Technological Change and Gender Wage Gaps in the U.S. Service Industry

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The CES Production Function is given by:

\[ Q_t = A_t \left[ \sum_{j=1}^{J} \alpha_{jt} L_{jt}^\rho + \left( 1 - \sum_{j=1}^{J} \alpha_{jt} \right) K_t^\rho \right]^{\frac{\phi}{\rho}} \]
CES Production Function

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- \( Q_t \) is a measure of output in quarter \( t \)

\( A_t \) is a scale factor that captures neutral technological change

\( L_{jt} \) represents employment in quarter \( t \) of the \( j \)th category of labor input

\( J \) is the number of distinct labor inputs defined by gender and four occupational categories within the U.S. service industry

\( K_t \) is a measure of non-labor inputs in quarter \( t \)

\( \alpha_{jt} \) is a productivity-parameter function that captures technological change by measuring the savings in one factor input relative to the others

\( \phi \) is the returns to scale parameter

\( \rho = \sigma_1 / \sigma \), where \( \sigma \) is the elasticity of substitution among inputs
CES Production Function

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- \alpha_{jt} \text{ is a productivity-parameter function that captures technological change by measuring the savings in one factor input relative to the others}
- \phi \text{ is the returns to scale parameter}
- \rho = \frac{\sigma - 1}{\sigma}, \text{ where } \sigma \text{ is the elasticity of substitution among inputs}
Cost Minimization

\[ MP_{L_{jt}} = \phi A_t^{\frac{p}{\phi}} \alpha_{jt} L_{jt}^{\rho - 1} Q_{t}^{1 - \frac{p}{\phi}} \]  

(1)

\[ MP_{K_t} = \phi A_t^{\frac{p}{\phi}} \left[ 1 - \sum_{j=1}^{J} \alpha_{jt} \right] K_t^{\rho - 1} Q_{t}^{1 - \frac{p}{\phi}}. \]  

(2)
Cost Minimization

\[ MP_{Ljt} = \phi A_t^{\rho} \alpha_{jt} L_{jt}^{\rho - 1} Q_t^{1 - \frac{\rho}{\phi}} \]  
\[ MP_{Kt} = \phi A_t^{\rho} \left[ 1 - \sum_{j=1}^{J} \alpha_{jt} \right] K_t^{\rho - 1} Q_t^{1 - \frac{\rho}{\phi}}. \]  
\[ \frac{MP_{Ljt}}{MP_{Lht}} = \frac{w_{jt}}{w_{ht}}, \quad j \neq h \]  
\[ \frac{MP_{Kt}}{MP_{Ljt}} = \frac{r_t}{w_{jt}}. \]
Inverse Relative Input Demand Functions

Upon substituting (1) and (2) into (3) and (4) and normalizing relative to the $h^{th}$ labor input (i.e. $L_{ht}$, and $w_{ht}$), one obtains

$$\frac{\alpha_{jt} L_{jt}^{\rho-1}}{\alpha_{ht} L_{ht}^{\rho-1}} = \frac{w_{jt}}{w_{ht}}, \quad j \neq h.$$
Inverse Relative Input Demand Functions

- Upon substituting (1) and (2) into (3) and (4) and normalizing relative to the $h^{th}$ labor input (i.e. $L_{ht}$, and $w_{ht}$), one obtains

$$\frac{\alpha_{jt} L_{jt}^{\rho-1}}{\alpha_{ht} L_{ht}^{\rho-1}} = \frac{w_{jt}}{w_{ht}}, \quad j \neq h.$$ 

- Taking the log of the above relations yields the following set of inverse relative input demand functions:

$$\ln \left( \frac{w_{jt}}{w_{ht}} \right) = \ln \left( \frac{\alpha_{jt}}{\alpha_{ht}} \right) + (\rho - 1) \ln \left( \frac{L_{jt}}{L_{ht}} \right), \quad j \neq h \quad (5)$$

$$\ln \left( \frac{r_t}{w_{ht}} \right) = \ln \left( \frac{1 - \sum_{j=1}^{J} \alpha_{jt}}{\alpha_{ht}} \right) + (\rho - 1) \ln \left( \frac{K_t}{L_{ht}} \right). \quad (6)$$
Specification of non-neutral technological change

\[
\alpha_{jt} = \frac{e^{\alpha_{j0} + \alpha_{j1} \left( \frac{1}{t} \right) + \epsilon_{jt}}}{1 + \sum_{j=1}^{J} e^{\alpha_{j0} + \alpha_{j1} \left( \frac{1}{t} \right) + \epsilon_{jt}}}, \quad j = 1, \ldots, J
\]

