Economics 501B Final Exam
University of Arizona
Fall 2014

1. Provide a proof of the following theorem: Let $\left(\widehat{p}, (\widehat{x}^i)_{i\in N}\right)$ be a Walrasian equilibrium for an economy $\left((u^i, \mathring{x}^i)\right)_{i\in N}$. If each u^i is locally nonsatiated, then $(\widehat{x}^i)_{i\in N}$ is a Pareto optimal allocation for the economy $\left((u^i, \mathring{x}^i)\right)_{i\in N}$. [**Note:** You may use either duality theorem from consumer theory without providing a proof.]

2. Amy, Bev, and Cat care only about consuming electricity and simoleans. Moreover, Amy wishes to consume electricity only in the morning, Bev wishes to consume electricity only in the afternoon, and Cat wishes to consume electricity only at night. Their marginal rates of substitution between simoleans and electricity consumption are given by the expressions

$$MRS_A = 10 - x_A \qquad MRS_B = 8 - x_B \qquad MRS_C = 6 - x_C,$$

where x_i denotes the number of electricity-units i consumes (at her preferred time of day only). Electricity is produced by a constant-returns-to-scale technology: 18 simoleans will produce one unit of electricity *throughout the day and night* — *i.e.*, morning, afternoon, and nighttime electricity are joint products, and producing a unit at any time of day automatically yields a unit at other times of day as well. (For example, 36 simoleans produces two units of electricity throughout the day.) Each person's consumption of electricity is monitored via an electricity meter.

(a) Determine a Pareto optimal production level and allocation of electricity usage.

(b) Assume that electricity is produced by the town. The town wishes to use time-of-day pricing (*i.e.*, charging different prices at different times of day) to maximize consumer welfare. Of course, the town will also have to cover the cost of production. What prices should it charge at each time of day? Explain how these prices can be said to maximize welfare.

(c) Now suppose electricity is produced instead by a monopoly, which is required by law to charge a uniform price for electricity, the same price at all times of day. Determine the firm's profit-maximizing price and production level.

(d) Now suppose the monopoly is allowed to use time-of-day pricing. The firm still wishes to maximize its profit. Will the firm charge prices that lead to uniform consumption across all times of day, or not? Explain why your answer is the correct one.

(e) What prices will the firm charge if it uses time-of-day pricing to maximize its profit, as in (d)?

3. There are two goods (denote their quantities by x and y) and two consumers, Amy and Beth. No production is possible. Each consumer has the same preference, which is representable by the utility function $u(x, y) = \sqrt{x} + \sqrt{y}$. Note that the MRS function for each consumer is $\sqrt{y/x}$. Amy owns four units of the y-good but none of the x-good; Beth owns four units of the x-good but none of the y-good.

(a) Determine the Pareto frontier for this two-person economy.

(b) Determine the core.

4. Ann chooses according to the utility function

$$u^A(x_0, x_H, x_L) = x_0 + x_H - \frac{1}{60}x_H^2 + x_L - \frac{1}{20}x_L^2$$

and Beth chooses according to the utility function

$$u^B(x_0, x_H, x_L) = x_0 + x_H - \frac{1}{60}x_H^2 + 3x_L - \frac{1}{20}x_L^2$$

where

x_0 represents consumption "today,"
x_H represents consumption "tomorrow" in event H, and
x_L represents consumption "tomorrow" in event L.

Storage of the consumption good from today until tomorrow is not possible. Each person is endowed with 30 units of the good today; Ann will have 5 units tomorrow in state H and 25 units in state L; Beth will have 35 units in state H and 5 units in state L.

(a) Determine the Pareto optimal allocations.

(b) Determine the Arrow-Debreu equilibrium — *i.e.*, the Arrow-Debreu prices and allocation.

(c) Suppose there are two securities: a bond, each unit of which pays the holder $1 + r$ consumption units tomorrow whichever state occurs; and an insurance contract against state L, which pays the holder one consumption unit tomorrow if and only if state L occurs, and nothing if state H occurs. The interest rate r is determined in equilibrium. Use Arrow's security-pricing formula to determine the equilibrium interest rate r and the price p of each unit of the insurance contract. How many units of each security will each person buy or sell in equilibrium?

(d) Suppose that the only market is a credit market (*i.e.*, a market for borrowing and lending). There are no markets in which one can insure oneself against either of tomorrow's two possible events. Explain why this will generally yield a market outcome that is not Pareto efficient.