OPTIMAL INVESTMENT IN SCHOOLING: A TEST
OF THE HUMAN CAPITAL MODEL

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I. Introduction

Social and intellectual interest in income disparities have generated a great deal of research on the formation of personal income. For most adults, labor earnings are the dominant source of income. Consequently, economists have directed their attention toward identifying determinants of the earnings of individual workers. Among theories of earnings formation, human capital theory enjoys preeminence. In this paper we specify and estimate a human capital model that incorporates the essence of schooling-investment decisions that are based on individual wealth maximization, and that identifies demand and supply functions for formal schooling.

A significant innovation in the empirical application of human capital theory was achieved by Becker (1964, 1975), Becker and Chiswick (1966), and Mincer (1974). Their studies offered a simple specification of the earnings-schooling relationship in which the natural logarithm of earnings was a linear function of years of schooling for an individual. This specification is easily estimated, and the coefficient on schooling readily lends itself to interpretation as an internal rate of return.

One obvious shortcoming of the earlier empirical work with the simple schooling model was its neglect, due to a paucity of data, of the role of ability. More recently, however, data sources that include proxy measures for ability have permitted researchers to control explicitly for its effects on earnings and schooling. Chief among these studies are Griliches (1976, 1977), Mason and Griliches (1972), Hause (1972), Lillard (1977), and Taubman (1975).

The leading impression left by these studies, however, is that the main difficulty with the simple schooling model is the bias caused by omission of
a variable. What is rarely recognized and seldom discussed in the empirical literature is that the simple schooling model is not identified, and that as a result, the estimated coefficient on schooling in the earnings equation has no economic interpretation at all, let alone as a rate of return.\textsuperscript{1} The implications of the identification problem are spelled out in Rosen (1973), which serves as the inspiration for the theoretical framework presented in Section II. In Section III we present an empirical specification of a schooling-model that is both identified and based on individual wealth maximization. Empirical results are presented in Section IV. Section V offers a summary and some conclusions.

II. Theoretical Framework

For the purposes of this paper, we concentrate exclusively on the schooling decision and treat it as though it were purely an investment activity. We first posit the existence of an earnings-transformation function that describes the conversion of schooling and ability into earnings:

\begin{equation}
Y = F(S, A),
\end{equation}

such that

\[ F_s, F_A, F_{SA} = F_{AS} > 0, F_{SS} < 0, \text{ and } F_s F_{SA} > F_A F_S, \]

where $Y$ is annual earnings, $S$ is years of schooling, and $A$ is some measure

\textsuperscript{1}Although Griliches (1977) noted that a missing variable was not the only shortcoming of the simple schooling model, he did not pursue the identification problem per se, but rather was concerned with the possible correlation of schooling with the error term in the earnings equation. Another attempt at formulating a more complete schooling model is found in Lazear (1977). That study focused on the consumption aspects of schooling, however, and employed a different model specification.
of ability.\textsuperscript{2,3}

We assume that all costs are foregone earnings and that the individual seeks to maximize the present value of lifetime earnings subject to the constraint imposed by equation (1).

For mathematical simplicity, we assume that the individual acts as if he or she faced an infinite horizon.\textsuperscript{4} Accordingly, the maximization problem is formally expressed by

\begin{equation}
\begin{aligned}
&\text{Max } V = \int_{-\infty}^{\infty} y e^{-iv} dv \\
&\text{s.t. } Y = F(S,A),
\end{aligned}
\end{equation}

where $V$ is the present value of lifetime earnings, $i$ is a fixed discounting rate of interest, and $v$ is an index.

Once the constraint is substituted into the objective (present value) function, the first-order conditions that obtain are familiar:

\begin{equation}
\begin{aligned}
&\text{r} = i, \\
&\text{where } r = \frac{\partial \ln F(S,A)}{\partial S}
\end{aligned}
\end{equation}

is the marginal rate of return.

