of the two conditions is overwhelmingly rejected once again ($t_{46} = 38.91$, $p < 0.0001$).

A comparison of the mean payoffs in group 2 shows that the subjects in Condition V5 earned, on average, 14.4 percent *less* than the subjects in Condition V3 (compare 1.80 to 1.54). Using the session as the unit of analysis, the null hypothesis that the three sessions in Condition V3 and the three in Condition V5 are drawn from the same population could not be rejected by the Mann-Whitney U test. This result is consistent with the equilibrium prediction. The parametric test also failed to reject the null hypothesis ($t_{94} = 1.20$, $p = 0.23$). We repeated all the analyses above for the individual payoffs in the last block of ten trials (trials 41-50) and obtained essentially the same results.

Table 2 displays the means and standard deviations of the individual payoffs by group, condition, and blocks of ten trials. The results are collapsed across the three sessions. Inspection of the two top rows shows that, under each condition, mean payoffs for group 1 maintain approximately the same value for all five blocks: mean payoffs of the subjects who only had to pass through the downstream bottleneck are remarkably stable over the 50 trials. In contrast, the

variability between the subjects, measured by the standard deviation, is seen to decline sharply across blocks from 1.23 in block 1 to 0.53 in block 5 of Condition V3, and from 1.54 to 0.47 in Condition V5. In contrast to the stability of group 1 payoffs, mean payoffs in group 2 (rows 3 and 4) increase steadily across blocks in both conditions, with the major change occurring between blocks 1 and 2. Similarly to group 1, but even more dramatically, the standard deviations of the individual payoffs decrease steadily across blocks from 2.67 in block 1 to 0.53 in block 5 of Condition V3 and from 3.28 to 0.74 in Condition V5.

--Insert Table 2 about here--

These observations are supported by two statistical analyses that we conducted separately for groups 1 and 2. For each subject, we first computed her mean payoff in each block of trials, 5 scores for each subject. Then, for each group separately, we subjected these scores to a 2×5 condition by block ANOVA with repeated measures on the block factor. When applied to the mean individual payoffs of the subjects in group 1, the ANOVA yielded a significant condition effect ($F(1, 46)=325.4, p<0.0001$), but no block ($F(4,184)=0.95, p=0.44$) or block by condition ($F(4,184)=1.72, p=0.15$) interaction effect. When applied to the mean individual payoffs in group 2, the analysis yielded a significant block effect ($F(4,376)=25.5, p<0.001$) but no condition ($F(1,94)=1.43, p=0.23$) or condition by block ($F(4,376)=0.64, p=0.64$) interaction effect. Taken together, these results confirm our previous conclusion that the mean payoffs of the subjects in group 1 hardly changed with experience. Moreover, they are accounted for rather well by the equilibrium solution. Mean payoffs of the subjects in group 2 (rows 3 and 4) steadily increased with experience in the direction of the equilibrium payoffs but stayed slightly below them. Importantly, the sharp and systematic decrease in the standard deviations of payoffs in

both conditions and in both groups is in agreement with the equilibrium solution that considers the subjects within group as homogenous (all players within the same group earn the same).

Aggregate Departure Times. Figure 4 exhibits the predicted and observed cumulative frequency distributions of departure time aggregated across all subjects within each group. Because of the evidence for learning in the first two blocks of group 2 (see Table 2), the departure times are computed across the 25 trials in the second half of the experiment, namely, trials 26-50. The cumulative frequency distributions are displayed separately for each of the six sessions. Figure 4 indicates no major differences between the three sessions of each of the two conditions. The only noticeable deviation, compared to the equilibrium solutions, is that the subjects in group 1 (who had to pass through a single bottleneck) departed home later than predicted in condition V3; the observed cumulative frequency distributions are to the *right* of the equilibrium predicted linear line. Even though the increased capacity of the upstream bottleneck in condition V3 mostly neutralizes the cost advantage of the group 1 users (in the Nash equilibrium solution), they (group 1) still appear to believe that they have an advantage because they only have to pass through one bottleneck and, therefore, leave later than the theory would predict. Comparison with the results displayed in Figs. 3(a) and (b) shows that the major effect of expanding the capacity of the downstream bottleneck is, as the theory predicts, to shift the equilibrium departure time distributions for group 1 earlier: $t_1^0 = 6:38$ and $t_1^e = 7:23$ in condition V3 compared to $t_1^0 = 7:18$ and $t_1^e = 7:45$ in condition V5. The observed mean departure times of group 1 were 7:16 and 7:37 in Conditions V3 and V5 respectively, and this difference from equilibrium was significant ($t=13.75$, $p<0.001$).

--Insert Fig. 4 about here--

Switching Departure Times. Dynamic theories of equilibrium explore the hypothesis that boundedly-rational players, who interact with one another repeatedly, converge to equilibrium behavior through a process of gradual adjustment (e.g., Drew Fudenberg and David Levine, 1998; Sergiu Hart and Andreu Mas-Collel, 2003; and Hart, 2005). Typical examples include the fictitious play learning process, regret matching, and the replicator dynamics from evolutionary biology. In our experiment, players may change their departure time from one round to another in an attempt to increase their payoff. We examine below whether they are successful in doing so. Players may also behave non-myopically, changing their departure times for strategic reasons. Particularly when the user population is small, as it is in our experiment, a subject may switch her departure time from trial t to trial $t+1$ in order to elicit a change in the distribution of departure times (e.g., by increasing congestion) and then exploit this change on trial $t+2$, if she correctly anticipates the population reaction to her previous decision.

Denote the departure times of subject j on rounds t ($t=1, 2, \dots, 49$) and $t+1$ by $DT_{j,t}$ and $DT_{j,t+1}$, respectively. We say that subject j *switches* her departure time between rounds t and $t+1$, if $|DT_{j,t}-DT_{j,t+1}| > \Delta$, where $\Delta \geq 0$. We define a switch in this way in order to allow for an arbitrary choice of the criterion for a “switch.” Figure 5 exhibits the frequency distribution of number of switches. The frequency of switches is computed with respect to the criterion $\Delta = 10$ minutes (the overall average switch size was 703 seconds and the overall median was 354 seconds so an intermediate value of $\Delta = 10$ was chosen in order to simultaneously capture meaningful large switches and at the same time keep as many as possible in the analysis). The results are displayed separately for each group across the three sessions of each condition. Figure 5 shows the distributions for each group under each of the two conditions. The median number of switches (out of a maximum of 36) is very consistent in 3 of the 4 cases, namely, 21, 19.5, and

20 for group 1 in Condition V3, group 2 in Condition V5, and group 2 in Condition V3, respectively. For group 1 in Condition V5, the median number of switches was only half these values at 10. From these distributions, it appears that switching frequency is very much in line with the travel cost *disadvantages* (for group 1) implied by the theoretical model as one moves from Condition V5 to Condition V3. The model predicts that group 2 travel costs should change only a small amount across the two conditions but that group 1 should do much better in Condition V5. It also predicts that the costs of group 1 and group 2 agents should be similar in Condition V3. What we appear to be seeing is that the frequency of switching of departure times is reflective of increased travel cost pressures. Group 1 members switch only half as much when they have a large travel cost advantage (condition V5 relative to V3). We formulate below and test two hypotheses about the switching process.

--Insert Fig. 5 about here--

The first hypothesis is in line with learning direction theory of Selten and Rolf Stoecker (1986). Learning direction theory predicts only the direction of change in choice of strategies; it applies to repeated interactive decision making tasks in which outcome information provided at the end of each period permits causal inferences about what might have been better. Evidence in support of this theory is presented in Selten (1998) and in Ryan D. Oprea, Daniel Friedman, and Steven T. Anderson (2007); see also Axel Ockenfels and Selten (2005) for an extension of this theory, called *impulse balance theory*. In the context of our experiment, Hypothesis 1 states that the absolute magnitude of the switch in departure time between trials t and $t+1$, denoted by $\Delta_{j,t} = |DT_{j,t} - DT_{j,t+1}|$, is negatively related to the difference $d_{j,t}$ between subject j 's payoff on trial t , denoted by $r_{j,t}$, and the *maximum* payoff in her group on that trial, denoted by $\max r_{i,t}$: thus, $d_{j,t} =$

$\max r_{i,t} - r_{j,t}$. Hypothesis 1 refers to the absolute magnitude of the switch because a player may adjust her departure time by departing earlier or later than before

Hypothesis 1 was tested by computing for each subject j the correlation between $\Delta_{j,t}$ and $d_{j,t}$. All the 72 correlations in Condition V3 were positive. Similarly, of the 72 correlations in Condition V5, all were positive. Based on these results, we conclude that the less a subject earned in comparison to her group's maximum in trial t the more she tended to switch her departure time in trial $t+1$.

Hypothesis 2 states that such switching of departure time (i.e. switching that is negatively correlated to relative payoff) pays off. To test Hypothesis 2, we proceeded as follows. Using the criterion of $\Delta = 10$ minutes for defining a switch, we identified for each subject all the pairs of adjacent trials where a switch occurred from round t to round $t+1$. Then, we computed for each pair the (counterfactual) payoff that the subject *would have earned* on trial $t+1$ if she did not switch, assuming that all other players in the session behaved as they actually did. Call this payoff a *counterfactual payoff* and denote it by $cr_{j,t+1}$. Next, we computed for each subject two means: the mean of her observed payoffs on trials $t+1$ (following a switch), which we denote by R_j , and the mean of her counterfactual payoffs on the same trials, denoted by CR_j . Hypothesis 2 states that $R_j \neq CR_j$. The null hypothesis $R_j = CR_j$ was tested by a series of two-sided t -tests for each session and each group within session. The results are presented in Table 3, which shows that the null hypothesis was rejected in all but one case. In support of Hypothesis 2, switching paid off with stronger effects observed in Condition V3 than in Condition V5.