(7)

\[
\alpha_{J+1,t} = 1 - \sum_{j=1}^{J} \alpha_{jt} = \frac{1}{1 + \sum_{j=1}^{J} e^{\alpha_{j0} + \alpha_{j1} \left( \frac{1}{t} \right) + \epsilon_{jt}}},
\]

(8)
Specification of non-neutral technological change

$$\alpha_{jt} = \frac{e^{\alpha_{j0} + \alpha_{j1} \left( \frac{1}{t} \right) + \epsilon_{jt}}}{1 + \sum_{j=1}^{J} e^{\alpha_{j0} + \alpha_{j1} \left( \frac{1}{t} \right) + \epsilon_{jt}}}, \ j = 1, ..., J$$ (7)

$$\alpha_{J+1,t} = 1 - \sum_{j=1}^{J} \alpha_{jt} = \frac{1}{1 + \sum_{j=1}^{J} e^{\alpha_{j0} + \alpha_{j1} \left( \frac{1}{t} \right) + \epsilon_{jt}}}$$, (8)

- $0 < \alpha_{jt} < 1$, $\sum_{j=1}^{J+1} \alpha_{jt} = 1$
Specification of non-neutral technological change

\[
\alpha_{jt} = \frac{e^{\alpha_{j0} + \alpha_{j1} \left( \frac{1}{t} \right) + \epsilon_{jt}}}{1 + \sum_{j=1}^{J} e^{\alpha_{j0} + \alpha_{j1} \left( \frac{1}{t} \right) + \epsilon_{jt}}} , \quad j = 1, \ldots, J 
\]  \hspace{1cm} (7)

\[
\alpha_{J+1,t} = 1 - \sum_{j=1}^{J} \alpha_{jt} = \frac{1}{1 + \sum_{j=1}^{J} e^{\alpha_{j0} + \alpha_{j1} \left( \frac{1}{t} \right) + \epsilon_{jt}}} , \hspace{1cm} (8)
\]

- \( 0 < \alpha_{jt} < 1 \), \( \sum_{j=1}^{J+1} \alpha_{jt} = 1 \)

- \( \epsilon_{jt} \) is a random error term distributed \( (0, \sigma_{\epsilon}^2) \)
Stochastic inverse relative input demand functions

\[
\ln \left( \frac{w_{jt}}{w_{ht}} \right) = \beta_{jh0} + \beta_{jh1} \frac{1}{t} + (\rho - 1) \ln \left( \frac{L_{jt}}{L_{ht}} \right) + \epsilon_{jht}, \quad j \neq h, \quad (9)
\]

\[
\ln \left( \frac{r_t}{w_{ht}} \right) = \beta_{kh0} + \beta_{kh1} \frac{1}{t} + (\rho - 1) \ln \left( \frac{K_t}{L_{ht}} \right) + \epsilon_{kht}, \quad (10)
\]
\[ \ln \left( \frac{w_{jt}}{w_{ht}} \right) = \beta_{jh0} + \beta_{jh1} \frac{1}{t} + (\rho - 1) \ln \left( \frac{L_{jt}}{L_{ht}} \right) + \epsilon_{jht}, \ j \neq h, \quad (9) \]

\[ \ln \left( \frac{r_t}{w_{ht}} \right) = \beta_{kh0} + \beta_{kh1} \frac{1}{t} + (\rho - 1) \ln \left( \frac{K_t}{L_{ht}} \right) + \epsilon_{kht}, \quad (10) \]

\[ \beta_{jh0} = \alpha_{j0} - \alpha_{h0}, \ \beta_{jh1} = \alpha_{j1} - \alpha_{h1}, \ \epsilon_{jht} = \epsilon_{jt} - \epsilon_{ht} \] with \( j \neq h, \) and