This is the solution to the Austrian problem -- that is, in deciding when to

\textsuperscript{2}The assumption $F \cdot F_{SA} > F_{AS}$ is a necessary and sufficient condition for the marginal rate of return to schooling to increase with ability, and hence for the demand for schooling to increase with ability. In the special case of a linearly homogeneous transformation function, this property implies that the elasticity of substitution between schooling and ability is less than one.

\textsuperscript{3}As Rosen (1973) points out, the transformation function is derived from a production function of knowledge whose arguments are schooling and ability. The units of knowledge (human capital) are multiplied by a constant market rental rate on human capital to yield earnings. The production function itself is derived from a learning function that governs the rate at which knowledge can be produced from prior schooling and ability.

\textsuperscript{4}For normal working lives -- that is, 30 to 40 years -- and discount rates in the neighborhood of 10 percent, the error introduced by this approximation is small.
terminate the investment, finding the point at which the rate of growth in value is equal to the discounting rate of interest.\(^5\)

The solution to the maximization problem also fits within the framework of conventional demand and supply analysis.\(^6\) The individual's demand function for investment in schooling is obtained from the first derivative of the transformation function with respect to schooling (ability is assumed fixed for the individual):

\[(4) \quad r = r(S,A),\]

or equivalently

\[S^d = S^d(i,A),\]

where \(S^d\) is the level of schooling demanded at each rate of interest for a given level of ability.

The individual's supply function for schooling investment is obtained from the present-value function. Simple manipulation of the present value expression yields

\[(5) \quad \ln Y = \ln(iV) + iS.\]

The individual's supply curve for schooling investment is arrived at by

\(^5\)Because an individual invests in a wage, which produces income only if he or she then works, the schooling-investment decision actually entails the simultaneous choice of years of schooling and lifetime labor (and consequently leisure) hours. See Lindsay (1971). It seems likely, therefore, that the marginal disutility of work would cause the individual to choose years of schooling that would less than maximize the present value of lifetime earnings. This would explain the result in Lazear (1977) that schooling appears to be an inferior good, which it may or may not be. We assume that any differences between the schooling that would maximize the present value of earnings and that which would maximize lifetime utility are small.

\(^6\)This is essentially the method used in Becker (1967).
differentiating (5) with respect to \( S \). Although the discounting rate of interest does not vary with \( S \) and is therefore fixed to the individual, it can vary across individuals. The rate of interest represents the marginal opportunity cost of an additional year of schooling, and this can vary across individuals. Thus we can specify the supply curve of schooling investment as infinitely elastic in terms of the rate of interest with respect to the level of schooling, but shifting among individuals as a result of variation in family resources:

\[
(6) \quad i = i(X),
\]

where \( X \) is a vector of family characteristics. From the first-order condition (3) and equations (4) and (6) we can solve for the reduced-form equation for the optimal level of schooling desired:

\[
(7) \quad S^* = f(X, \Lambda).
\]

A geometric representation of the model is shown in figure 1. In the upper half of the figure the logarithmic form of the transformation function equation (1) is represented by the concave curve. The present-value function (in logs) consists of a family of straight lines defined by equation (5). Each line represents the locus of \( \ln Y \) and \( S \) combinations that yield the same present value at a given rate of interest. Upward and leftward, each line represents successively higher present values. The optimal level of schooling desired, \( S^* \), is the level that maximizes the present discounted value of lifetime earnings. This occurs at the point of tangency between the earnings-transformation curve and a present-value line. Thus the solution is given
\[ \ln Y = \ln (\ln (i^*) + iS) \]

\[ \ln Y = \ln [F(S, A)] \]

Figure 1
by \((S^*, Y^*, V^*)\). At the optimal schooling level, the marginal rate of return is equal to the rate of interest. This is represented in the lower half of Figure 1 by the intersection of the downward-sloping demand curve and the infinitely elastic supply curve.