Taken together, these two hypotheses suggest that switching is a component in the process by which agents participating in decentralized decision making tasks evolve towards departure times that are, in the aggregate, close to those of the mixed strategy Nash equilibrium. The

results displayed in Table 2 showing increase in mean payoffs and decrease in variability of payoffs across blocks of trials with a gradual convergence to the equilibrium payoffs imply that round-to-round adjustments to departure times must be a factor. As shown in the next section, however, individual behavior varies widely and is not supportive of the claim that all individuals play Nash strategies. It appears likely that some agents very quickly find an attractive departure time and do little or no switching at all (and earn consistently high payoffs).

--Insert Table 3 about here--

Individual Differences. The adjustment process postulated by dynamic theories of equilibrium may account for gross patterns of behavior on the aggregate level but fail to account for decisions over time of many of the individual subjects. Therefore, it is imperative to examine closely the individual data. Figure 6 (parts A and B) exhibits the departure times of all the 24 subjects in session 1 of Condition V3. The patterns displayed by the individual subjects in the other five sessions are similar. Figure 6a displays the results for the eight subjects in group 1, and Fig. 6b for the sixteen subjects in group 2. Players are numbered from 1 through 24 (group 1: 1-8, group 2: 9-24). The horizontal axis presents the round number (1-50) and the vertical axis shows the departure time (measured in seconds after 5:00 a.m.). In each plot we draw two horizontal lines that correspond to the equilibrium starting (t_j^o) and ending (t_j^e) departure times (in the case of group 2, we replace t_2^e with the last departure time that results in the player not being late, t^f). Figures 6c and 6d exhibit the cumulative distribution functions for all group 1 and group 2 subjects across all 50 trials for this same set of data (condition V3, session 1).

--Insert Figs. 6a, 6b, 6c, and 6d about here--

Inspection of Figs. 6a and 6b shows a few subjects who hardly changed their departure time across all the 50 rounds (Subject 3 in group 1, and, with a few exceptions, Subjects 15 and 23

in group 2) or a substantial number of rounds typically toward the end of the session (Subjects 17, 22, and 24 in group 2). For several subjects, we observe local patterns of departure times that decrease in small steps over successive trials (Subject 4 in rounds 6-14 and 16-25; Subject 5 in rounds 6-14 and 17-21 ; Subject 11 in rounds 8-13; Subject 18 in rounds 10-14, 18-27, 23-25, 39-42, and 46-49). Other subjects display patterns of play that are more difficult to classify. Further inspection of Fig. 6a reveals that all eight subjects departed too late on several rounds. In agreement with Fig. 4 that displays the aggregate departure times, the individual mean departure times are closer to t_I^e than to t_I^o . In contrast, a few subjects in group 2 (Fig. 6b) departed too early on several rounds. As is clear for the individual plots in Figs. 6a and 6b and in the widely-varying distributions in Figs. 6c and 6d, there is little evidence in support of mixed-strategy equilibrium play on the individual level. Some individuals, particularly in group 2, exhibit Nash behavior but some select a departure time early in the process and deviate very little from that time.

IV. Discussion

Equilibrium models that invoke strong assumptions of homogeneity of group members and common knowledge of rationality acquire added value when they succeed in accounting for behavioral regularities in what seems to be counterintuitive behavior. As Arnott and Small (1994) have shown in their expository paper, several paradoxical results have been known to be present in transportation research, all of them resulting from selfish behavior of network users who ignore the negative externalities they impose on others. Experimental evidence has been reported in the last few years in an attempt to answer the question whether these transportation paradoxes are only of theoretical interest with no practical application or they are manifested in actual behavior. Morgan, Orzen and Sefton (2006) have reported experimental evidence in

support of the implications of the PKD Paradox. Even more striking is the experimental evidence that strongly supports the implications of the Braess Paradox (e.g., Rapoport et al., in press; Morgan, Orzen and Sefton, 2006; and Rapoport et al., 2006). All of these studies have focused on changes in behavior resulting from exogenous changes in the cost structure imposed on the network (the PKD Paradox) or from adding or deleting one or more links in the traffic network (the Braess Paradox). Our study extends previous investigation of paradoxes in networks from choice of routes when departure times are fixed to choice of departure times when the structure of the network is fixed but capacities vary. In this more challenging environment involving the coordination of departure times, behavior again supports theory and the predicted paradox emerges. As capacity of the upstream bottleneck is *increased*, social cost *rises*.

Behavioral regularities observed in our study on the aggregate level are broadly consistent with mixed-strategy equilibrium play. With more experience in playing the iterated bottleneck game, mean payoffs approach the theoretical predictions, between-subject variability in mean payoff declines sharply, and departure time distributions are well approximated by the symmetric mixed-strategy equilibrium distributions. The major exception is the tendency of the subjects in the advantaged group (group 1), whose members have to pass through a single bottleneck only, to depart from home later than predicted in the situation, case V3, in which their advantage is more apparent than real. In this case, the capacity of upstream bottleneck (which only group 2 members must pass through) is set at a high level which, according to the theory, should nearly neutralize the advantage of group 1. However, Group 1 members depart later than the theory would predict, acting as if they had a larger advantage. Finding and testing explanations for this phenomenon is left for further research.

On the individual level, the behavioral patterns are quite chaotic with some subjects changing their departure time quite often (e.g., Subject 7 in Fig. 6a) and others (e.g., Subject 3 in Fig. 6a) changing them hardly at all. We find no evidence that the patterns of choice made by most subjects, although non-deterministic, are broadly consistent with individual play of mixed strategy. Nevertheless, the aggregated group behavior is accounted for quite well by the mixed-strategy equilibrium suggesting that while we cannot predict a particular player's behavior using the Nash equilibrium it is still useful for forecasting the population choices.

Our results also show that subjects respond to their payoff outcome in the previous trial. The magnitude of switches in departure time from one round to the next is significantly correlated with a subject's own payoff (in comparison to her group's best payoff) on the preceding trial. For most subjects, switching departure time seems to be a critical component of the process by which the aggregate choices move in the direction of equilibrium play. As subjects gain experience with the game, the variance in payoff decreases dramatically and approaches the equilibrium prediction. This low variance may be viewed as a manifestation of the notion of equilibrium in which no one group member can do much better than the others in her group. Accounting for the dynamics of play by which subjects reach this state using parameterized reinforcement-based learning models seems to be a profitable direction for future research.

REFERENCES

- Arnott, Richard, Andre De Palma, and C. Robin Lindsey** (1990). “Economics of Bottlenecks”. *Journal of Urban Economics*, **2**: 111-130.
- Arnott, Richard, Andre De Palma, and C. Robin Lindsey** (1993a). “Properties of dynamic traffic equilibrium involving bottlenecks, including a paradox and metering”. *Transportation Science*, **27**, 148-160.
- Arnott, Richard, Andre De Palma, and C. Robin Lindsey** (1993b). “A structural model of peak-period congestion: A traffic bottleneck with elastic demand”. *American Economic Review*, **83**, 161-179.
- Arnott, Richard, Andre De Palma, and C. Robin Lindsey** (1999). “Information and stochastic capacity and demand in the bottleneck model”. *European Economic Journal*, **43**, 523-548.
- Arnott, Richard and Small, Kenneth** (1994). “The economics of traffic congestion”. *American Scientist*, **82**, 446-455.
- Avineri, Erel** (2006). “Measuring and simulating altruistic behavior in group travel choice decisions”. University of East Anglia, unpublished manuscript.
- Avineri, Erel and Joseph N. Prashker** (2005). “Sensitivity to travel time variability: Travelers’ learning perspective”. *Transportation Research*, **13C**, 157-183.
- Braess, Dietrich** (1968). “Über ein paradoxon der verkehrsplanung“. *Unternehmensforschung*, **12**, 258-268.
- Daganzo, Carlos F.** (1985). “The uniqueness of a time-dependent equilibrium distribution of arrivals at a single bottleneck”. *Transportation Science*, **19**, 29-37.

- Denant-Boemont, Laurent and Romain Petiot** (1999). “Information value and sequential decision-making in a transport context: An experimental study”. Laboratoire d’Economie des Transports, Universite Lumiere Lyon 2, unpublished manuscript.
- Fudenberg, Drew and David K. Levine** (1998). *Theory of Learning in Games*. Cambridge, MA: MIT Press.
- Gabuthy, Yannick, Matthieu Neveu and Laurent Denant-Boement** (2004). “Structural model of peaked-period congestion: An experimental study”. Working paper, Groupe d’Analyse de Théorie Economique, Ecully, France.
- Hart, Sergui** (2005). “Adaptive heuristics”. *Econometrica*, **73**, 1401-1430.
- Hart, Sergui and Andreu Mas-Collel** (2003). “Regret-based dynamics”. *Games and Economic Behavior*, **45**, 375-394.
- Helbing, Dirk** (2004). “Dynamic decision behavior and optimal guidance through information services: Models and experiments”. In M. Schreckenberg and R. Selten (Eds.), *Human Behavior and Traffic Networks*. Berlin: Springer, pp. 48-95.
- Helbing, Dirk, Martin Schönhof, and Daniel Kern** (2002). “Volatile decision dynamics: Experiments, stochastic description, intermittency control, and traffic optimization”. *New Journal of Physics*, **4**, 1-33.
- Hendrickson, Chris T. and George Kocur** (1981). “Schedule delay and departure time decisions in a deterministic model”. *Transportation Science*, **15**, 62-77.
- Iida, Yasunori, Takamasa Akiyama and Takashi Uchida** (1992). Experimental analysis of dynamic route choice behavior. *Transportation Research*, **26B**, 17-32.
- Knight, Frank H.** (1924). “Some Fallacies in the Interpretation of Social Cost”. *Quarterly Journal of Economics*, **38**, 582-606.

