Economics 501B Final Exam
University of Arizona
Fall 2017

1. For each of the following propositions, state whether the proposition is true or false. If true, provide a proof (or at least indicate how a proof could be constructed). If false, provide a counterexample and verify that it is a valid counterexample.

(a) If each consumer's utility function is

(a1) continuous,

(a2) convex (*i.e.*, upper-contour sets are convex), and

(a3) weakly increasing (*i.e.*, if $\widetilde{x}_k \geqq x_k$ for each good k, then $u(\widetilde{\mathbf{x}}) \geqq u(\mathbf{x})$),

then any Walrasian equilibrium allocation is Pareto optimal.

(b) If every consumer has a lexicographic preference, then there is no Walrasian equilibrium.

2. When we have a parametric family of optimization problems $P(\theta)$ for parameter values θ in some set Θ of possible parameter values, we're usually interested in the solution function (or the solution correspondence) for the set $\{P(\theta) \,|\, \theta \in \Theta\}$, and we're often interested in the value function as well. An application of this idea arises in the concept of Pareto optimality. The simplest case is a "two by two exchange economy," where there are only two goods and only two consumers, production is not possible and there are no externalities — and where we use the Edgeworth box diagram to graphically depict some of our economic concepts. Assume that both consumers' preferences are representable by continuous, strictly increasing, strictly concave utility functions.

Give a careful explanation of how the following concepts are related to one another in such an "Edgeworth box" economy: the optimization problems $P(\theta)$ and what economic parameters are playing the role of theta; the solution function or correspondence; the Pareto allocations; the graph of the Pareto allocations in the box; the value function; and the utility frontier (also called the Pareto frontier). Why do we refer to the value function and never to the value correspondence?

3. Amy and Bev both have flower gardens. Their gardens are positioned in such a way that Amy can see Bev's garden as well as her own, and Amy therefore derives "utility" both from Bev's garden and her own garden. But Bev can't see Amy's garden, so she derives utility only from her own garden. Their preferences are represented by the utility functions

$$u^A(x_A, y_A, x_B) = y_A + 12x_A - \tfrac{1}{2}x_A^2 + 6x_B - \tfrac{1}{2}x_B^2 \quad \text{and}$$
$$u^B(x_B, y_B) = y_B + 8x_B - \tfrac{1}{2}x_B^2 \ ,$$

where y_i is i's consumption of dollars and x_i is the size of i's garden, in square meters. The cost of a garden is four dollars per square meter, and each woman is endowed with 100 dollars.

(a) Write down a maximization problem for which the solutions are the Pareto allocations.

(b) Derive the first-order conditions that characterize the solution(s) of the problem in (a).

(c) Determine the Pareto optimal allocations.

(d) Determine the utility (*i.e.*, Pareto) frontier.

(e) Express the first-order conditions in terms of marginal rates of substitution, and suggest prices and per-unit taxes or subsidies that would yield a Pareto allocation as an equilbrium if Amy and Bev are both price-takers, even if there is no way for Amy to purchase flowers for Bev's garden.

4. Two Manhattan pretzel vendors must decide where to locate their pretzel carts along a given block of Fifth Avenue, represented by the unit interval $I = [0,1] \subseteq \mathbb{R}$ — *i.e.*, each vendor chooses a location $x_i \in [0,1]$. The profit of each vendor i depends continuously on *both* vendors' locations — *i.e.*, the profit functions $\pi_i : I \times I \to \mathbb{R}$ are continuous for $i = 1, 2$. Furthermore, each π_i is concave (but not strictly concave) in x_i.

Define an equilibrium in this situation to be a joint action $\widehat{x} = (\widehat{x}_1, \widehat{x}_2) \in I^2$ that satisfies both

$$\forall x_1 \in I : \pi_1(\widehat{x}) \geqq \pi_1(x_1, \widehat{x}_2) \qquad \text{and} \qquad \forall x_2 \in I : \pi_2(\widehat{x}) \geqq \pi_2(\widehat{x}_1, x_2).$$

In other words, an equilibrium consists of a location for each vendor, with the property that each one's location is best for him given the other's location.

Prove that an equilibrium exists. If you're unable to prove this for the case in which each π_i is merely concave, assume they're both *strictly* concave in x_i and prove the result in that case.

5. In a two-period, one-good model where S is the set of possible states, one of which will occur after period zero and prior to period one, let $\mathbf{p} \in \mathbb{R}^S$ be a state-contingent price-list; let D be an $S \times K$ securities-returns matrix; and let $\mathbf{q} = \mathbf{p}D \in \mathbb{R}^K$. The following proposition appears in our lecture notes (where S denotes the number of states as well as the set of states):

Proposition: Let

$$A = \{(z_0, \mathbf{z}) \in \mathbb{R}^{1+S} \mid z_0 + \mathbf{p} \cdot \mathbf{z} = 0\} \text{ and}$$
$$B = \{(z_0, \mathbf{z}) \in \mathbb{R}^{1+S} \mid \exists \mathbf{y} \in \mathbb{R}^K : z_0 + \mathbf{q} \cdot \mathbf{y} = 0 \text{ and } \mathbf{z} = D\mathbf{y}\}.$$

If $rank\, D = S$, then $A = B$.

(a) The $(1 + S)$-tuples $(z_0, \mathbf{z})$ represent net consumption bundles and the K-tuples $\mathbf{y}$ represent holdings of securities. Describe what the proposition tells us in economic terms, and describe the role the proposition plays in establishing the relation between an Arrow-Debreu contingent-claims-markets equilibrium and an equilibrium in Arrow's model of securities and spot markets.

(b) Provide a proof of the proposition.