Now consider the situation that exists when one allows abilities and discounting rates of interest to vary across individuals. In a cross section of individuals, the scatter of points corresponding to the log of earnings and schooling completed are generated by the tangencies between varying transformation curves and present-value lines. These data points correspond to the intersections of the underlying investment demand and supply curves. Figure 2 depicts the situation. It is clear that the specification of the simple schooling model,

\[
\ln Y_j = b_0 + b_1 S_j + \mu_j
\]

(8) \(\ln Y_j = b_0 + b_1 S_j + \mu_j\)

(where \(Y_j\) represents the earnings of the \(j^{th}\) worker), amounts to fitting a line to a set of equilibrium points generated from the tangencies between present-value lines and earnings-schooling transformation curves. As a consequence the coefficient \(b_1\) has no economic meaning because the model is not identified.\(^7\) Nevertheless, its interpretation in previous studies as

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\(^7\)There are two, but highly implausible, conditions under which the simple schooling model of equation (8) could be identified. First, suppose everyone were equally able and faced a transformation function in which \(\ln Y = \ln Y_0 + rS\), where \(Y_0\) represents earnings in the absence of formal schooling. This equation would be identified if present-value curves (in logs) were convex and differed across individuals. Tangencies between convex present-value curves and the semi-log transformation function would correspond to intersections of various upward-sloping investment-supply curves with a single perfectly elastic investment-demand function at \(r\). A second possibility is that equation (8) is identified as equation (5) by assuming that all face the same rate of interest but possess differing concave transformation curves that are coincidentally tangent at various points along the same present-value line. Thus it is exceedingly difficult to make a convincing case for identifying equation (8) as either \(\ln Y = \ln Y_0 + rS\) or \(\ln y = \ln (iV^*) + iS\).
some sort of average rate of return has been reinforced by repeated estimates, that are not only positive and statistically significant, but are more or less plausible as well.

III. Empirical Specification

Our stochastic approximation to equation (1) in logs is given by

\[(9) \quad \ln Y_j = b_0 + b_1 S_j + b_2 A_j S_j + b_3 S_j^2 + \mu_j,\]

where \( A_j \) is some measure of ability, \( \mu_j \) is a disturbance term, and \( b_1, b_2 > 0, b_3 < 0. \)

The schooling-investment demand function is obtained from the marginal rate of return function:

\[(10) \quad r_j = b_1 + b_2 A_j + 2b_3 S_j, \quad \text{where} \quad r_j = \frac{\partial \ln Y_j}{\partial S_j}.\]

We specify the schooling-investment supply relationship to be a linear function of permanent family income and family size:

\[(11) \quad i_j = a_0 + a_1 I_j + a_2 N_j + \varepsilon_j,\]

where \( I_j \) is permanent family income, \( N_j \) is size of family, \( \varepsilon_j \) is a disturbance term, and \( a_1 < 0, a_2 > 0. \)

The greater the permanent income of an individual's parental family, ceteris paribus, the greater the resources available for schooling investment, hence the lower the marginal opportunity cost and discounting rate of interest faced by the individual. The larger one's family, ceteris paribus, the less the resources available for schooling investment and hence the greater the marginal opportunity cost and discounting rate of interest.

Our reduced-form schooling equation is obtained by substituting equations (10) and (11) into the equilibrium condition.
\( r_j = i_j \)

and solving for the level of schooling:

\( S_j = \gamma_0 + \gamma_1 I_j + \gamma_2 N_j + \gamma_3 A_j + \omega_j, \)

where \( \gamma_0 = (a_0 - b_1)/2b_3 \), \( \gamma_1 = a_1/2b_3 > 0 \), \( \gamma_2 = a_2/2b_3 < 0 \),
\( \gamma_3 = -b_2/2b_3 > 0 \), \( \omega_j = \varepsilon_j/2b_3 \), and \( \sigma^2 = \sigma^2/4b_3^2 \).