- Knödel, W.** (1969). "Graphentheoretische Methoden und ihre". *Anwendungen*. Berlin: Springer-Verlag, pp. 56-59.
- Kuwahara, Masao** (1990). "Equilibrium queueing patterns at a two-tandem bottleneck during the morning peak". *Transportation Science*, **24**, 217-229.
- Morgan, John, Hendrik Orzen, and Martin Sefton** (2006). "Network architecture and traffic flows". Hass School of Business, University of California, unpublished manuscript.
- Murchland, John D.** (1970). "Braess's paradox of traffic flow". *Transportation Research*, **4**, 391-394.
- Ockenfels, Axel and Reinhard Selten** (2005). "Impulse balance equilibrium and feedback in first price auctions". *Games and Economic Behavior*, **51**, 155-170.
- Oprea, Ryan D. Daniel Friedman, and Steven T. Anderson** (2007). "A laboratory investigation of deferred options". Economics Dept. Univ. of California, Santa Cruz, unpublished manuscript.
- Rapoport, Amnon, Tamar Kugler, Subhasish Dugar, and Eyran Gisches** (in press). "Braess Paradox in the Laboratory: Experimental Study of Route Choice in Traffic Networks with Asymmetric Costs". In T. Kugler, J. C. Smith, T. Connolly, and Y. J. Son (Eds.), *Decision Modeling and Behavior in Uncertain and Complex Environments*. New York: Springer.
- Rapoport, Amnon, Vincent Mak, and Rami Zwick** (2006). "Navigating congested networks with variable demand: Experimental evidence". *Journal of Economic Psychology*, **27**, 648-666.

- Schneider, Kerstin and Joachim Weimann** (2004). “Against all odds: Nash equilibria in road pricing experiment”. In M. Schreckenberg and R. Selten (Eds.), *Human Behavior and Traffic Networks*. Berlin: Springer, pp. 133-153.
- Selten, Reinhard** (1998). “Features of experimentally observed bounded rationality”. *European Economic Review*, **42**, 413-436.
- Selten, Reinhard, Thorsten Chmura, Thomas Pitz, Sebastian Kube, and Martin Schreckenberg** (2004). “Experimental investigation of day-to-day route-choice behavior and network simulations of Autobahn traffic in North Rhine-Westphalia”. In Martin Schreckenberg and Reinhard Selten (Eds.), *Human Behavior and Traffic Networks*. Berlin: Springer, pp. 1-21.
- Selten, Reinhard, Thorsten Chmura, Thomas Pitz, Sebastian Kube, and Martin Schreckenberg** (2007). “Commuters route choice behavior”. *Games and Economic Behavior*, **58**, 394-406.
- Selten, Reinhard and Rolf Stoecker** (1986). “End behavior in sequences of finite Prisoner’s Dilemma supergames”. *Journal of Economic Behavior and Organization*, **7**, 47-70.
- Smith, Mike J.** (1983). “The existence and calculation of traffic equilibria”. *Transportation Research*, **17B**, 291-303.
- Vickrey, William S.** (1969). “Congestion theory and transport investment”. *American Economic Review*, **59**, 251-261.
- Zieglmeyer, Anthony, Frederic Koessler, Kene Boun My, and Laurent Denant-Boèmont** (in press). “Road traffic congestion and public information: An experimental investigation”. *Journal of Transport Economics and Policy*.

APPENDIX A

Derivation of ADL Equilibrium Conditions with Finite Late Cost (for $\bar{C}_1 < \bar{C}_2$ and $s_2 > s_d$).

The following two lemmas will be used in the derivations to follow:

Lemma A: Once the first user in group 2 departs, there is always a queue at the upstream bottleneck until all group 1 users have departed.

Proof: Once the users from group 1 begin to depart (at t_1^o) a queue must exist at the upstream bottleneck; otherwise, a group 2 user could depart at the same time as someone from group 1 and incur an identical cost violating $\bar{C}_1 < \bar{C}_2$. Before group 1 users begin departing, if the upstream queue vanishes then a group 2 user could depart and incur lower costs than the previous user to depart (no delay at upstream queue, no delay at downstream queue (because $s_2 > s_d$), no late cost, and no greater cost of being early).

Corollary: The downstream bottleneck has a queue over the entire period from the first departure until all group 1 users are have departed.

Proof: Because $s_2 > s_d$ and because, from Lemma A, the upstream queue is not empty, there always will be a queue at the downstream bottleneck.

Lemma B: The downstream bottleneck has a queue from the time of the first departure until the time of the last departure, t_{max} .

Proof: The corollary above covers the period up to the last departure from group 1. Between t_1^e and t^* if the downstream queue becomes empty then a user from group 1 could depart and incur a lower cost (arrive with no delay, no late cost, and a lower cost of being early than anyone in group 1 who departed earlier). This violates the equilibrium condition (3). Between the times t^* and t_{max} the downstream queue must not be empty or a member from group 2 could depart and

arrive with no delay (the upstream queue must also be empty because $s_2 > s_d$) and hence incur a lower cost than the user who arrives at t_{max} again violating (3).

Derivation of each equation:

Equation (5) $t_2^o = t_{max} - (n_1 + n_2)/s_d$: From Lemma B, the downstream queue is never empty over this period and $n_1 + n_2$ users flow through the bottleneck.

Equation (4) $t_{max} = t_2^e = t^* + \beta(n_1 + n_2)/s_d(\beta + \gamma)$: The first user from group 2 departs at t_2^o and arrives with no delay incurring a cost of $\beta(t^* - t_2^o)$. The last user from group 2 departs at t_{max} (and arrives then) incurring a cost of $\gamma(t_{max} - t^*)$. Equilibrium requires that these costs be equal hence $t_{max} - t^* = \beta(t^* - t_2^o)/\gamma$. Combining this condition with (5) above results in (4).

Equation (6) $d_2^o = \frac{\alpha}{\alpha - \beta}s_d$: Initially, group 2 users begin to depart at a rate d_2^o . The first of these incurs a cost of $\beta(t^* - t_2^o)$ and one who leaves at time t spends $t_q = \frac{(d_2^o - s_d)(t - t_2^o)}{s_d}$ in queues (by Lemma 2) and arrives $t^* - t - t_q$ early. Equating the costs of these users yields equation (6).

Equation (7) $t^f = \frac{\alpha + \gamma}{\alpha}t^* - \frac{\gamma}{\alpha}t_{max}$: Some group 2 user must arrive exactly at t^* , otherwise another user could depart fractionally later and incur a lower cost. If this user departed at t^f then his cost is $\alpha(t^* - t^f)$. This cost must, by the equilibrium condition, be equal to the cost of the last arrival from group 2 (who arrives at t_{max} with no delay) which is $\gamma(t_{max} - t^*)$. Equating these costs and rearranging terms leads to (7).

Equation (8) $d_2^f = \frac{\alpha}{\alpha + \gamma}s_d$: [Derivation is analogous to that of (6) above.]

Equation (9) $t_1^e = t_2^o + \frac{n_2 - s_d(t_{\max} - t^*)}{s_2}$: The number of group 2 users who arrive late is

$s_d(t_{\max} - t^*)$ because the downstream queue is never empty hence the last group 2 member to arrive on time departs the upstream queue at $t_2^o + \frac{n_2 - s_d(t_{\max} - t^*)}{s_2}$. This must be the departure

time of the last user from group 1 to arrive on time.

Equation (10) $t_1^o = t_1^e - \frac{n_1}{d_1^o}$: For all group 1 users to have the same cost (as required for

equilibrium), the marginal value of delaying departure by an incremental amount Δt must be 0.

Such a delay increases queue delay costs by $\alpha \frac{(d_1(t) + s_2 - s_d)}{s_d} \Delta t$ where $d_1(t)$ is the departure

rate of group 1 users at t . Likewise, the delay reduces early arrival costs by

$$\beta \left(\Delta t + \frac{(d_1(t) + s_2 - s_d)}{s_d} \Delta t \right).$$

Summing these two expressions and equating to 0 yields $d_1(t) = \frac{\alpha}{(\alpha - \beta)} s_d - s_2$, which is a

constant. As a result, group 1 members depart at the constant rate $d_1^o = \frac{\alpha}{(\alpha - \beta)} s_d - s_2$ and

$$t_1^o = t_1^e - \frac{n_1}{d_1^o}.$$

Equation (11) $\bar{C}_1 = \alpha \left(\frac{\gamma(n_1 + n_2)}{s_d(\beta + \gamma)} - \frac{n_2}{s_2} + \frac{\beta(n_1 + n_2)}{s_d(\beta + \gamma)} \right)$: The last member of group 1 arrives at t^* .

Hence, $\bar{C}_1 = \alpha(t^* - t_1^e)$. Substituting (9) for t_1^e and rearranging $t_{\max} - t^* = \beta(t^* - t_2^o) / \gamma$ from the proof of (5) and substituting for t^* , gives (11).

Equation (12) $\bar{C}_2 = \beta \left(\frac{\gamma(n_1 + n_2)}{s_d(\beta + \gamma)} \right)$: Rearranging equation (4), we have

$$t_{\max} - t^* = \beta(n_1 + n_2) / s_d(\beta + \gamma).$$

Multiplying both sides by γ gives, on the LHS of the equation, the cost of the last group 2 user to arrive and, with the equilibrium condition, implies (12).

APPENDIX B

DEPARTURE TIME EXPERIMENT – INSTRUCTIONS TO PARTICIPANTS

Introduction

Welcome to an experiment on the choice of departure times. During this experiment you will be asked to make a large number of decisions and so will the other participants. Your decisions, as well as the decisions of the other participants, will determine your monetary payoff according to the rules that will be explained shortly. The money that you earn during the experiment will be paid to you in cash at the end of the session.