\( j = 1, ..., J \) for equation (9)
Stochastic inverse relative input demand functions

\[
\ln \left( \frac{w_{jt}}{w_{ht}} \right) = \beta_{jh0} + \beta_{jh1} \frac{1}{t} + (\rho - 1) \ln \left( \frac{L_{jt}}{L_{ht}} \right) + \epsilon_{jht}, \quad j \neq h, \quad (9)
\]

\[
\ln \left( \frac{r_t}{w_{ht}} \right) = \beta_{kh0} + \beta_{kh1} \frac{1}{t} + (\rho - 1) \ln \left( \frac{K_t}{L_{ht}} \right) + \epsilon_{kht}, \quad (10)
\]

- \( \beta_{jh0} = \alpha_{j0} - \alpha_{h0}, \beta_{jh1} = \alpha_{j1} - \alpha_{h1}, \epsilon_{jht} = \epsilon_{jt} - \epsilon_{ht} \) with \( j \neq h \), and \( j = 1, \ldots, J \) for equation (9)
- \( \beta_{kh0} = -\alpha_{h0}, \beta_{kh1} = -\alpha_{h1}, \epsilon_{kht} = -\epsilon_{ht} \) for equation (10).
Stochastic inverse relative input demand functions

\[
\ln \left( \frac{w_{jt}}{w_{ht}} \right) = \beta_{jh0} + \beta_{jh1} \frac{1}{t} + (\rho - 1) \ln \left( \frac{L_{jt}}{L_{ht}} \right) + \epsilon_{jht}, \quad j \neq h, \quad (9)
\]

\[
\ln \left( \frac{r_t}{w_{ht}} \right) = \beta_{kh0} + \beta_{kh1} \frac{1}{t} + (\rho - 1) \ln \left( \frac{K_t}{L_{ht}} \right) + \epsilon_{kht}, \quad (10)
\]

- \( \beta_{jh0} = \alpha_{j0} - \alpha_{h0} \), \( \beta_{jh1} = \alpha_{j1} - \alpha_{h1} \), \( \epsilon_{jht} = \epsilon_{jt} - \epsilon_{ht} \) with \( j \neq h \), and \( j = 1, \ldots, J \) for equation (9).

- \( \beta_{kh0} = -\alpha_{h0} \), \( \beta_{kh1} = -\alpha_{h1} \), \( \epsilon_{kht} = -\epsilon_{ht} \) for equation (10).

- non-neutral technological change is captured by the coefficients on \( \frac{1}{t} \).

A direct measure might be \( 1/RD_t \).
Let the wage $w_{jt}^m$ of male workers in quarter $t$, occupation $j$ be equated to the corresponding marginal revenue product:

$$w_{jt}^m = MR_t \cdot MP_{jt}^m.$$

Let the wage $w_{jt}^f$ of female workers in quarter $t$, occupation $j$ be equated to the corresponding marginal product, discounted by an unexplained wage differences index ($d_{jt}$):

$$w_{jt}^f = MR_t \cdot MP_{jt}^f (1 + d_{jt}).$$

The relative wage equations for female workers in occupation $j$, (relative to male workers in occupation $h$) will reflect potential unexplained wage differences, and can be written as

$$\ln \frac{w_{jt}^f}{w_{ht}^m} = \ln \frac{MP_{jt}^f}{MP_{ht}^m} \cdot \ln (1 + d_{jt}).$$
Unexplained wage gaps

- Let the wage $w_{jt}^m$ of male workers in quarter $t$, occupation $j$ be equated to the corresponding marginal revenue product:

$$w_{jt}^m = MR_t \cdot MP_{Ljt}^m.$$ 

- Let the wage $w_{jt}^f$ of female workers in quarter $t$, occupation $j$ be equated to the corresponding marginal product, discounted by an unexplained wage differences index ($d_{jt}$):

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$$w_{jt}^f = \frac{MR_t \cdot MP_{jt}^f}{(1 + d_{jt})}.$$  

The relative wage equations for female workers in occupation $j$, (relative to male workers in occupation $h$) will reflect potential unexplained wage differences, and can be written as

$$\ln \left( \frac{w_{jt}^f}{w_{ht}^m} \right) = \ln \left( \frac{MP_{jt}^f}{MP_{ht}^m} \right) - \ln (1 + d_{jt}).$$
Unexplained wage gaps

