Econ 481, 482 prerequisite tools: review notes

Econ 339/Econ 276 tools

Let X be a normally distributed random variable such that $E(X) = u_x$ and $E(X - u_x)^2 = \sigma_x^2$ (variance).

Let $X_1, ..., X_T$ denote a random sample of size T.

Estimator of the mean of X: $\hat{u}_x = \bar{X} = \dfrac{\sum_{t=1}^{T} X_t}{T}$.

Unbiased estimator: $E(\bar{X}) = u_x$

Estimator of the variance: $\hat{\sigma}_x^2 = \dfrac{\sum_{t=1}^{T} \left(X_t - \bar{X}\right)^2}{T - 1}$

Standard error of $\bar{X}$: $\sigma_{\bar{x}} = \dfrac{\sigma_x}{\sqrt{T}}$

Estimated standard error of $\bar{X}$: $\hat{\sigma}_{\bar{x}} = \dfrac{\hat{\sigma}_x}{\sqrt{T}}$

t statistic: $\dfrac{\bar{X}}{\hat{\sigma}_{\bar{x}}} \sim t_{T-1}$

Hypothesis testing: H_0: $u_x = c$, H_1: $u_x \neq c$

$\left| \dfrac{\bar{X} - c}{\hat{\sigma}_{\bar{x}}} \right| > t_{T-1}^{0.975} \Rightarrow$ reject H_0 at the 5% level of significance for a two-tailed test

Math tools for Econ 361

$\ell n\,(zx) = \ell n(z) + \ell n(x)$ and $\ell n\left(\frac{z}{x}\right) = \ell n(z) - \ell n(x)$

Let $y = ax^{n}$, then $\frac{dy}{dx} = nax^{n-1}$

Let $y = a$, then $\frac{dy}{dx} = 0$

Let $y = a\ell n\,(x)$, then $\frac{dy}{dx} = \frac{a}{x}$

Let $y = zx$, then $\frac{dy}{dx} = z + x\left(\frac{dz}{dx}\right)$

Let $y = ax^{n}z^{b}$, then
$$\begin{aligned} dy &= \frac{\partial y}{\partial x}dx + \frac{\partial y}{\partial z}dz \\ &= nax^{n-1}z^{b}dx + bax^{n}z^{b-1}dz \end{aligned}$$

Econ 361 tools

Total revenue: $TR = PQ$

Average revenue: $AR = \frac{TR}{Q} = P$

Marginal revenue: $MR = \frac{d\,(TR)}{dQ} = P + Q\left(\frac{dP}{dQ}\right)$

Point elasticity of demand: $\eta_{qp} = \left(\frac{dQ}{dP}\right)\left(\frac{P}{Q}\right)$

Arc elasticity of demand: $\eta_{qp} = \left(\frac{\Delta Q}{\Delta P}\right)\left(\frac{P_1 + P_2}{Q_1 + Q_2}\right)$, where $\Delta Q = Q_2 - Q_1$ and $\Delta P = P_2 - P_1$.

Total Cost: $TC = C\,(Q)$

Average Cost: $AC = \frac{TC}{Q}$

Marginal Cost: $MC = \frac{d\,(TC)}{dQ}$

Production function: $Q = F(K, L)$

Average products of capital and labor: $AP_k = \frac{Q}{K}$, $AP_l = \frac{Q}{L}$

Marginal products of capital and labor: $MP_k = \frac{\partial Q}{\partial K}$, $MP_l = \frac{\partial Q}{\partial L}$

Marginal rate of technical substitution: $MRTS_{kl} = -\frac{dK}{dL} = \frac{MP_l}{MP_k}$

Elasticity of substitution: $\sigma_{kl} = \frac{\%\Delta\,(K/L)}{\%\Delta\,(w/r)}$, where w is the wage rate and r is the rental rate on capital.

Utility function: $U = G(x, y)$

Marginal utilities of x and y: $MU_x = \frac{\partial U}{\partial x}$, $MU_y = \frac{\partial U}{\partial y}$

Marginal rate of substitution: $MRS_{yx} = -\frac{dy}{dx} = \frac{MU_x}{MU_y}$

Utility function for n goods: $U = G(x_1, ..., x_n)$

Budget constraint: $y = p_1 x_1 + ... + p_n x_n = \sum_{i=1}^{n} p_i x_i$

Demand function for good 1: $x_1 = d_1(p_1, ..., p_n, y)$

Pure income effect on the demand for good 1: $\frac{\partial x_1}{\partial y} \gtreqless 0$

Slutsky equation for good 1:

$$\frac{\partial x_1}{\partial p_1} = \left(\frac{\partial x_1}{\partial p_1}\right)_{u=\bar{u}} - x_1^* \frac{\partial x_1}{\partial y}$$

where $\frac{\partial x_1}{\partial p_1} \gtreqless 0$ is the slope of the ordinary (uncompensated) demand curve, $\left(\frac{\partial x_1}{\partial p_1}\right)_{u=\bar{u}} < 0$ is the substitution effect (slope of the compensated demand curve), and $-x_1^* \frac{\partial x_1}{\partial y} \lesseqgtr 0$ is the income effect of the price change.