Please read the instructions carefully. If you have any questions, please raise your hand and someone will come to assist you.

From now on communication between the participants is prohibited. If the participants communicate with one another by any shape or form, the experiment will be terminated.

The Departure Time Task

This experiment attempts to simulate the basic features of departure time decisions that people traveling to a common destination face every day. It is fully computerized. You will be making your decisions by clicking on the appropriate buttons on the screen. A total of 24 persons are participating in this experiment (i.e., 23 participants in addition to you). During the experiment, you will participate in a series of **50** identical rounds. In each round, each participant will be asked to choose a time to depart for a common destination (for example, the time you leave for work every morning, a school class, a movie, or an airplane flight) knowing that waiting line delays may occur along the way. As in real life, there will be a value associated with reaching your destination on time and costs associated with waiting in line along the way and for either arriving too early (all participants wish to arrive by a common time; namely, 8:30 am) or arriving too late (after 8:30 am). These costs will be explained to you in the next section. On each trial, the participants will be paid a set amount for reaching the destination, and the costs they incur in getting there will be subtracted from their earnings. It is possible to lose money on any one trial if those costs are high.

Description of the Network

Please consult Figure A1 at the end of the instructions. Figure A1 displays a picture of the computer screen on your PC that will be presented to you on each trial. We'll now explain it. The diagram at the top of the screen illustrates the routes that the participants travel to reach their common destination in each round of the experiment. There are two groups of participants (called '**Blue**' and '**Green**'). The line on the screen just below the diagram tells you which group you belong to for all 50 rounds of the experiment. The participant illustrated in Figure A1 is a member of the '**Green**' group.

In traveling to their destination, members of the Blue group (8 participants) face only a single delay or bottleneck (drawn as a square on the diagram and labeled "Delay B"), whereas members of the **Green** group (16 participants) face an additional bottleneck (drawn as a square and labeled "Delay A") prior to reaching bottleneck "B" that is common to both groups. (You may think of

real examples of such bottlenecks such as sections of construction on roadways, ticket booths at the movie theatre, or baggage check-in and security stations at airports).

Bottlenecks slow down traffic and may cause queues to form. Participants in this experiment face a fixed service time to get through each bottleneck:

- **5** minutes at bottleneck A
- and **5** minutes at bottleneck B.

In addition, they may also have to wait in line before being served, if other network users arrived there before them and are still being served or waiting in line. For example, if two persons arrive at bottleneck B only seconds apart, then the first person to arrive will encounter a delay of 5 minutes at B (assuming no one was there before her), and the second person will encounter a delay of nearly 10 minutes (the time waiting in line for the first person to be served plus her own service time). We assume in this experiment that it takes no time to travel between delay points or between the final delay point and the destination.

The reward and costs associated with the travel are also shown in Figure A1. They include:

- the delay costs at each bottleneck - **10** points per minute for each of bottlenecks A and B,
- the cost of arriving early - **6** points per minute,
- the cost of arriving late - **83** points per minute,
- the value associated with reaching the destination - **1000** points.

In addition, summary information on the participant's earnings to date is shown at the top right corner.

Procedure

Once you decide on a departure time for the current round, you will enter your chosen time using the keypad on the screen (see Figure A1 again). Note that all decisions are made with the mouse; you will not need the keyboard. To enter the hour of departure, click the number desired on the keypad on the screen. Similarly, to enter minutes and seconds, click the corresponding cells either 'min' or 'sec' (the cell will turn yellow), and then click the desired numbers on the keypad. Errors can be erased by clicking the 'C' button on the keypad. Clicking the 'confirm' button indicates that you are satisfied with the chosen time. A confirmation text box will then appear on the screen to allow you a final opportunity to change your departure time.

Interpreting the Results

After all the participants have made their departure time choices, a results screen will appear. Figure 2 at the end of the instructions shows an example of this screen.

A summary of the delays experienced by a participant and the resulting costs and payoff are shown in the left-hand panel of the results screen (see Figure A2). The table on the right-hand panel shows the departure times and payoffs for all 24 participants. As seen in the left panel in the example shown in Figure A2, the participant departed at 7:20:00 and, as a member of the **Green** group, she had to pass through both bottlenecks. She encountered 4 persons ahead of her

at delay A (the other 5 have already been served and moved on) and waited 20 minutes and 39 seconds (including her own 5 minute service time).

*This delay cost her **206.5** points (10 points per minute).*

At delay B, she encountered 5 people ahead of her and waited 30 minutes.

*This delay cost her **300** points.*

She then arrived at destination at 8:10:39 (again, note that it takes no time to travel between delay points or between the final delay point and the destination), which was about 20 minutes early.

*This early arrival cost her **116.1** points (6 points per minute).*

As a result, at the end of this round this participant was left with a payoff of **377.4** points after all costs were subtracted from the **1000** point payoff for getting to the destination.

Payoff = 1000 – 206.5 – 300 – 116.1 = 377.4 points.

In the table for all participants on the right-hand side of the screen, this participant's times are highlighted in yellow as they will be during the experiment. Checking this table, you can find out the departure times of the other participants and the payoffs associated with each departure time (note that the times are color coded to identify participants in the **Green** and **Blue** groups). For example, a participant in the **Green** group departed at 7:22:27 and had a delay at bottleneck A of about 33 minutes and another at B of 40 minutes and hence arrived over 5 minutes late at 8:35:39. His total payoff was a loss of over 200 points for this round largely because of the high cost of being late (83 points per minute).

You may find it helpful to think of the current experiment in terms of two groups of people choosing when to leave for an event, such as a concert, where one group already has tickets (**Blue** group) and will face only delays in getting through the security line-up at the gate to the stadium. The other group (**Green** group) has to line up to buy a ticket first, and then go through the same security gate. All the participants of both groups want to minimize time spent waiting in line and idle time spent by arriving too early. However, they also do not want to be late for the concert as this has a much higher cost for them.

End of Experiment

After completing all **50** rounds, a summary screen will display the total points you have accumulated and the corresponding earnings in dollars. (Points will be converted to money at the rate 750 points=\$1.00). During the experiment, the cumulative number of points and dollar payoff that you have earned will be displayed on each of the individual screens. Please remain at your desk until asked to come forward and receive payment for the experiment.

Please place the instructions on the table in front of you to indicate that you have completed reading them. The experiment will begin shortly. Initially **five (5) unpaid** practice rounds will be conducted to familiarize you with the process, and then the **50** paid rounds will follow. Please

remember that no communication is allowed during the experiment. If you encounter any difficulties please raise your hand and you will be responded to by the experimenter.

Figure A1: Decision Screen

Round:
 Total score: points
 Total payoff: \$

Blue group
8 people

Green group
16 people

Delay A
Service time: 5 minutes/person
Service cost: 10 points/minute

Delay B
Service time: 5 minutes/person
Service cost: 10 points/minute

Goal arrival time: 8:30:00

Early arrival cost: 6 points/minute
 Late arrival cost: 83 points/minute

You are in the **green** group

Destination value: points

Enter your departure time: (hr : min : sec)

0	1	2	3	4	
5	6	7	8	9	C

Please enter your departure time

Figure A2: Results Screen

Round:
 Total score: points
 Total payoff: \$

Your Results

Destination value is: points

You departed at:

At the delay A there were: people ahead of you in line
 Your delay was: which cost: points

At the delay B there were: people ahead of you in line
 Your delay was: which cost: points

You arrived at: which cost: points

Your total payoff is: points

Complete Results of all Participants

Departure	Delay A	Delay B	Arrival	Payoff
06:01:00	00:05:00	00:05:00	06:11:00	66
06:49:08		00:05:00	06:54:08	374.8
06:55:39	00:05:00	00:05:00	07:05:39	393.9
06:55:52	00:09:47	00:05:00	07:10:39	376.07
06:56:14	00:14:25	00:05:00	07:15:39	359.73
06:58:00	00:17:39	00:10:00	07:25:39	337.4
07:00:23	00:20:16	00:15:00	07:35:39	321.23
07:06:23	00:19:16	00:20:00	07:45:39	341.23
07:11:00	00:19:39	00:20:00	07:50:39	367.4
07:12:16		00:08:23	07:20:39	500.07
07:13:00	00:22:39	00:25:00	08:00:39	347.4
07:19:00		00:11:39	07:30:39	527.4
07:20:00	00:20:39	00:30:00	08:10:39	377.4
07:21:29	00:24:10	00:30:00	08:15:39	372.23
07:21:58	00:28:41	00:35:00	08:25:39	337.07
07:22:27	00:33:12	00:40:00	08:35:39	-200.95
07:22:52		00:17:47	07:40:39	526.07
07:29:47	00:30:52	00:40:00	08:40:39	-592.62
07:33:56	00:31:43	00:40:00	08:45:39	-1016.12
07:34:51		00:20:48	07:55:39	585.9
07:38:23		00:27:16	08:05:39	581.23
07:48:00		00:32:39	08:20:39	617.4
07:51:00		00:39:39	08:30:39	549.55
07:51:28	00:19:11	00:40:00	08:50:39	-1305.78

Table 1. Means and standard deviations of payoffs (in dollars) by session, condition, and group