- It follows that

\[
\ln\left(\frac{w_{jt}^f}{w_{ht}^m}\right) + \ln\left(1 + d_{jt}\right) = \ln\left(\frac{MP_{jt}^f}{MP_{ht}^m}\right) = \beta_{jh0}^f + \beta_{jh1}^m + (\rho - 1) \ln\left(\frac{L_{jt}^f}{L_{ht}^m}\right) + \epsilon_{jht}^f,
\]

where \(\beta_{jh0}^f = \alpha_{j0}^f - \alpha_{h0}^m\), \(\beta_{jh1}^f = \alpha_{j1}^f - \alpha_{h1}^m\), and \(\epsilon_{jht}^f = \epsilon_{jt}^f - \epsilon_{ht}^m\) for \(j, h = 1, \ldots, 4\).
Unexplained wage gaps

- It follows that

\[
\ln\left(\frac{w_{jt}^f}{w_{ht}^m}\right) + \ln(1 + d_{jt}) = \ln\left(\frac{MP_{jt}^f}{MP_{ht}^m}\right) = \beta_{jh0} + \frac{\beta_{jh1}}{t} + (\rho - 1) \ln\left(\frac{L_{jt}^f}{L_{ht}^m}\right) + \epsilon_{jht}^{fm},
\]

where \(\beta_{jh0} = \alpha_{j0}^f - \alpha_{h0}^m\), \(\beta_{jh1} = \alpha_{j1}^f - \alpha_{h1}^m\), and \(\epsilon_{jht}^{fm} = \epsilon_{jt}^f - \epsilon_{ht}^m\) for \(j, h = 1, \ldots, 4\).

- The wage equations for female workers in occupation \(j\), relative to the wage of female workers in occupation \(h\), with \(j \neq h\) can be written as

\[
\ln\left(\frac{w_{jt}^f}{w_{ht}^f}\right) = \ln\left(\frac{MP_{jt}^f}{MP_{ht}^f}\right) + \ln(1 + d_{ht}) - \ln(1 + d_{jt})
\]
It follows that

\[ \ln\left(\frac{w_{jt}^f}{w_{ht}^f}\right) - \ln(1 + d_{ht}) + \ln(1 + d_{jt}) = \ln\left(\frac{MP_{jt}^f}{MP_{ht}^f}\right) \]

\[ = \beta_{j0}^{ff} + \frac{\beta_{j1}^{ff}}{t} + (\rho - 1) \ln\left(\frac{L_{jt}^f}{L_{ht}^f}\right) + \epsilon_{jht}^{ff}, \]

where \( \beta_{j0}^{ff} = \alpha_{j0}^f - \alpha_{h0}^f, \beta_{j1}^{ff} = \alpha_{j1}^f - \alpha_{h1}^f, \) and \( \epsilon_{jht}^{ff} = \epsilon_{jt}^f - \epsilon_{ht}^f \) for \( j, h = 1, \ldots, 4. \)
It follows that

\[ \ln\left(\frac{w_{jt}^f}{w_{ht}^f}\right) - \ln(1 + d_{ht}) + \ln(1 + d_{jt}) = \ln\left(\frac{MP_{jt}^f}{MP_{ht}^f}\right) \]

\[ = \beta_{jh0}^{ff} + \frac{\beta_{jh1}^{ff}}{t} + (\rho - 1) \ln\left(\frac{L_{jt}^f}{L_{ht}^f}\right) + \epsilon_{jht}^{ff}, \]

where \( \beta_{jh0}^{ff} = \alpha_{j0}^f - \alpha_{h0}^f, \beta_{jh1}^{ff} = \alpha_{j1}^f - \alpha_{h1}^f, \) and \( \epsilon_{jht}^{ff} = \epsilon_{jt}^f - \epsilon_{ht}^f \) for \( j, h = 1, \ldots, 4. \)

The wage equations for female workers in occupation \( j, \) relative to the non-labor input can be expressed as

\[ \ln\left(\frac{w_{jt}^f}{r_t}\right) = \ln\left(\frac{MP_{jt}^f}{MP_{Kt}}\right) - \ln(1 + d_{jt}) \]
Unexplained wage gaps

