Dutta's "Commons" Example in Chapter 7

The example has two features:

- A common resource – it's freely available to everyone
- The resource is "exhaustible" – any use depletes the total available in the future

There are two players, labeled $i = 1$ and $i = 2$.

There are y units of the resource, which will last for only two periods – say, two years.

Player i's "consumption stream" is denoted by (c_i, c_i')

Each person has a preference (indifference map) for two-year "consumption streams," represented by the utility function

$$u_i(c_i, c_i') = c_i c_i'. \qquad (*)$$

Note that Dutta uses instead the utility function $u_i(c_i, c_i') = \mathit{log}\ c_i + \mathit{log}\ c_i'$. That function has exactly the same indifference map as the function $u_i(c_i, c_i') = c_i c_i'$ -- in other words, a person represented by either one of these utility functions has the same preference. (Can you see why that's so?)

Each player knows at the outset that when the second period arrives the two players will divide equally whatever remains of the resource; i.e., whatever their choices c_1 and c_2 are at period 1, the resource remaining at period 2 will be $y – (c_1 + c_2)$, and they will each end up with one-half that amount:

$$c_1' = c_2' = ½ [y – (c_1 + c_2)].$$

Each player therefore knows what his payoff – i.e., his utility, according to the utility function (*) – will be as a function of just the profile (c_1, c_2) of the two players' consumption choices in the first period. For example, player 1's payoff will be

$$\pi_1(c_1, c_2) = u_1(c_1, c_1') = c_1c_1' = c_1(½)[y – (c_1 + c_2)] = ½ [(y – c_2)c_1 - c_1^2] = ½ (y – c_2)c_1 - ½ c_1^2$$

The derivative of player 1's payoff function is therefore $½ (y – c_2) - c_1$, and the derivative is zero when

$$\mathbf{c_1 = ½ (y – c_2) ,}$$ which is therefore player 1's best response function.

Similarly, player 2's best response function is

$$\mathbf{c_2 = ½ (y – c_1) .}$$

Solving the two best response function equations simultaneously yields the Nash equilibrium profile of choices,

$$\mathbf{(c_1 , c_2) = (1/3 y , 1/3 y) .}$$

The Welfare Optimum:

In order to determine the profile that maximizes joint welfare (utility), note that the two players are completely identical to one another, so we focus on the profiles in which they each take the same action and, more important, in which they each receive the same utility -- i.e., the actions that satisfy $c_1 = c_2$. In other words, we consider the profiles in which each player gets a total of 1/2 y to divide between consumption in periods 1 and 2. This is now a familiar consumer maximization problem from Intermediate Microeconomics:

Maximize $u_i(c_i, c_i') = c_i c_i'$ subject to the constraint $c_i + c_i' = 1/2\ y$.

The solution is $c_i = c_i' = 1/4\ y$.

In the Nash equilibrium, then, each person uses too much of the resource in the first period: 1/3 y instead of 1/4 y.

A More Specific, Numerical Version of the Example

Suppose that $y = 60$ in Dutta's example: there are 60 units of the resource available.

Then the players' reaction functions are

$$\mathbf{c_1 = 30 - \tfrac{1}{2} c_2 \quad \text{and} \quad c_2 = 30 - \tfrac{1}{2} c_1 .}$$

The Nash equilibrium is

$$\mathbf{(c_1 , c_2) = (20 , 20) ;}$$

each player uses 20 units of the resource in the first period, and consumes 10 units in the second period. Each player therefore obtains a utility level of

$$\mathbf{u = (20)(10) = 200 .}$$

But each could attain the maximum utility of $u = (15)(15) = 225$ by using only 15 units apiece in the first period, leaving 30 units to divide equally (15 units each) in the second period. Note that this is the solution of the individual maximization problem

Maximize $u_i(c_i, c_i') = c_i c_i'$ subject to the constraint $c_i + c_i' = 30$.