		Trials 1-50				Trials 41-50			
		Group 1		Group 2		Group 1		Group 2	
		“Blue” group		“Green” group		“Blue” group		“Green” group	
		Cond.	Cond.	Cond.	Cond.	Cond.	Cond.	Cond.	Cond.
		V3	V5	V3	V5	V3	V5	V3	V5
Session 1	Mean	3.44	5.52	2.04	1.87	3.05	5.41	2.45	2.43
	SD	(0.38)	(0.26)	(0.87)	(0.77)	(0.50)	(0.68)	(0.44)	(0.44)
Session 2	Mean	3.32	5.59	1.88	1.65	3.73	5.88	2.42	2.00
	SD	(0.49)	(0.23)	(0.76)	(0.96)	(0.40)	(0.22)	(0.42)	(0.82)
Session 3	Mean	3.36	5.34	1.48	1.11	3.40	5.69	2.17	2.04
	SD	(0.38)	(0.50)	(1.00)	(1.51)	(0.42)	(0.07)	(0.64)	(0.79)
Across Sessions	Mean	3.38	5.55	1.80	1.54	3.39	5.66	2.35	2.16
	SD	(0.42)	(0.39)	(0.91)	(1.17)	(0.52)	(0.46)	(0.52)	(0.73)
Equil		3.13	5.70	2.48	2.32	3.13	5.70	2.48	2.32

Table 2. Means and standard deviations of payoffs (in dollars) across sessions by group, condition, and block

Group	Cond.		Bl. 1	Bl. 2	Bl. 3	Bl. 4	Bl. 5	Total	Equil.
1	V3	Mean	3.49	3.67	3.19	3.14	3.39	3.37	3.13
<i>n</i> =24		SD	(1.23)	(0.95)	(0.74)	(0.79)	(0.53)	(0.42)	
1	V5	Mean	5.31	5.58	5.52	5.77	5.66	5.55	5.70
<i>n</i> =24		SD	(1.54)	(0.39)	(0.18)	(0.39)	(0.47)	(0.39)	
2	V3	Mean	0.60	2.10	1.74	2.22	2.35	1.80	2.48
<i>n</i> =48		SD	(2.67)	(1.63)	(1.25)	(0.74)	(0.53)	(0.91)	
2	V5	Mean	0.08	1.68	1.84	1.96	2.16	1.54	2.32
<i>n</i> =48		SD	(3.28)	(1.55)	(1.05)	(1.38)	(0.74)	(1.17)	

Table 3. Mean observed payoffs (in points) following a switch in departure time compared to mean counterfactual payoffs for the same trials by condition, session, and group.

		Condition V3			Condition V5		
	Group	Observed	Counterf.	<i>p</i>	Observed	Counterf.	<i>p</i>
Session	1	364.73	169.71	**	553.83	419.71	**
1	2	198.58	-46.72	***	226.19	90.53	***
Session	1	339.18	153.18	***	596.54	493.39	*
2	2	189.77	-72.96	***	144.53	-64.14	***
Session	1	359.17	106.06	***	528.01	459.25	0.107
3	2	208.06	-178.57	***	25.10	-171.02	***
Across	1	354.36	142.98	***	559.46	457.45	***
Sessions	2	198.80	-99.41	***	131.94	-48.21	***

* $p < 0.05$

** $p < 0.01$

*** $p < 0.001$

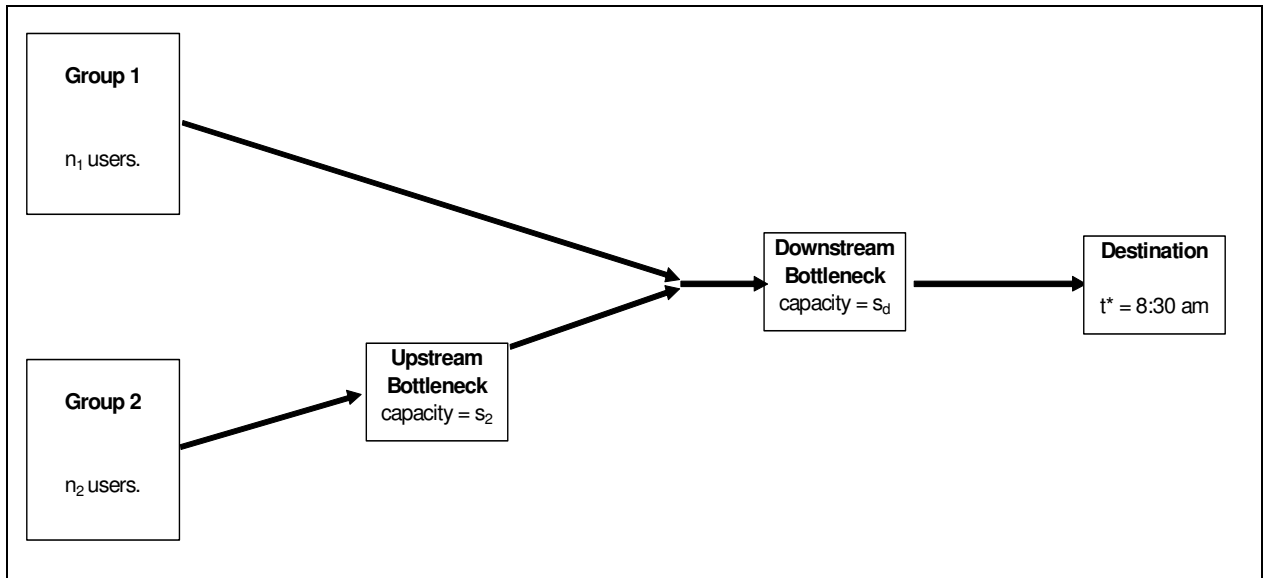


Fig. 1: Traffic Network with Two Groups and Two Bottlenecks. Group 1 passes through the downstream bottleneck with capacity s_d , and group 2 passes through the upstream and downstream bottlenecks with respective capacities s_2 and s_d .

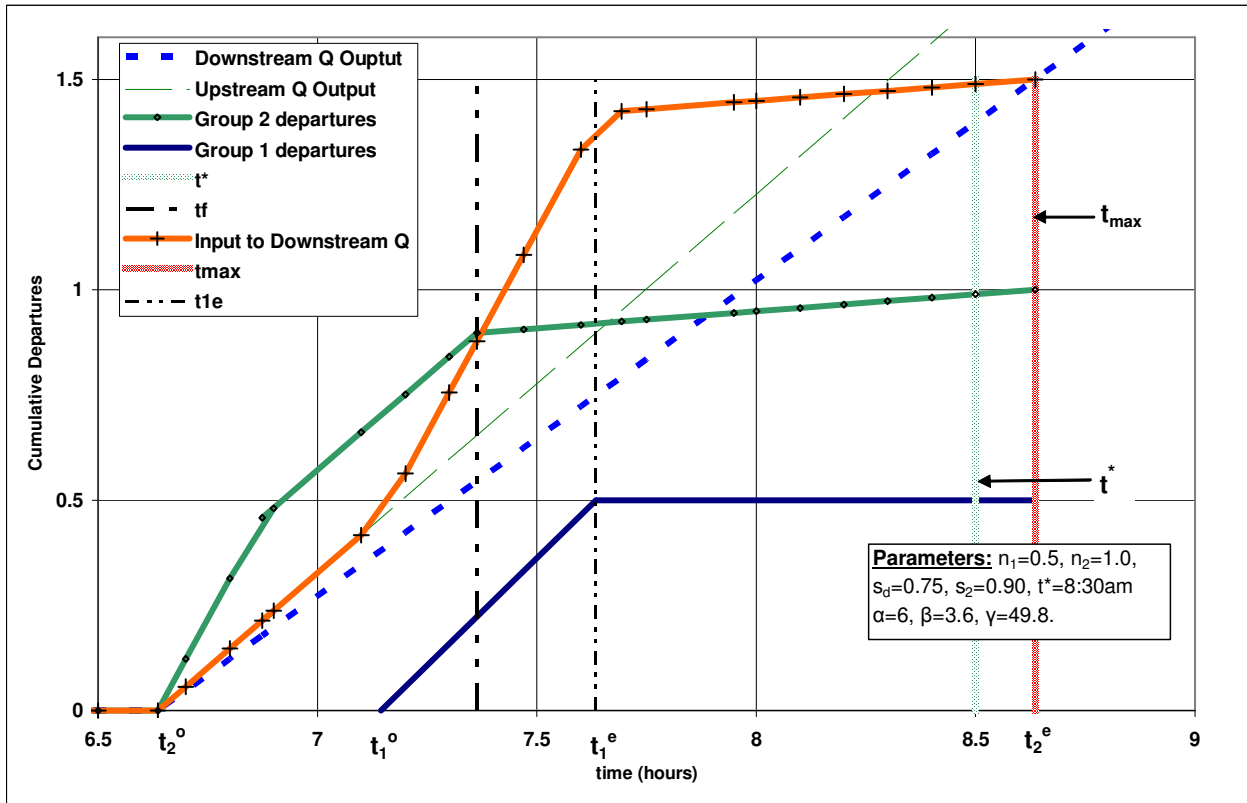


Fig. 2: Distribution of Cumulative Departure Times under Equilibrium Conditions for ADL model with late arrivals permitted (finite late cost).