- It follows that

$$\ln \left( \frac{w_{jt}^f}{r_t} \right) + \ln (1 + d_{jt}) = \ln \left( \frac{MP_{jt}^f}{MP_{Kt}} \right)$$

$$= \beta_{jk0}^f + \frac{\beta_{jk1}^f}{t} + (\rho - 1) \ln \left( \frac{L_{jt}^f}{K_t} \right) + \epsilon_{jkt}^f,$$

where $$\beta_{jk0}^f = \alpha_{j0}^f$$, $$\beta_{jk1}^f = \alpha_{j1}^f$$, and $$\epsilon_{jkt}^f = \epsilon_{jt}^f$$ for $$j = 1, \ldots, 4$$. 
Unexplained wage gaps

- It follows that

\[
\ln\left(\frac{w_{jt}^f}{r_t}\right) + \ln\left(1 + d_{jt}\right) = \ln\left(\frac{MP_{jt}^f}{MP_{Kt}}\right)
\]

\[
= \beta_{jk0}^f + \frac{\beta_{jk1}^f}{t} + (\rho - 1) \ln\left(\frac{L_{jt}^f}{K_t}\right) + \epsilon_{jkt}^f,
\]

where \(\beta_{jk0}^f = \alpha_{j0}^f\), \(\beta_{jk1}^f = \alpha_{j1}^f\), and \(\epsilon_{jkt}^f = \epsilon_{jt}^f\) for \(j = 1, \ldots, 4\).

- The wage equations for male workers in occupation \(j\), relative to the non-labor input can be expressed as

\[
\ln\left(\frac{w_{jt}^m}{r_t}\right) = \ln\left(\frac{MP_{jt}^m}{MP_{Kt}}\right)
\]

\[
= \beta_{jk0}^m + \frac{\beta_{jk1}^m}{t} + (\rho - 1) \ln\left(\frac{L_{jt}^m}{K_t}\right) + \epsilon_{jkt}^m,
\]

where \(\beta_{jk0}^m = \alpha_{j0}^m\), \(\beta_{jk1}^m = \alpha_{j1}^m\), and \(\epsilon_{jkt}^m = \epsilon_{jt}^m\) for \(j = 1, \ldots, 4\).
Unexplained wage gaps

- Separate wage equations for each occupation and gender can be estimated using monthly micro data on individual characteristics (schooling, potential experience, potential experience squared).
Unexplained wage gaps

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  - individual sampling weights are used.
Unexplained wage gaps

- Separate wage equations for each occupation and gender can be estimated using monthly micro data on individual characteristics (schooling, potential experience, potential experience squared).
  - individual sampling weights are used
  - the monthly data are pooled for each quarter.

Consider \( \ln w_{mljt} = X_{mjt} \hat{\beta}_m + \upsilon_{mljt} \), where \( X_{mjt} \) is the vector of individual characteristics of male workers.

Similarly, consider the wage equation for a female worker \( l_i \), in occupation \( j \), quarter \( t \),
\[ \ln w_{fjt} = X_{fjt} \hat{\beta}_f + \upsilon_{fjt}, \] (11)
where \( X_{fjt} \) is the vector of individual characteristics of female workers.
Unexplained wage gaps

- Separate wage equations for each occupation and gender can be estimated using monthly micro data on individual characteristics (schooling, potential experience, potential experience squared).
  - Individual sampling weights are used.
  - The monthly data are pooled for each quarter.
- Consider first the wage equation for a male worker \( l \), in occupation \( j \), quarter \( t \),

\[
\ln (w_{jtl}^m) = X_{jtl}^m \hat{\beta}_j^m + \nu_{jtl}^m,
\]

where \( X_{jtl}^m \) is the vector of individual characteristics of male workers.
Unexplained wage gaps

- Separate wage equations for each occupation and gender can be estimated using monthly micro data on individual characteristics (schooling, potential experience, potential experience squared).
  - Individual sampling weights are used
  - The monthly data are pooled for each quarter.

Consider first the wage equation for a male worker $l$, in occupation $j$, quarter $t$,

$$\ln (w_{jtl}^m) = X_{jtl}^m \hat{\beta}_{jt}^m + \nu_{jtl}^m,$$

where $X_{jtl}^m$ is the vector of individual characteristics of male workers.