Unfortunately, the schooling-investment demand and supply functions cannot be estimated directly because marginal rates of return and discounting rates of interest are not directly observable. Consequently, the estimated parameters of the investment demand function, equation (10), are obtained from ordinary least squares (OLSQ) estimation of the earnings function, equation (9). These estimated parameters then imply estimated marginal rates of return to schooling, \( \hat{r}_j \), for each individual. From the equilibrium condition, equation (12), we impose the condition

\( \hat{r}_j = \hat{i}_j \)

and estimate the investment supply function, equation (11), using the estimated marginal rates of return as values of the dependent variable.

The reduced-form schooling relationship, equation (13), can be estimated in unrestricted form by OLSQ. Alternatively, the restrictions on the reduced form parameters and residual variance implied by equations (10)-(12) and listed below equation (13) can be imposed in the form of indirect estimation of equation (13) through appropriate substitution of the estimated parameters of equations (10) and (11). These restrictions on the reduced-form schooling coefficients and residual variance are jointly tested below with a \( \chi^2 \) test based on a likelihood ratio test.
An important consideration in estimating the schooling model defined by equations (9), (10), (11), and (13), is that actual earnings data reflect the effects of post-school investments in work experience as well as schooling and ability. To ignore those effects would bias the OLSQ estimator of equation (9) to the extent that schooling and experience are correlated in cross-section data. In dealing with this issue we adopt a procedure similar to that employed by Mincer (1974) in the estimation of the simple schooling model of equation (8) and its variants. The sample is stratified into work-experience groups and equation (9) is estimated separately for each of those cohorts. That cohort for which the earnings regression corresponding to equation (9) exhibits the smallest residual variance is then selected as providing the best estimate of the schooling model.

The rationale behind this procedure is that the observed earnings of the selected cohort will more closely correspond to what their earnings would have been based only on schooling and ability -- that is, in the absence of post-school investments. For example, the observed earnings of those with little experience will tend to be less than their earnings based solely on schooling and ability because of the foregone earnings attributable to on-the-job training (OJT). Similarly, the observed earnings of those with more experience will tend to exceed their earnings based solely on schooling and ability because of the returns to OJT. At the overtaking year of experience, an individual's observed earnings will be identical, apart from random disturbances, to his or her earnings based solely on schooling and ability. If the overtaking period were the same for everyone, it would be a relatively simple matter to identify the
overtaking cohort and estimate the schooling model with the data for this group. However, it is reasonable to suppose that the overtaking period varies across individuals. Consequently, we control for the influences of post-school investments by estimation of a full interaction model in which the earnings function (equation 9) is estimated separately for each experience cohort. The regression results corresponding to the experience cohort for which the schooling and ability variables are associated with the smallest residual variance in the log of earnings are adopted as our best estimates of the schooling model.

IV. Empirical Results

Data for this study are taken from the National Longitudinal Survey (NLS) of Young Men 14-24 in 1966. The subsample selected for the study consists of white males who were not enrolled in school in 1969, who earned at least $100 that year, and who reported their highest educational level completed.

Two alternative proxies for ability were used: the individual’s Intelligence Quotient (IQ) and the Knowledge of World of Work (KWW) score. The latter is essentially a measure of one’s grasp of the content of various occupations and their associated remuneration.\(^8\) Our proxy variable for parental permanent family income is the Duncan index of socioeconomic

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\(^8\)It could be argued that these ability variables are influenced by educational level and are therefore endogenous. This point is addressed in Griliches (1976) and Lazear (1977). Both of these studies attempt to purge IQ and KWW of the schooling influence through regression techniques. In our study we have maintained the presumption that IQ and KWW are exogenous. Any simultaneity between schooling and these ability measures would tend to bias the OLSQ estimator of \(\gamma_3\) in the schooling equation.
Status of the head of the family when the respondent was 14 years old.