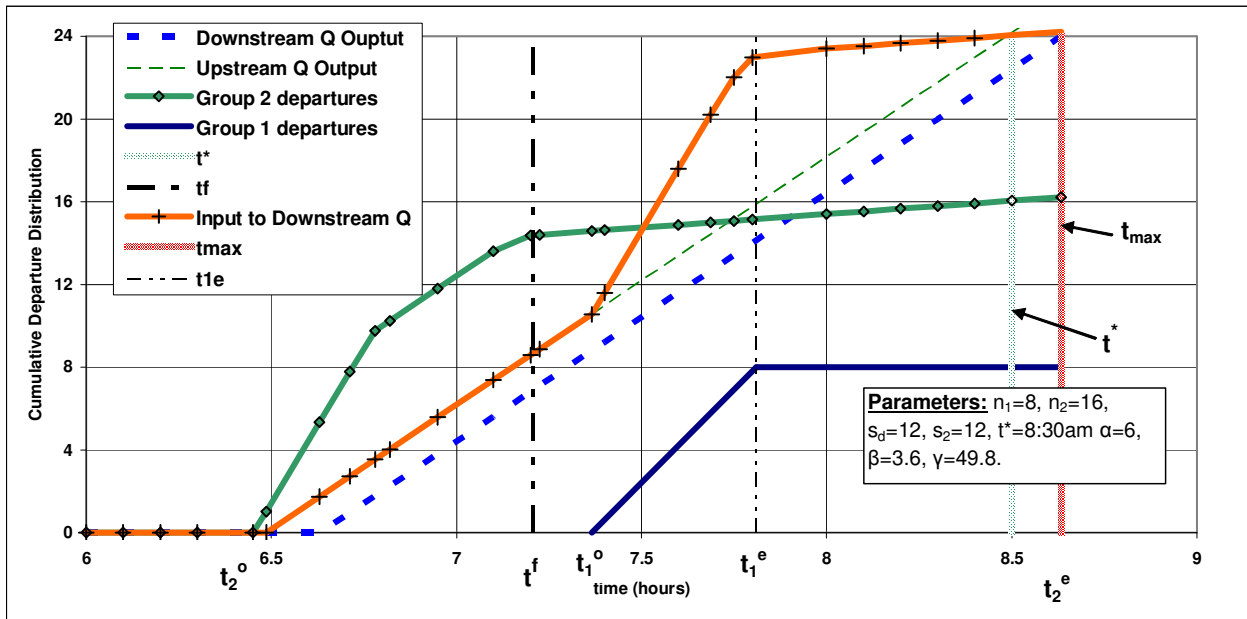


Fig. 3(a): Equilibrium Conditions with Restricted Upstream Capacity

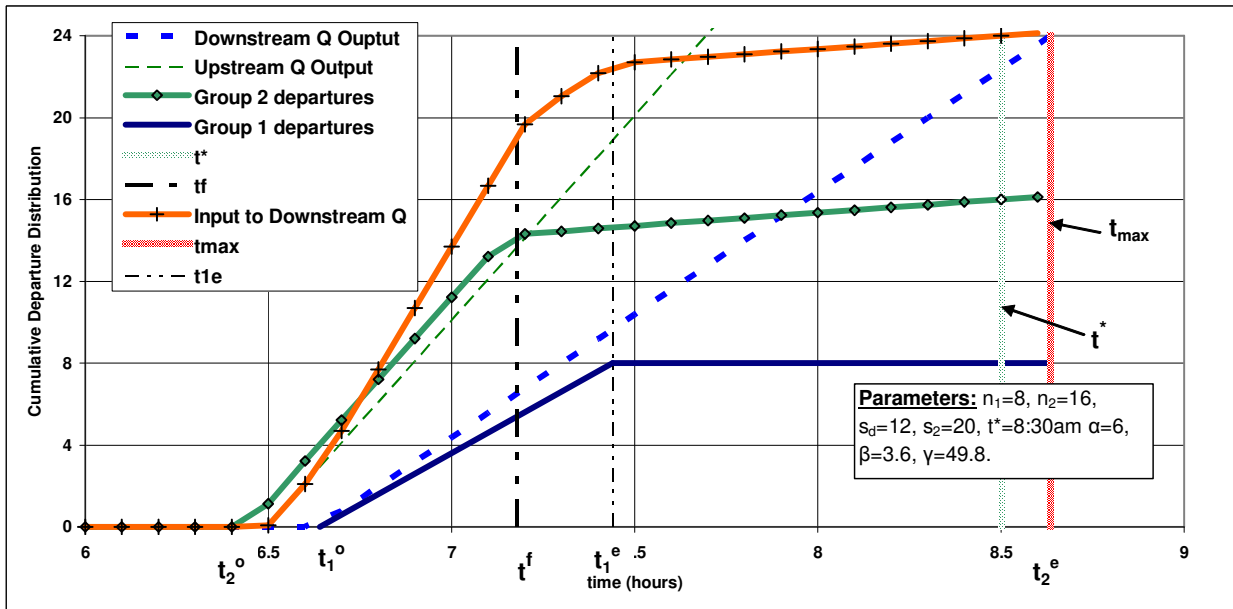
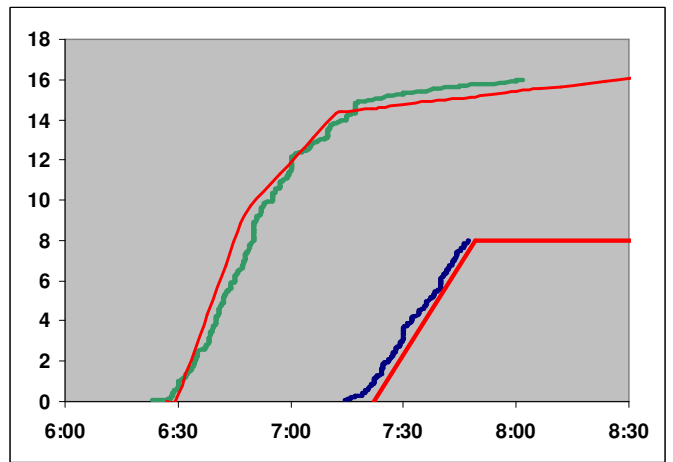
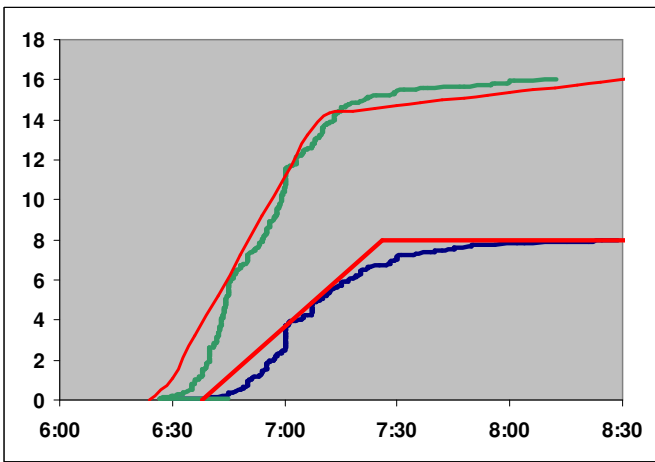
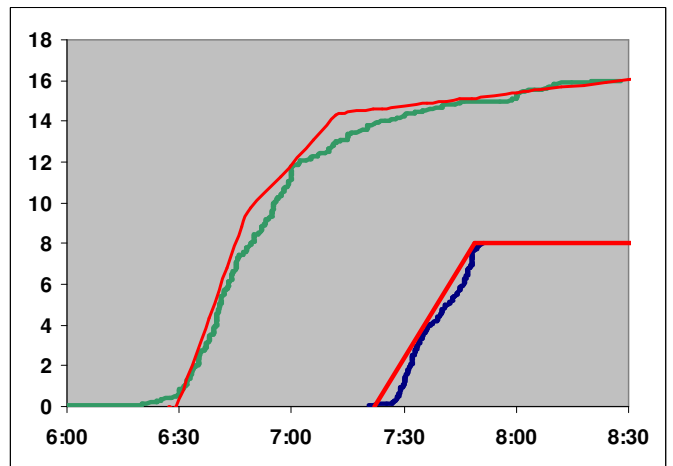
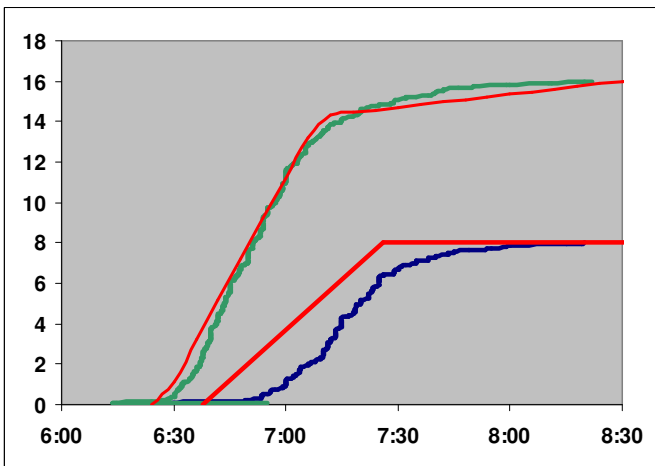
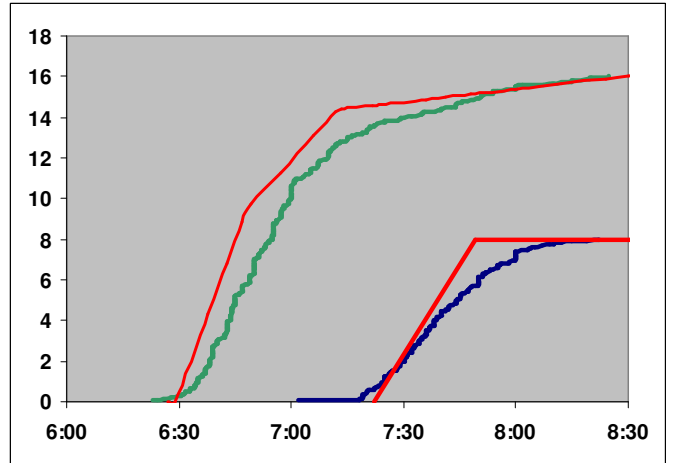
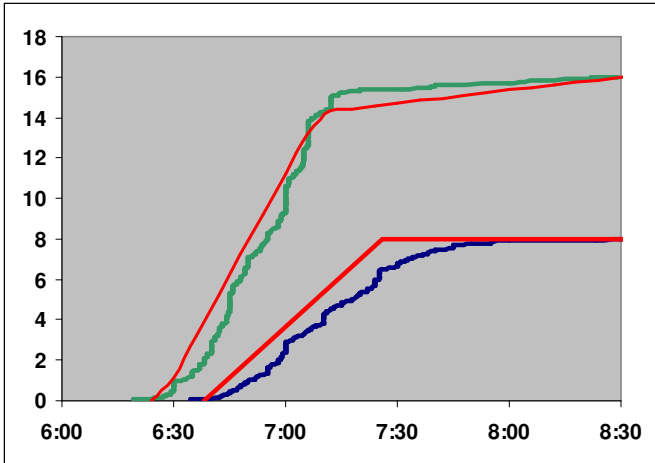


