Economics 501B Final Exam
University of Arizona
Fall 2016

1. Two firms (Firm 1 and Firm 2) each sell spring water, directly from the source: no matter how much they sell, the cost to them is zero. The market demand functions for their water are

$$q_1 = 60 - 2p_1 + p_2 \qquad \text{and} \qquad q_2 = 60 + p_1 - 2p_2,$$

where q_i denotes the number of gallons Firm i sells and p_i denotes the price (in dollars) Firm i charges for each gallon. Each firm chooses its price to maximize its profit (which, because costs are zero, is equivalent to its revenue).

(a) Assume that each firm takes its rival's price as given, and determine their Bertrand reaction functions, draw the reaction functions in a diagram, and determine the Bertrand equilibrium prices, quantities, and profits (revenues).

(b) Are the two firms' products identical, or are they differentiated from one another? Explain how you can tell whether they're identical or differentiated.

For the remainder of this problem it may be helpful to know that the following two matrices are inverses of one another, as you can easily check:

$$\begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix} \qquad \text{and} \qquad \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

(c) Now suppose that instead of taking its rival's price as given, each firm takes its rival's output as given. Determine the Cournot reaction functions, draw the reaction functions in a diagram, and determine the Cournot equilibrium quantities, prices, and profits (revenues).

(d) Now suppose that Firm 2 is charging $p_2 = \$24$ per gallon and is selling $q_2 = 36$ gallons. Assume that Firm 1 takes the quantity $q_2 = 36$ as given. Determine Firm 1's residual demand function, and draw its residual demand curve and its marginal revenue curve in a single diagram. Depict Firm 1's profit-maximizing decision in the diagram.

(e) Now assume that Firm 1 instead takes Firm 2's price $p_2 = \$24$ as given. Determine Firm 1's residual demand curve, and draw its residual demand curve and its marginal revenue curve in a single diagram. Depict Firm 1's profit-maximizing decision in the diagram.

2. Suppose we have a model of the economy in which there are only two periods, $t = 0$ and $t = 1$ ("today" and "tomorrow"), and in which there is only a single good, which we'll call simoleons (or you can call it dollars if you like). Consumers are endowed with some units of the good today, and will be endowed again with some units tomorrow, but their endowments tomorrow will depend (in a way that's known today) on which one of three states of the world will have occurred after today and before tomorrow: $s = H$, or $s = M$, or $s = L$. Let $S = \{H, M, L\}$. Consumers have differing state-dependent preferences over the space $\mathbb{R}^4_+$ of consumption bundles $\mathbf{x}^i = (x^i_0, x^i_H, x^i_M, x^i_L)$. No production or storage is possible. Assume that the unique Arrow-Debreu complete contingent claims equilibrium prices are

$$p_H = \frac{1}{8}, \quad p_M = \frac{1}{4}, \quad p_L = \frac{1}{4}.$$

In (a), (b), and (c), below, you're given three alternative sets of securities. The three components of a security d_k are the number d_{sk} of simoleons that a unit of the security will pay to the holder tomorrow in each of the three states; *i.e.*,

$$d_k = \begin{bmatrix} d_{kH} \\ d_{kM} \\ d_{kL} \end{bmatrix}.$$

In (a), (b), and (c) determine each of the following, if it's possible; if it's not, explain why not:

- The equilibrium prices ψ_k of each of the securities.
- The equilibrium interest rate.
- How many units y_k of each of the securities a consumer would need to hold in order to ensure that she will receive the state-dependent payout $(z_H, z_M, z_L) = (1, 2, 2)$.
- How much it will cost her today to ensure she will receive $(1, 2, 2)$ tomorrow.
- If for one of the securities market structures there is more than one list $\mathbf{y}$ of holdings y_k that will achieve the payout $(1, 2, 2)$, indicate one of the additional vectors $\mathbf{y}$ that will attain $(1, 2, 2)$ and determine how much that will cost the consumer today.

(a) $\quad d_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $d_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

(b) $\quad d_1 = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}, \quad d_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad d_3 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$.

(c) $\quad d_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad d_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad d_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad d_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

3. Suppose there are two consumers and that $\left(\widehat{\mathbf{p}}, (\widehat{\mathbf{x}}^1, \widehat{\mathbf{x}}^2)\right) \in \mathbb{R}^\ell_{++} \times \mathbb{R}^{2\ell}_+$ is a Walrasian equilibrium for preferences $\succsim^1$ and $\succsim^2$ on $\mathbb{R}^\ell_+$ and endowments $\mathring{\mathbf{x}}^1$ and $\mathring{\mathbf{x}}^2$ in $\mathbb{R}^\ell_+$. Assuming only that each preference is a complete and locally nonsatiated (LNS) preorder of $\mathbb{R}^\ell_+$, prove that the allocation $(\widehat{\mathbf{x}}^1, \widehat{\mathbf{x}}^2)$ is in the core. (Note that the preferences might not be differentiable, or continuous, or quasiconcave, etc., and they might not be representable by utility functions.) Before giving your proof, give the definition of a Walrasian equilibrium and the definition of the core for this two-consumer economy. If you find it easier to assume there are only two goods, it's OK to assume that.