Similarly, consider the wage equation for a female worker $l$, in occupation $j$, quarter $t$,

$$\ln (w_{jtl}^f) = X_{jtl}^f \hat{\beta}_{jt}^f + \nu_{jtl}^f,$$

where $X_{jtl}^f$ is the vector of individual characteristics of female workers.
Unexplained wage gaps

The wage decomposition for workers in occupation $j$, quarter $t$, is given by

$$\ln(w_{jt}^m) - \ln(w_{jt}^f) = (\bar{X}_{jt}^m - \bar{X}_{jt}^f)\hat{\beta}_{jt}^m + \bar{X}_{jt}^f(\hat{\beta}_{jt}^m - \hat{\beta}_{jt}^f),$$

where the first right hand side term represents the wage gap due to differences in skills and the second term represents the unexplained wage gap.
The wage decomposition for workers in occupation $j$, quarter $t$, is given by

$$\ln(w_{jt}^m) - \ln(w_{jt}^f) = (\bar{X}_{jt}^m - \bar{X}_{jt}^f) \hat{\beta}_{jt}^m + \bar{X}_{jt}^f (\hat{\beta}_{jt}^m - \hat{\beta}_{jt}^f),$$

where the first right hand side term represents the wage gap due to differences in skills and the second term represents the unexplained wage gap.

A measure of unexplained differences can be obtained as

$$\ln(1 + d_{jt}) = \bar{X}_{jt}^f (\hat{\beta}_{jt}^m - \hat{\beta}_{jt}^f)$$

$$= \ln \left( \frac{w_{jt}^m}{w_{jt}^f} \right) - (\bar{X}_{jt}^m - \bar{X}_{jt}^f) \hat{\beta}_{jt}^m.$$
Unexplained wage gaps

• $\bar{X}_{jt}^{f(m)}$ is the sample weighted average of female (male) workers’ characteristics, $\bar{X}_{jt}^{f(m)} = \sum_{l_f} (X_{jtl}^{f(m)}) \times weight_{jtl}^{f(m)}$, and $weight_{jtl}^{f(m)}$ is the sampling weight.
Unexplained wage gaps

- $\bar{X}_{jt}^f(m)$ is the sample weighted average of female (male) workers’ characteristics, $\bar{X}_{jt}^f(m) = \sum_{l_f} (X_{jtl}^f(m)) \times weight_{jtl}^f(m)$, and $weight_{jtl}^f(m)$ is the sampling weight.

- Because the unexplained gap is a residual after netting out the effects of gender differences in characteristics, wage equations were only estimated for males.
User cost of capital and measurement of nonlabor inputs

\[ P_t Q_t = w_t L_t + r_t K_t \]  

\[ r_t = (i_t + \delta_t) p_{dt} \]

\( i_t \) is the quarterly 3-month T-bill rate from the Federal Reserve Statistical Release of Historical Data
\( \delta_t \) is the depreciation rate
\( p_{dt} \) is a price deflator for private fixed investment

The \( K_t \) series is obtained residually from (12):

\[ K_t = \left( P_t Q_t - w_t L_t \right) r_t \]
User cost of capital and measurement of nonlabor inputs

\[ P_t Q_t = w_t L_t + r_t K_t \] (12)

\[ r_t = (i_t + \delta_t) pd_t, \]
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\[ P_t Q_t = w_t L_t + r_t K_t \]  \hspace{1cm} (12)

\[ r_t = (i_t + \delta_t) p_d t, \]

- \( i_t \) is the quarterly 3-month T-bill rate from the Federal Reserve Statistical Release of Historical Data
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User cost of capital and measurement of nonlabor inputs

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- \( \delta_t \) is the depreciation rate
- \( pd_t \) is a price deflator for private fixed investment
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\[ K_t = \frac{(P_t Q_t - w_t L_t)}{r_t} \]
Identification

- Empirical issues
Empirical issues

- cross-equation restrictions on $\rho$ and other parameters
Identification

Empirical issues

- cross-equation restrictions on $\rho$ and other parameters
- endogeneity of the relative input ratios
Identification

- **Empirical issues**
  - cross-equation restrictions on $\rho$ and other parameters
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- **Instruments**
Identification

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- **Instruments**
  - the ratio of year-round, economy wide, employed women to employed men ($L_t^f / L_t^m$)
Identification

