Econ 481, 482 prerequisite tools: review notes

Econ 339/Econ 276 tools

Let \( X \) be a normally distributed random variable such that \( E(X) = u_x \) and \( E(X - u_x)^2 = \sigma_x^2 \) (variance).

Let \( X_1, ..., X_T \) denote a random sample of size \( T \).

Estimator of the mean of \( X \): \( \hat{u}_x = \bar{X} = \frac{\sum_{t=1}^{T} X_t}{T} \).

Unbiased estimator: \( E(\bar{X}) = u_x \)

Estimator of the variance: \( \hat{\sigma}_x^2 = \frac{\sum_{t=1}^{T} (X_t - \bar{X})^2}{T - 1} \)

Standard error of \( \bar{X} \): \( \sigma_{\bar{X}} = \frac{\sigma_x}{\sqrt{T}} \)

Estimated standard error of \( \bar{X} \): \( \hat{\sigma}_{\bar{X}} = \frac{\hat{\sigma}_x}{\sqrt{T}} \)

\( t \) statistic: \( \frac{\bar{X}}{\sigma_{\bar{X}}} \sim t_{T-1} \)

Hypothesis testing: \( H_0: u_x = c, \quad H_1: u_x \neq c \)

\( \left| \frac{\bar{X} - c}{\sigma_{\bar{X}}} \right| > t_{0.975}^{T-1} \Rightarrow \text{reject } H_0 \text{ at the 5% level of significance for a two-tailed test} \)
Math tools for Econ 361

\( \ln(zx) = \ln(z) + \ln(x) \) and \( \ln \left( \frac{z}{x} \right) = \ln(z) - \ln(x) \)

\( \ln(x^b) = b \ln(x) \)

Let \( y = ax^n \), then \( \frac{dy}{dx} = nax^{n-1} \)

Let \( y = a \), then \( \frac{dy}{dx} = 0 \)

Let \( y = a\ln(x) \), then \( \frac{dy}{dx} = \frac{a}{x} \)

Let \( y = zx \), then \( \frac{dy}{dx} = z + x \left( \frac{dz}{dx} \right) \)

Let \( y = ax^n z^b \), then
\[
\frac{dy}{dx} = \frac{\partial y}{\partial x} \frac{dx}{dx} + \frac{\partial y}{\partial z} \frac{dz}{dx}
\]
\[= nax^{n-1} z^b dx + bax^n z^{b-1} dz \]

Let \( y = ae^{bx} \), then \( \frac{dy}{dx} = bae^{bx} \)

Econ 361 tools

Total revenue: \( TR = PQ \)

Average revenue: \( AR = \frac{TR}{Q} = P \)

Marginal revenue: \( MR = \frac{d(TR)}{dQ} = P + Q \left( \frac{dP}{dQ} \right) \)

Point elasticity of demand: \( \eta_{qp} = \left( \frac{dQ}{dP} \right) \left( \frac{P}{Q} \right) \)
Arc elasticity of demand: \( \eta_{qp} = \left( \frac{\Delta Q}{\Delta P} \right) \left( \frac{P_1 + P_2}{Q_1 + Q_2} \right) \), where \( \Delta Q = Q_2 - Q_1 \) and \( \Delta P = P_2 - P_1 \).

Total Cost: \( TC = C(Q) \)

Average Cost: \( AC = \frac{TC}{Q} \)

Marginal Cost: \( MC = \frac{d(TC)}{dQ} \)

Production function: \( Q = F(K, L) \)

Average products of capital and labor: \( AP_k = \frac{Q}{K}, \ AP_l = \frac{Q}{L} \)

Marginal products of capital and labor: \( MP_k = \frac{\partial Q}{\partial K}, \ MP_l = \frac{\partial Q}{\partial L} \)

Marginal rate of technical substitution: \( MRTS_{kl} = -\frac{dK}{dL} = \frac{MP_l}{MP_k} \)

Elasticity of substitution: \( \sigma_{kl} = \frac{\% \Delta (K/L)}{\% \Delta (w/r)} \), where \( w \) is the wage rate and \( r \) is the rental rate on capital.

Utility function: \( U = G(x, y) \)

Marginal utilities of \( x \) and \( y \): \( MU_x = \frac{\partial U}{\partial x}, \ MU_y = \frac{\partial U}{\partial y} \)

Marginal rate of substitution: \( MRS_{yx} = -\frac{dy}{dx} = \frac{MU_x}{MU_y} \)

Utility function for \( n \) goods: \( U = G(x_1, ..., x_n) \)

Budget constraint: \( y = p_1x_1 + \ldots + p_nx_n = \sum_{i=1}^{n} p_ix_i \)
Demand function for good 1: \( x_1 = d_1(p_1, ..., p_n, y) \)

Pure income effect on the demand for good 1: \( \frac{\partial x_1}{\partial y} < 0 \)

Slutsky equation for good 1:

\[
\frac{\partial x_1}{\partial p_1} = \left( \frac{\partial x_1}{\partial p_1} \right)_{\bar{u}} - x_1^* \frac{\partial x_1}{\partial y}
\]

where \( \frac{\partial x_1}{\partial p_1} \gg 0 \) is the slope of the ordinary (uncompensated) demand curve, 
\( \left( \frac{\partial x_1}{\partial p_1} \right)_{\bar{u}} < 0 \) is the substitution effect (slope of the compensated demand curve), and 
\( -x_1^* \frac{\partial x_1}{\partial y} \ll 0 \) is the income effect of the price change.