We stratified our sample by employment-experience cohorts, in contrast to the potential labor force experience used in Mincer (1974). Because Mincer used census data, which have no information on previous employment history, he had to rely on current age minus the age upon leaving school as a proxy experience variable. This proxy suffers because potential experience exceeds actual experience by the length of time spent out of the labor force. And labor force experience exceeds employment experience by the length of time-spent unemployed but searching for employment. Fortunately, the NLS data provide information on employment histories. In order to control for the effects of on-the-job training on observed earnings, we stratify by actual employment experience, rather than by potential experience or by labor force experience.

An individual’s employment experience during any year covered by the NLS is calculated as the number of weeks employed over the year times the number of hours per week usually worked during the year, all divided by 2,080 (40 hours per week times 52 weeks per year). Cumulative employment experience over the period is simply the sum of the standardized years of employment experience. For those who left school before the beginning of the survey period, total employment experience is estimated as the ratio of in-period employment to potential full-time employment times the number of years between the termination of schooling and the end of the survey. Naturally, measured years of employment experience do not correspond to chronological time. One’s total hours employed since the termination of schooling are merely measured in terms of standardized years. This amounts to rescaling the unit of measure of employment experience.
Thus, for example, five standardized years of employment experience almost always corresponds to more than five years of elapsed chronological time since the termination of schooling.

Equation (9), the earnings function, was estimated for various experience cohorts using the KWW and IQ ability measures in alternative specifications. Our best results were obtained with the cohort corresponding to five to seven years of employment experience. Estimated coefficients were of the theoretically expected signs and were significant at conventional levels of statistical significance. Estimated residual variance achieved a local minimum for this cohort as evidenced by the standard errors of estimate reported in Table 1. Although the standard errors of estimate are lower for the last two cohorts in the sample, the regression coefficients generally lacked statistical significance and were of the wrong signs. Consequently, we confine our discussion to estimates of the schooling model for the cohort with five to seven years experience.

Tables 2, 3, and 4 report the estimated equations of the model. In Table 2 we report the earnings equations estimated with both the KWW and IQ ability proxies as well as the simple schooling model. Our estimated schooling-investment demand and supply functions are presented in Table 3. The estimated marginal rate of return to schooling evaluated at the mean is about 10 percent. Since our estimation procedure constrains the model to be in equilibrium, the estimated marginal rate of interest is also 10 percent at the mean. This compares with the 13 percent rate of return estimated from the simple model, equation (8). Mean years of schooling were 11.5. Our estimates of the interest elasticity of demand for schooling range from 0.47 to 0.27, indicating a relatively inelastic demand at the mean.
### TABLE 1

Standard Errors of Estimate for Equation (9)

<table>
<thead>
<tr>
<th>Years of Employment Experience</th>
<th>KWV</th>
<th>IQ</th>
<th>Sample Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>0.591</td>
<td>0.596</td>
<td>366</td>
</tr>
<tr>
<td>2-3</td>
<td>0.549</td>
<td>0.556</td>
<td>282</td>
</tr>
<tr>
<td>3-4</td>
<td>0.503</td>
<td>0.511</td>
<td>110</td>
</tr>
<tr>
<td>4-5</td>
<td>0.553</td>
<td>0.567</td>
<td>87</td>
</tr>
<tr>
<td>5-6</td>
<td>0.579</td>
<td>0.569</td>
<td>106</td>
</tr>
<tr>
<td>5-7</td>
<td>0.517</td>
<td>0.506</td>
<td>199</td>
</tr>
<tr>
<td>6-7</td>
<td>0.415</td>
<td>0.405</td>
<td>95</td>
</tr>
<tr>
<td>7-8</td>
<td>0.625</td>
<td>0.624</td>
<td>92</td>
</tr>
<tr>
<td>8-9</td>
<td>0.652</td>
<td>0.658</td>
<td>77</td>
</tr>
<tr>
<td>9-10</td>
<td>0.316</td>
<td>0.323</td>
<td>71</td>
</tr>
<tr>
<td>10-12</td>
<td>0.360</td>
<td>0.387</td>
<td>86</td>
</tr>
</tbody>
</table>
TABLE 2