- **Empirical issues**
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- **Instruments**
  - the ratio of year-round, economy wide, employed women to employed men ($L_t^f / L_t^m$)
  - 3-month T-bill rates ($i_t$)
Empirical issues

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- endogeneity of the relative input ratios

Instruments

- the ratio of year-round, economy wide, employed women to employed men ($L_t^f / L_t^m$)
- 3-month T-bill rates ($i_t$)
- $\frac{1}{t}$ or $1/RD_t$
Avoidance of the “overidentification“ problem necessitates estimating \(^{\binom{9}{2}} = 36\) equations for all possible wage differential pairings with cross-equation restrictions that uniquely identify the estimated parameters.

Because any 8 equations can span the remaining 28 equations, internal consistency requires that additional cross-equation restrictions be imposed on the constant term and the coefficient of the time variable. These constraints insure invariance of the estimated coefficients to the choice of any 8 equations. The residual variance/covariance matrix will be singular because the error terms will be perfect linear combinations of one another. Therefore, Three Stage Least Squares estimation cannot be performed for the system of 36 demand equations. Two Stage Least Squares (2SLS) is used to estimate the system jointly with appropriate cross-equation restrictions.
Estimation strategy

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Two Stage Least Squares (2SLS) is used to estimate the system jointly with appropriate cross-equation restrictions.
Table 1: Definition of Occupation Variables

<table>
<thead>
<tr>
<th>Occupational Categories</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Occ 1</em>  Managerial and Professional Specialty</td>
</tr>
<tr>
<td><em>Occ 2</em>  Technical, Sales and Administrative Support</td>
</tr>
<tr>
<td><em>Occ 3</em>  Service Occupations and Precision Production, Craft and Repair</td>
</tr>
<tr>
<td><em>Occ 4</em>  Operators, Fabricators and Laborers, Farming, Forestry and Fishing</td>
</tr>
</tbody>
</table>

Table 2: Definition of Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w^f_{jt}$</td>
<td>Hourly wage of full time female employees, occupation j, quarter t [dollars]</td>
</tr>
<tr>
<td>$w^m_{jt}$</td>
<td>Hourly wage of full time male employees, occupation j, quarter t [dollars]</td>
</tr>
<tr>
<td>$L^f_{jt}$</td>
<td>Full time female employees, occupation j, quarter t [thousands]</td>
</tr>
<tr>
<td>$L^m_{jt}$</td>
<td>Full time male employee, occupation j, quarter t [thousands]</td>
</tr>
<tr>
<td>$r_t$</td>
<td>Non-labor Input factor price in quarter t</td>
</tr>
<tr>
<td>$K_t$</td>
<td>Non-labor Input, in quarter t [thousand 2000 dollars]</td>
</tr>
<tr>
<td>$i_t$</td>
<td>3-month T-bill rate</td>
</tr>
<tr>
<td>$RD_t$</td>
<td>Total R&amp;D expenditure for quarter t [million 2000 dollars]</td>
</tr>
</tbody>
</table>
Table 3: Summary Statistics of Main Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>No.Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{1t}$</td>
<td>0.791</td>
<td>0.164</td>
<td>92</td>
</tr>
<tr>
<td>$w_{2t}$</td>
<td>0.783</td>
<td>0.189</td>
<td>92</td>
</tr>
<tr>
<td>$w_{3t}$</td>
<td>0.786</td>
<td>0.431</td>
<td>87</td>
</tr>
<tr>
<td>$w_{4t}$</td>
<td>0.895</td>
<td>0.070</td>
<td>92</td>
</tr>
<tr>
<td>$L_{1t}$</td>
<td>1,694,543</td>
<td>374,661.6</td>
<td>92</td>
</tr>
<tr>
<td>$L_{2t}$</td>
<td>1,195,887</td>
<td>157,752.6</td>
<td>92</td>
</tr>
<tr>
<td>$L_{3t}$</td>
<td>908,408.7</td>
<td>96,894.8</td>
<td>92</td>
</tr>
<tr>
<td>$L_{4t}$</td>
<td>85,589.4</td>
<td>13,714.7</td>
<td>92</td>
</tr>
<tr>
<td>$L^m_{1t}$</td>
<td>1,602,240</td>
<td>240,879.3</td>
<td>92</td>
</tr>
<tr>
<td>$L^m_{2t}$</td>
<td>343,033.3</td>
<td>77,278.38</td>
<td>92</td>
</tr>
<tr>
<td>$L^m_{3t}$</td>
<td>794,278.2</td>
<td>83,561.8</td>
<td>92</td>
</tr>
<tr>
<td>$L^m_{4t}$</td>
<td>231,720.2</td>
<td>39,422.2</td>
<td>92</td>
</tr>
<tr>
<td>$r_t$</td>
<td>0.055</td>
<td>0.006</td>
<td>92</td>
</tr>
<tr>
<td>$K_t$</td>
<td>58,448.3</td>
<td>28,594.7</td>
<td>92</td>
</tr>
<tr>
<td>$i_t$</td>
<td>0.016</td>
<td>0.007</td>
<td>92</td>
</tr>
<tr>
<td>$RD_t$</td>
<td>33,426</td>
<td>8,417.57</td>
<td>92</td>
</tr>
</tbody>
</table>

