Overuse of a Common Resource:
A Two-player Example

- There are two fishermen who fish a common fishing ground – a lake, for example
- Each can choose either

  $x_i = 1$ (light fishing; for example, use one boat), or
  $x_i = 2$ (heavy fishing; for example, use two boats).

- Each player's payoff depends both on
  -- his own choice, x_i and
  -- the total amount of fishing that is done, i.e., $X = x_1 + x_2$
  (the more the lake is fished, the more the fish population is depleted)

Player i's payoff: $\boldsymbol{\pi_i(x_1, x_2) = (5 - X)\, x_i}$

- This yields the following payoff table:

	$x_2 = 1$	$x_2 = 2$
$x_1 = 1$	3 , 3	2 , 4
$x_1 = 2$	4 , 2	2 , 2

- For each player, the strategy $x_i = 1$ is weakly dominated by $x_i = 2$.
- The game is almost a Prisoners' Dilemma game, but not quite:
  The strategies that yield the good joint outcome are dominated for each player.
  But the domination is weak, not strong, as in a PD game.

In order to make the game comparable to the Common Resource game with *many* players that we're going to study momentarily, we'll change the example slightly: we assume that the payoff is only 1, instead of 2, to a player who chooses $x_i = 1$ when the other player chooses $x_i = 2$:

	$x_2 = 1$	$x_2 = 2$
$x_1 = 1$	3 , 3	1 , 4
$x_1 = 2$	4 , 1	2 , 2

This change makes it a Prisoners' Dilemma game. The strongly dominant strategy for each player is $x_i = 2$, which yields them only 2 payoff units apiece, not the 3 units they could receive if each chose $x_i = 1$.

We played this game in class, in two ways:

- First, each person in class chose either $x_i = 1$ or $x_i = 2$, with each person's payoff determined by a random matching into pairs.
- Second, two students were selected, and they played the game eight times in succession.

In-Class Games

Playing the game only once:

- Each student chose either $x_i = 1$ or $x_i = 2$.
- Students were randomly matched into pairs
- Each student's payoff was determined from the two players' choices
- 29 students chose $x_i = 2$,   5 students chose $x_i = 1$.
- One pair was selected, and one person in the pair was selected to receive her payoff, in dollars.
- When all subjects were paired (after class), the results were as follows:

  In 13 pairs, both players chose $x_i = 2$
  In 3 pairs, one player chose $x_i = 2$, the other chose $x_i = 1$
  In 1 pairs, both players chose $x_i = 1$

In-Class Games

Playing the game many times:

- Two students were selected, to play against one another many times in succession
- At the end, one of the players would be selected by a coin flip to be paid his accumulated payoffs, in quarters (i.e., each payoff unit was worth 25 cents)
- The results:

	1	2	3	4	5	6	7	8		
Bryan's Choices	1	1	2	2	2	2	2	2		
Stephen's Choices	1	2	1	1	2	1	1	2		
Bryan's Payoffs	3	1	4	4	2	4	4	2	24 total =	$6.00
Stephen's Payoffs	3	4	1	1	2	1	1	2	15 total =	$3.75

- Bryan and Stephen "cooperated" on only the first play.
- More often, in repeated PD games, we see cooperation in many periods.
- Here is a link to data from an in-class repeated PD game played in 1996:

  http://www.u.arizona.edu/~mwalker/Game4.htm

- Four of the pairs cooperated on almost every play. The other six pairs cooperated rarely, if at all.

A Common Resource Example with Many Players

- There are 40 players, about the same as the number of students in the course.
- We'll play this game in class shortly
- There are 100 units of a common resource.
- Each player can use either one unit of the resource ($x_i = 1$) or two units ($x_i = 2$).
- Use X to denote the total amount of the resource they use: $X = x_1 + x_2 + \ldots + x_{40}$
- A player's payoff π_i is determined as follows:

$$\pi_i = Rx_i \qquad \text{where } R = 100 - X, \text{ i.e., the unused amount of the resource}$$
$$= (100 - X)\, x_i$$
$$= (100 - S_{-i} - x_i)\, x_i$$
$$= (100 - S_{-i})x_i - x_i^2$$

- Thus, $\pi_i = 99 - S_{-i}$ if $x_i = 1$ and $\pi_i = 2\,(98 - S_{-i}) = 196 - 2S_{-i}$ if $x_i = 2$.
- It's easy to show from the above two payoff expressions that the strategy $x_i = 1$ is strongly dominated by the strategy $x_i = 2$, since S_{-i} cannot be larger than 78. It's also clear in the payoff table on the following page that the strategy $x_i = 1$ is strongly dominated by the strategy $x_i = 2$.

The Dominant-Strategy and Nash Equilibrium Prediction

- The following table gives a player's payoffs for any combination of
  - -- strategies chosen by the other players (summarized by the total they use, S_{-i})
  - -- a strategy chosen by the player in question (either $x_i = 1$ or $x_i = 2$).

Si	39	40	41	…	50	…	60	…	70	…	77	78
100 - Si	61	60	59	…	50	…	40	…	30	…	23	22
If xi = 1	60	59	58	…	49	…	39	…	29	…	22	21
If xi = 2	118	116	114	…	96	…	76	…	56	…	42	40

- The strategy $x_i = 1$ is strongly dominated by the strategy $x_i = 2$.
- Nash equilibrium (in dominant strategies):
  - -- Each player chooses $x_i = 2$.
  - -- $X = 80$ units of the resource are used, leaving $R = 20$ units unused.
  - -- Each player's payoff is $\pi_i = \$(20)(2) = \$40$.
  - -- Total profit is $\Pi = \$1600$.

The Pareto or Social Optimum

- Maximize $\Pi = \pi_1 + \pi_2 + \ldots + \pi_{40}$

  $= (100 - X)x_1 + (100 - X)x_2 + \ldots + (100 - X)x_{40}$

  $= (100 - X)\, X$

  $= 100X - X^2$

- The derivative is $\Pi' = 100 - 2X$
- The derivative is zero at $X = 50$
- The social optimum is therefore
  - -- 30 players choose $x_i = 1$
  - -- 10 players choose $x_i = 2$
- Maximum joint profit is P = (50)($50) = $2500.
- The Nash equilibrium (in dominant strategies) is a far worse outcome.
- With 40 players instead of only two, it is much less likely that cooperation can be sustained – i.e., we should probably expect most of the players to choose $x_i = 2$.

Externalities, Common Resources and Property Rights

When a fisherman chooses $x_i = 2$ instead of $x_i = 1$, this has an effect upon all the other fishermen. We call this effect an "externality." The institution of "common property" provides no mechanism by which this externality is brought to bear on the individual decision-maker – i.e., the marginal cost than an individual fisherman *does* bear is much less than the marginal cost his decision actually imposes on *all* the fishermen.

But suppose instead that someone – for example, one of the fishermen – *owns* the fishing ground, with exclusive rights to fish it and/or to sell (or give) the right to do some fishing to other fishermen. The owner will sell fishing rights (or do his own fishing) just to the level at which the fishing ground's profitability is maximized – a *socially efficient outcome* (if the market for fish is competitive).

Of course, if entry into the "fishing ground" market is not free, the owner may earn "rent" – above-normal profit. This suggests that we could auction off or tax the ownership rights.

Exercise: How do the Nash equilibrium and efficient outcomes change if x_i has a positive marginal cost – say, $\pi_i = (100 - X)x_i - cx_i$?