Fig. 3(b): Equilibrium Conditions with Expanded Upstream Capacity



Condition V3

Condition V5

Fig. 4. Observed and Predicted Cumulative Departure times in Trials 26-50 by Session and Condition (Smooth Lines Represent Predicted Equilibrium Distributions)

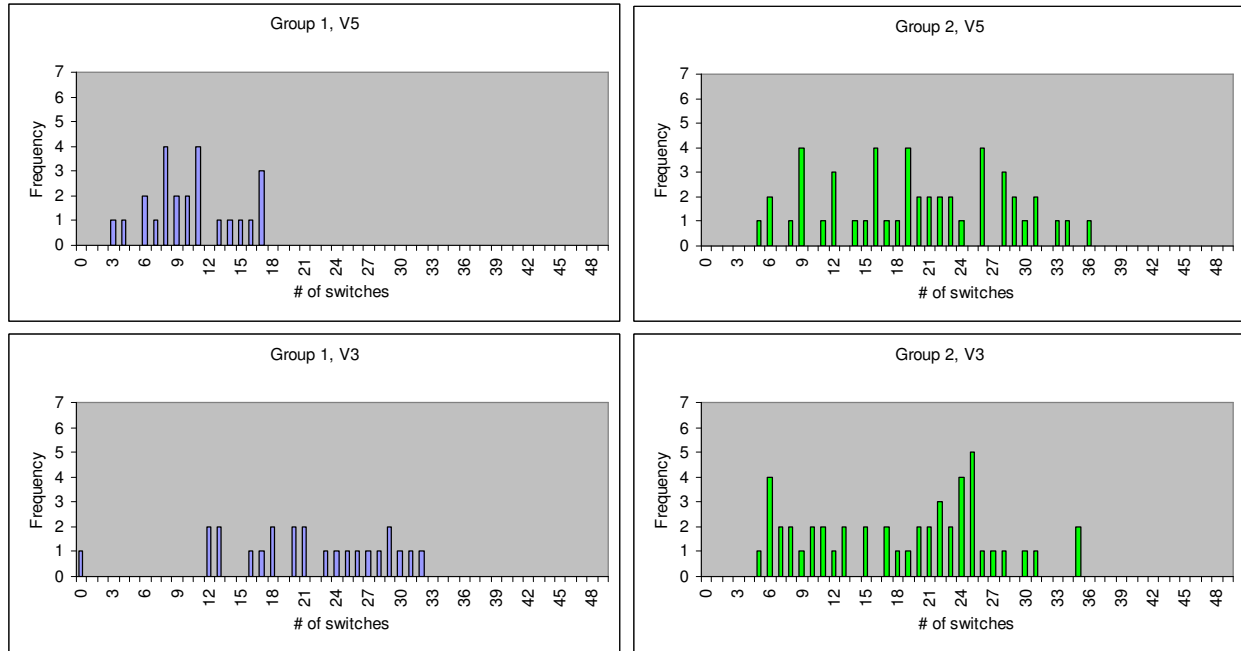


Fig. 5. Frequency Distributions of Number of Switches by Condition and Group

Fig. 6a. Individual Departure Times for All Eight Subjects in Group 1 of Condition V3.

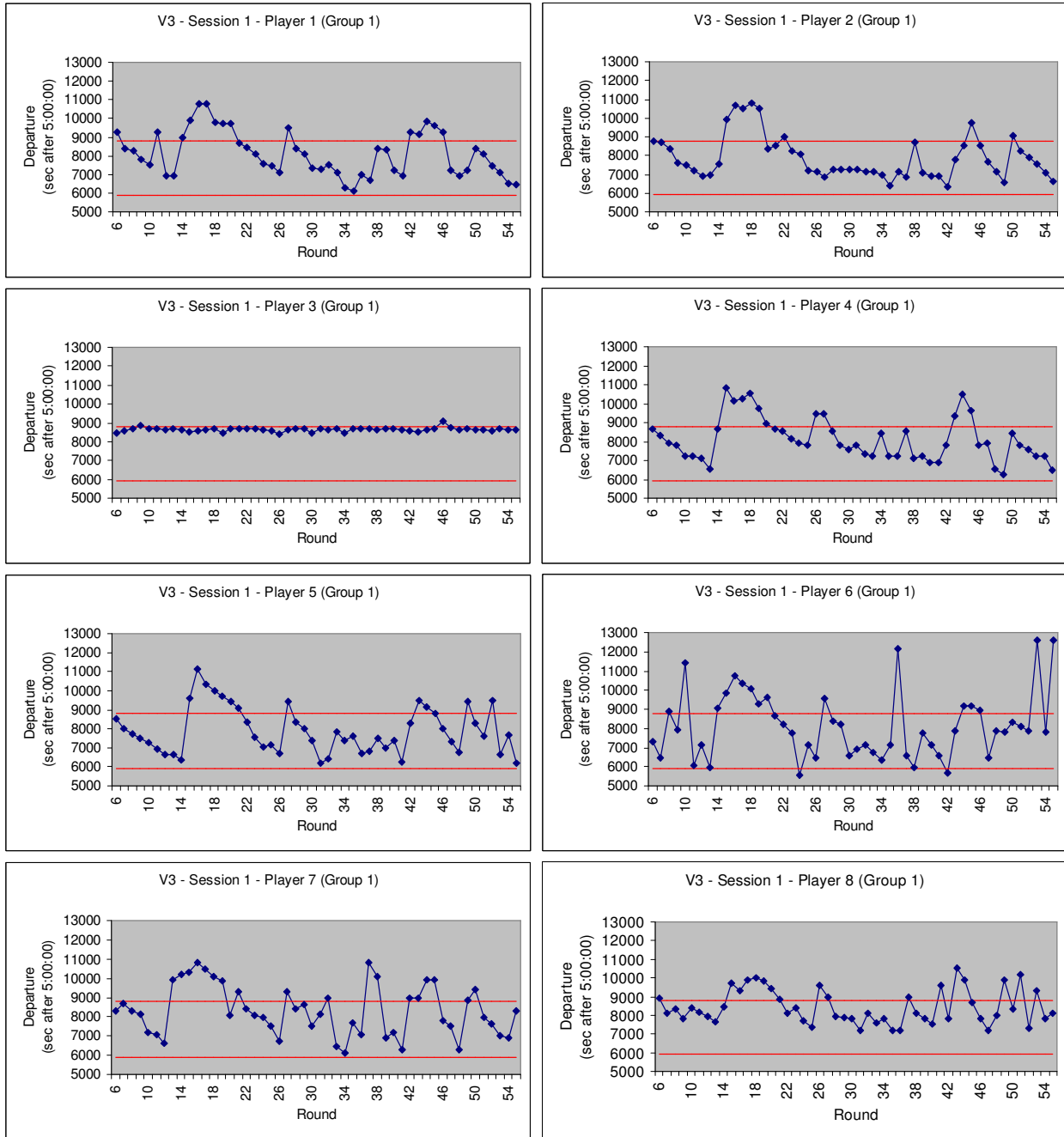
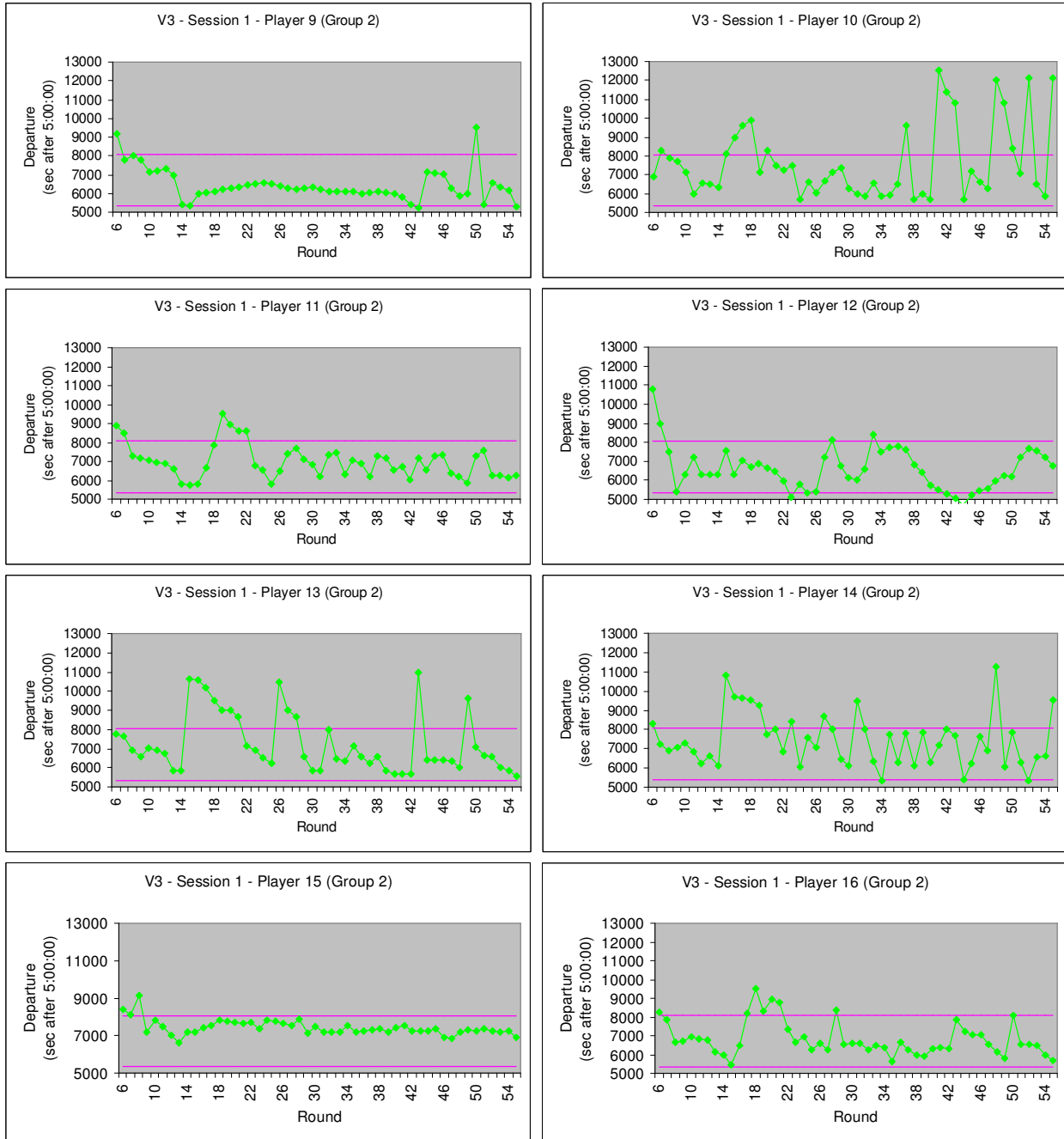


