Economics 501B Midterm Exam
University of Arizona
Fall 2016

1. Here is an alternative duality theorem to the one we've stated and proved: If $u : \mathbb{R}^\ell_+ \to \mathbb{R}$ is a strictly quasiconcave function and $\widehat{\mathbf{x}}$ maximizes u on the set $\{\mathbf{x} \in \mathbb{R}^\ell_+ \,|\, \mathbf{p} \cdot \mathbf{x} \leqq \mathbf{p} \cdot \widehat{\mathbf{x}}\}$ then $\widehat{\mathbf{x}}$ minimizes $\mathbf{p} \cdot \mathbf{x}$ on the set $\{\mathbf{x} \in \mathbb{R}^\ell_+ \,|\, u(\mathbf{x}) \geqq u(\widehat{\mathbf{x}})\}$. Using this new duality theorem, provide a proof of this alternative version of the First Welfare Theorem, stated for only two consumers:

Assume that $u^i : \mathbb{R}^\ell_+ \to \mathbb{R}$ is strictly quasiconcave for $i = 1, 2$ and let $\widehat{\mathbf{p}} \in \mathbb{R}^\ell_+$, $\mathring{\mathbf{x}} = (\mathring{\mathbf{x}}^1, \mathring{\mathbf{x}}^2) \in \mathbb{R}^{2\ell}_+$, and $\widehat{\mathbf{x}} = (\widehat{\mathbf{x}}^1, \widehat{\mathbf{x}}^2) \in \mathbb{R}^{2\ell}_+$. If $(\widehat{\mathbf{p}}, \widehat{\mathbf{x}})$ is a Walrasian equilibrium for the economy $\big((u^1, \mathring{\mathbf{x}}^1), (u^2, \mathring{\mathbf{x}}^2)\big)$, then $\widehat{\mathbf{x}}$ is a Pareto allocation.

2. There are two goods, both of which are valued for consumption. Good Y can be produced by using good X as input. There are two firms producing Y; each can produce $2z$ units of Y by using z units of X. There also two consumers: each consumer owns one of the firms (she receives all the profit from the firm she owns), and each consumer's preference is described by the utility function $u(x, y) = xy$. Each consumer is endowed with 8 units of X and no Y. The firms purchase the X good from the consumers to use as input, and they sell their output to the consumers. Both firms and both consumers are price-takers in each market.

(a) Determine the competitive (Walrasian) equilibrium: the prices; the input z_j used and output q_j produced by each firm; each firm's revenue, cost, and profit; and the consumption bundle (x_i, y_i) of each consumer i.

(b) Write down the (P-max) problem for this economy and verify that the equilibrium is a solution of the problem.

3. There are two goods and three people in the economy, and all three people have the same utility function: $u(x, y) = xy$. Persons #1 and #2 are each endowed with the bundle $(12, 0)$, and Person #3 is endowed with the bundle $(0, 12)$. In each of the following cases determine whether the given allocation is in the economy's core. If it is, verify that it is; and if it's not, find an allocation with which some coalition can unilaterally make each of its members better off.

(a) $(x_1, y_1) = (8,2)$, $(x_2, y_2) = (8,2)$, $(x_3, y_3) = (8,8)$

(b) $(x_1, y_1) = (8,4)$, $(x_2, y_2) = (8,4)$, $(x_3, y_3) = (8,4)$

(c) $(x_1, y_1) = (4,2)$, $(x_2, y_2) = (6,3)$, $(x_3, y_3) = (14,7)$