Data source: 1979-2001 Quarterly CPS data and NSF R&D data.
Table 5: The Relation between Technological Change and the Gender Wage Gaps in the U.S. Service Industry: Two Stage Least Squares Estimates of Inverse Relative Input Demand Functions taking into Account the Unexplained Gender Wage Gaps

<table>
<thead>
<tr>
<th>Independent Variables</th>
<th>( \ln\left(\frac{MP_f}{MP_m}\right) )</th>
<th>( \ln\left(\frac{MP_{f2}}{MP_{m2}}\right) )</th>
<th>( \ln\left(\frac{MP_{f3}}{MP_{m3}}\right) )</th>
<th>( \ln\left(\frac{MP_{f4}}{MP_{m4}}\right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-.398*** (.017)</td>
<td>.392*** (.018)</td>
<td>-.909** (.033)</td>
<td>-.428*** (.022)</td>
</tr>
<tr>
<td>( \frac{1}{t} )</td>
<td>-.415*** (.086)</td>
<td>-.038 (.090)</td>
<td>-.039 (.117)</td>
<td>.052 (.050)</td>
</tr>
<tr>
<td>( \ln\left(\frac{L_f}{L_m}\right) )</td>
<td>-.323*** (.018)</td>
<td>-.323*** (.018)</td>
<td>-.323*** (.018)</td>
<td>-.323*** (.018)</td>
</tr>
</tbody>
</table>

\( \sigma = \frac{1}{(1-\rho)} \) 3.10

No. of Observations = 89

Note: Standard errors are in parentheses. *Statistically significant at .10 level; ** at the .05 level; *** at the .01 level (two tailed t-tests).
Table 7: The Relation between Technological Change and the Gender Wage Gaps in the U.S. Service Industry: Two Stage Least Squares Estimates of Inverse Relative Input Demand Functions taking into Account the Unexplained Gender Wage Gaps and using R&D as Direct Measure of Technological Change

<table>
<thead>
<tr>
<th>Independent Variables</th>
<th>ln(MP$_1^f$/MP$_1^m$)</th>
<th>ln(MP$_2^f$/MP$_2^m$)</th>
<th>ln(MP$_3^f$/MP$_3^m$)</th>
<th>ln(MP$_4^f$/MP$_4^m$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-.026 (.038)</td>
<td>.731*** (.066)</td>
<td>-1.018*** (.094)</td>
<td>-.714*** (.059)</td>
</tr>
<tr>
<td>1/(RD)</td>
<td>-1.510*** (.152)</td>
<td>-.797*** (.140)</td>
<td>-.271 (.197)</td>
<td>.460*** (.067)</td>
</tr>
<tr>
<td>ln((L_f^j/L_m^j))</td>
<td>-.398*** (.031)</td>
<td>-.398*** (.031)</td>
<td>-.398*** (.031)</td>
<td>-.398*** (.031)</td>
</tr>
</tbody>
</table>

\(\sigma = \frac{1}{(1-\rho)}\) 2.51

No. of Observations = 89

Note: Standard errors are in parentheses. *Statistically significant at .10 level; ** at the .05 level; *** at the .01 level (two tailed t-tests).