Earnings Functions for Cohort with 5-7 Years Experience
(Independent variable: Natural log of 1969 earnings)

<table>
<thead>
<tr>
<th>Independent Variables</th>
<th>Equation (8)</th>
<th>Equation (9)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>kWV</td>
</tr>
<tr>
<td>S</td>
<td>0.1341</td>
<td>0.3091</td>
</tr>
<tr>
<td></td>
<td>(7.50)</td>
<td>(2.67)</td>
</tr>
<tr>
<td>S·KWW</td>
<td>........</td>
<td>0.0009</td>
</tr>
<tr>
<td></td>
<td>........</td>
<td>(1.86)</td>
</tr>
<tr>
<td>S·IQ</td>
<td>........</td>
<td>........</td>
</tr>
<tr>
<td></td>
<td>........</td>
<td>........</td>
</tr>
<tr>
<td>S²</td>
<td>........</td>
<td>-0.0101</td>
</tr>
<tr>
<td></td>
<td>........</td>
<td>(-1.93)</td>
</tr>
<tr>
<td>Constant</td>
<td>7.2381</td>
<td>6.2083</td>
</tr>
<tr>
<td></td>
<td>(34.75)</td>
<td>(9.60)</td>
</tr>
</tbody>
</table>

| R²                    | 0.22         | 0.25         | 0.28         |

* t values are in parentheses.
### TABLE 3

Schooling-Investment Demand and Supply Functions for Cohort with 5-7 Years Experience

(Independent variable: $\hat{r}$)

<table>
<thead>
<tr>
<th>Independent Variables</th>
<th>DEMAND, Equation (10)</th>
<th>SUPPLY, Equation (11)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>KWW</td>
<td>IQ</td>
</tr>
<tr>
<td>KWW</td>
<td>0.0009</td>
<td>......</td>
</tr>
<tr>
<td>IQ</td>
<td>......</td>
<td>0.0009</td>
</tr>
<tr>
<td>S</td>
<td>-0.0202</td>
<td>-0.0308</td>
</tr>
<tr>
<td>Duncan Index</td>
<td>......</td>
<td>......</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Family Size</td>
<td>......</td>
<td>......</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.3091</td>
<td>0.3593</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| $R^2$                  | ......      | ......    | 0.15       | 0.16      |
| $s^2_{\varepsilon}$    | ......      | ......    | 0.0013     | 0.0032    |
| $\frac{\bar{r}}{\bar{i}}$ | ......  | ......    | 0.1097     | 0.0945    |
| $\hat{n}_d$            | 0.47       | 0.27     | ......      | ......    |

$t$ values are in parentheses.

a. $s^2_{\varepsilon}$ = estimated residual variance.

b. $\bar{r}$ and $\bar{i}$ are the mean marginal rate of return and rate of interest, respectively.

c. $\hat{n}_d = -\left(\frac{\partial S}{\partial i}\right)\left(\frac{i}{S}\right) = -\left(0.5/b_3\right)\left(\frac{i}{S}\right)$. 


TABLE 4

Reduced-Form Schooling Equations
for Cohort with 5-7 Years Experience
(Dependent variable: Years of Schooling Completed)

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Equation (13), KWW</th>
<th>Equation (13), IQ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLSQ</td>
<td>Restricted</td>
</tr>
<tr>
<td>Duncan Index</td>
<td>0.0208</td>
<td>0.0248</td>
</tr>
<tr>
<td></td>
<td>(3.24)</td>
<td></td>
</tr>
<tr>
<td>Family Size</td>
<td>-0.2274</td>
<td>-0.2673</td>
</tr>
<tr>
<td></td>
<td>(-2.95)</td>
<td></td>
</tr>
<tr>
<td>KWW</td>
<td>0.1005</td>
<td>0.0446</td>
</tr>
<tr>
<td></td>
<td>(5.31)</td>
<td></td>
</tr>
<tr>
<td>IQ</td>
<td>.......</td>
<td>.......</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>8.0207</td>
<td>10.0396</td>
</tr>
<tr>
<td></td>
<td>(10.23)</td>
<td></td>
</tr>
</tbody>
</table>