Fig. 6b. Individual Departure Times for All Sixteen Subjects in Group 2 of Condition V3.



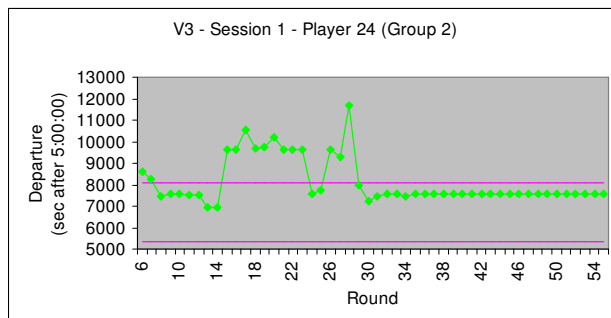
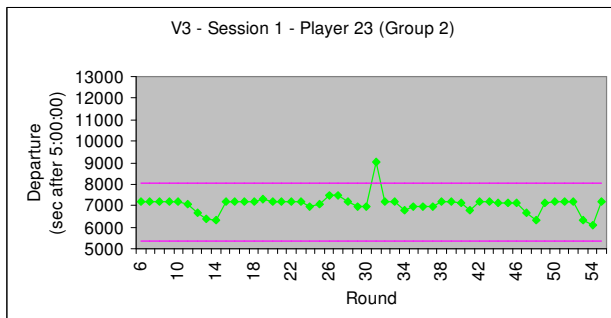
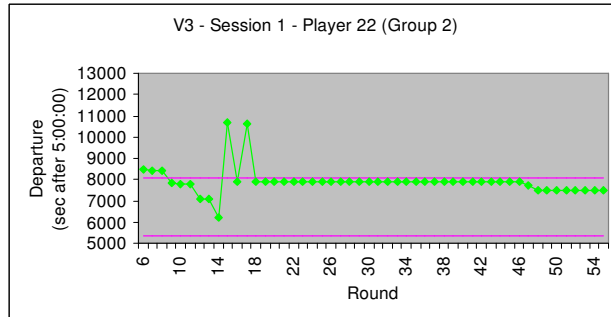
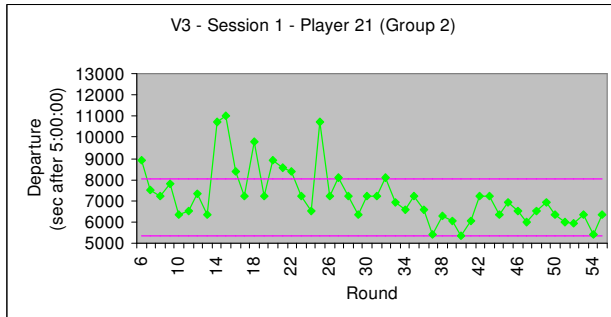
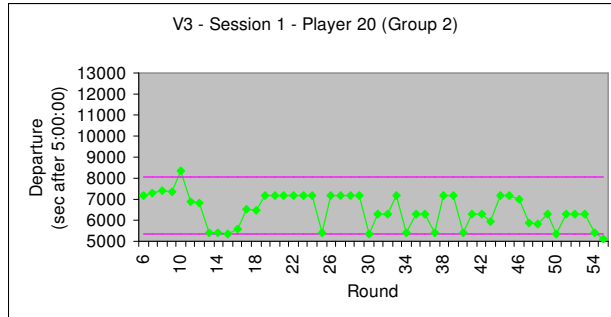
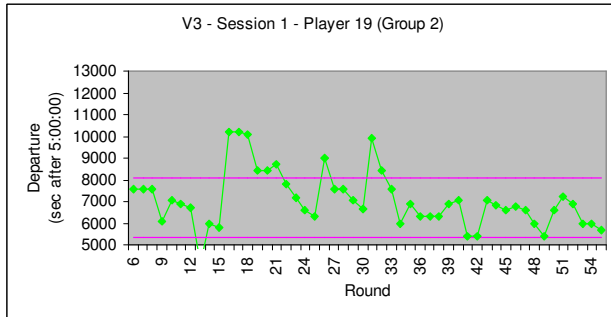
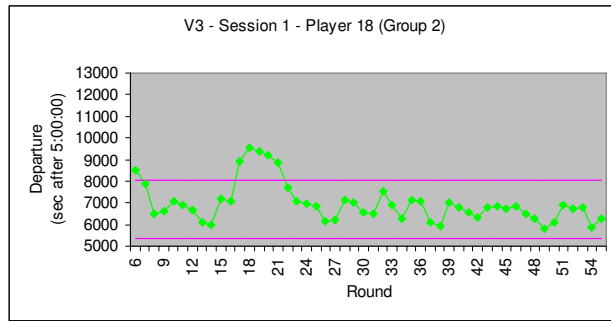
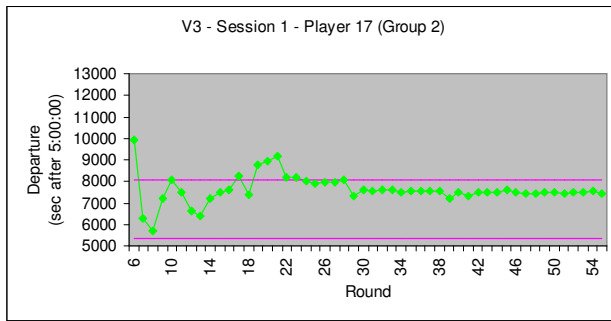


Fig. 6c. Individual Departure Time Distributions for All Eight Group 1 Subjects in Condition V3.

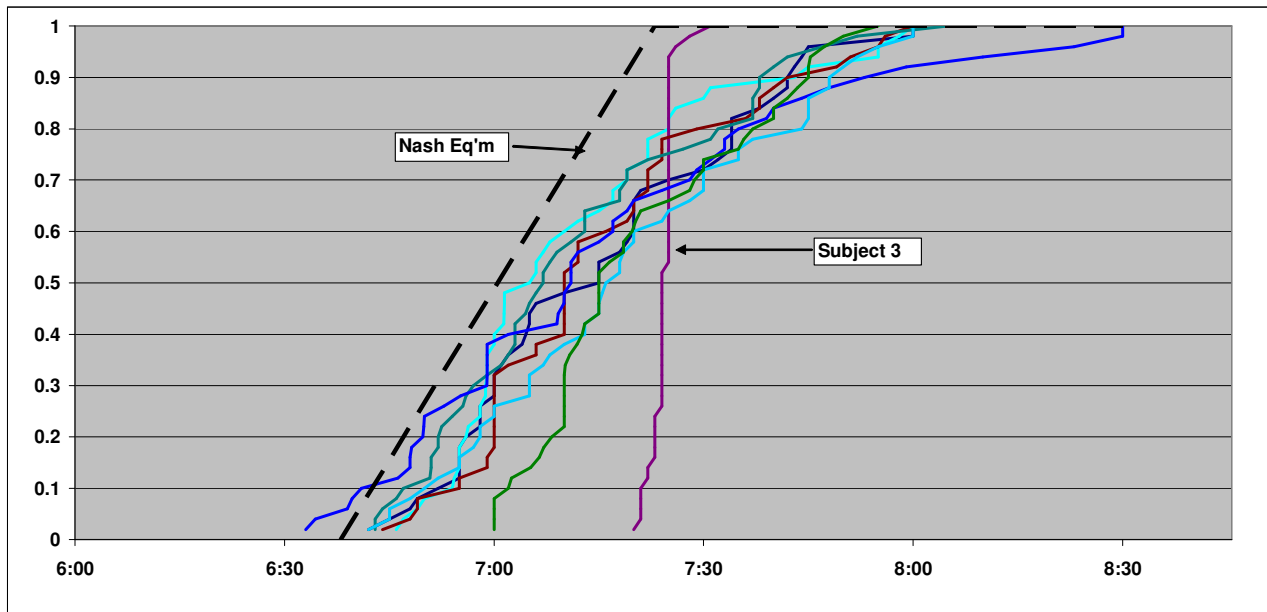


Fig. 6d. Individual Departure Time Distributions for All Sixteen Group 2 Subjects in Condition V3.

