First-Price Auction

Ann's value is $34 **Bob's value is $32**

Ann's Bid \ Bob's Bid	$31	*$32*	$33	$34	$35	$36	$37
$31	0 or 3, 0 or 1	0 , 0	0 , -1	0 , -2	0 , -3	0 , -4	0 , -5
$32	2 , 0	0 or 2, 0 or 0	0 , -1	0 , -2	0 , -3	0 , -4	0 , -5
$33	1 , 0	1 , 0	0 or 1, 0 or -1	0 , -2	0 , -3	0 , -4	0 , -5
$34	0 , 0	0 , 0	0 , 0	0 or 0, 0 or -2	0 , -3	0 , -4	0 , -5
$35	-1 , 0	-1 , 0	-1 , 0	-1 , 0	0 or -1, 0 or -3	0 , -4	0 , -5
$36	-2 , 0	-2 , 0	-2 , 0	-2 , 0	-2 , 0	0 or -2, 0 or -4	0 , -5
$37	-3 , 0	-3 , 0	-3 , 0	-3 , 0	-3 , 0	-3 , 0	0 or -3, 0 or -5

Are any strategies dominated?

A bid equal to your value yields a net gain of $0 no matter what the other bidder does.

Therefore:

(a) Bidding your value dominates all higher bids.

(b) Bidding your value is dominated by all lower bids.

All bids below your value (except the lowest allowable bid) are undominated. Why?

On the following page, all bids above $34 have been eliminated, and some bids below $34 have been added.

First-Price Auction

Ann's value is $34 **Bob's value is $32**

Ann's Bid \ Bob's Bid	$28	$29	$30	$31	*$32*	$33
$28	0 or 6, 0 or 4	0 , 3	0 , 2	0 , 1	0 , 0	0 , -1
$29	5 , 0	0 or 5, 0 or 3	0 , 2	0 , 1	0 , 0	0 , -1
$30	4 , 0	4 , 0	0 or 4, 0 or 2	0 , 1	0 , 0	0 , -1
$31	3 , 0	3 , 0	3 , 0	0 or 3, 0 or 1	0 , 0	0 , -1
$32	2 , 0	2 , 0	2 , 0	2 , 0	0 or 2, 0 or 0	0 , -1
$33	1 , 0	1 , 0	1 , 0	1 , 0	1 , 0	0 or 1, 0 or -1

What are the players' Best Response functions?

Are there any Nash equilibria? There are two: ($32,$31) and ($33,$32), but the second one requires Bob to choose a dominated strategy/bid.

<u>Important:</u> **We've been assuming each bidder *knows* the other's value for the item.**

What if Ann is not so sure of Bob's value?
What if she thinks it could be anything from $29 to $32, for example?
If she is unsure of Bob's value, then she will be unsure how much he will bid.
And if she's unsure how much Bob will bid, she'll be unsure how much she should bid.

As soon as we recognize the players' uncertainties, or lack of information, about one another, we see that we'll need a deeper analytical approach for such situations.

Some Simple Games
that have
Multiple Nash Equilibria

Battle of the Sexes

A Coordination Game in Which
the Players Prefer Different Equilibria

Jennifer and Brad each buy tickets to an event for next Saturday night.

Jennifer prefers a Leo De Caprio movie, Brad prefers an XFL football game.

But in any case, each prefers to be with the other.

Brad's Choice

Jennifer's Choice	XFL	Leo
XFL	2 , 5	-1 , -1
Leo	0 , 5	5 , 2

There are two Nash equilibria:
(XFL,XFL) and (Leo,Leo).

A Pure CoordinationGame

Al and Bill have agreed to go out to a D.C. watering hole to talk about the great ride of the last eight years.

But they neglect to mention *which* watering hole -- the Hawk and Dove, the Congressional Record, or Eddie's?

Of course, they prefer to drink together than drink alone.

		Bill's Choice		
		HD	**CR**	**E**
Al's Choice	**HD**	**1 , 1**	**0 , 0**	**0 , 0**
	CR	**0 , 0**	**1 , 1**	**0 , 0**
	E	**0 , 0**	**0 , 0**	**1 , 1**

There are three Nash equilibria:

(HD,HD), (CR,CR), and (E,E).

A Chicken Game

Sam and Boris are driving down the road toward a head-on collision. Will one of them concede?

… or …

Boris has missiles in Cuba, Sam threatens a blockade. Will one of them concede?

Boris's Choice

Sam's Choice	Concede	Fight
Concede	**0 , 0**	**-1 , 10**
Fight	**10 , -1**	**-20 , -20**

There are two Nash equilibria:

(Concede,Fight) and (Fight,Concede).