\[
R^2 = 0.27 \quad \text{OLSQ} \quad \text{OLSQ} \\
\frac{s^2_{\text{a/\text{w}}}}{s^2_{\text{w}}} = 3.1798 \quad 3.1860 \quad 3.3273 \quad 3.3733
\]

t values are in parentheses.

a. \(s^2_{\text{w}} = \text{estimated residual variance.}\)
Table 4 presents the OLSQ and restricted estimates of the reduced-form schooling equation. These estimates are quite close for both the coefficients and residual variances.

The results of statistical tests for the restrictions on the schooling equation are presented in Table 5. Specifically, we are testing whether the OLSQ estimates of the coefficients of equation (13) and the maximum likelihood estimate of the residual variance of equation (13) are jointly significantly different from the values implied by the restrictions derived from equations (10)-(12). The hypothesis we wish to test regarding the parameters is simply stated as

\[
H_0: \gamma = \gamma^0, \sigma^2_w = \sigma^2_{w0}, \quad H_a: \gamma \neq \gamma^0, \sigma^2_w \neq \sigma^2_{w0}
\]

where \( \gamma \) and \( \gamma^0 \) are 4 x 1 vectors corresponding, respectively, to the true coefficient values and values implied by the restrictions of the model on equation (13), \( \sigma^2_w \) and \( \sigma^2_{w0} \) are the residual variances corresponding, respectively, to the true residual variance and the value implied by the restrictions on equation (13).

A likelihood-ratio test is used to perform a joint test of the model restrictions on the coefficients and residual variances. The test statistic is given by \(-2 \ln \lambda\) whose distribution is assumed to be approximated by a \( \chi^2_5 \) where \( \lambda \) is the likelihood ratio for the above hypothesis with 5 parameter restrictions. The calculated \( \chi^2 \) values are reported in Table 5. The test values for both the KnW and IQ versions of the schooling equation are less than the critical \( \chi^2 \) value. Consequently, we cannot reject at the five percent level the hypothesis that the restrictions of the model are satisfied for the five to seven year experience cohort.
TABLE 5
Tests for Restrictions on the Schooling Equation
(Equation (13) for Cohort with 5-7 Years Experience)

<table>
<thead>
<tr>
<th>Critical value $x^2_{0.05}$</th>
<th>$x^2$ test values</th>
<th>KWW</th>
<th>IQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2_{0.05} = 11.1$</td>
<td></td>
<td>8.86</td>
<td>2.36</td>
</tr>
</tbody>
</table>

V. Conclusion and Summary

We have explored the nature of the identification problem in the oft-cited simple schooling model of human capital theory. An alternative model which is identified and has testable implications was estimated for a sample of young male workers drawn from the National Longitudinal Survey data. Our initial findings have been encouraging; they yielded estimated coefficients with the theoretically expected signs that are statistically significant. Also, we found evidence to support certain restrictions implied by the model on the parameters and residual variance of the schooling equation.

There remains a great deal of room for experimentation with functional forms of the equations, as well as for incorporating additional variables into the model. With the specification of equations nonlinear in the parameters, more sophisticated estimation techniques can be employed. Also, as better data become available with respect to measured ability and permanent family income, continued estimation and testing may be justified.
For the present, it is hoped that we have made some contribution toward putting the empirical examination of schooling and earnings on a sounder methodological footing. In addition such an approach redirects attention toward the determinants of individual schooling levels and thus away from the estimation of rates of return that can become an obsession when adhering to the specification of the simple schooling model.
REFERENCES


