Using Game Theory to Analyze Auctions

A look at the data from our in-class auctions:

The data can be found here: http://www.u.arizona.edu/~mwalker/431AuctionResults.pdf

Note that in each auction the set of values is identical, except for a constant increment added to or subtracted from each. The values for the English auction were drawn randomly from the numbers between \$35 and \$45, and then the values for each other auction were obtained by subtracting \$5, or \$10, or \$15 from each of the English auction values. This enables us to compare the four auctions as if they were conducted in exactly the same setting.

- An unusual result in the English auction: the high-value person "left money on the table." He or she had a value of \$44 and allowed the item to go for \$41.25. Normally, we would see the high-value person outbid the second-high-value person at \$42, and thereby obtain the item for \$42, the second highest value.
- In the graph of the two floppy disk sealed-bid auctions, we see that for any given value, the person with that value in the second-price auction almost always bid more than the corresponding person in the first-price auction (after taking account of the \$10 increment between values in the two auctions). First-price bids were almost all below bidders' values, second-price bids were often at or above value.
- Quite a few bids in the pennies auction were well above the actual value of the item.

Review of our In-Class Auctions

There were six auctions:

#1 English oral auction for floppy disk

#2 Dutch (descending) auction for floppy disk

#3 First-price sealed-bid auction for floppy disk

#4 Second-price sealed-bid auction for floppy disk

#5 First-price sealed-bid auction for bottle of pennies

#6 English oral auction for bottle of pennies

What features did they have in common, and what features differed from one to another?

Can we "classify" the auctions in any useful way?

Classifying Auctions

- In some auctions bids were sequential, and a bidder could bid more than one time
- In some the bids were simultaneous, and a bidder could bid only once

- In some a bidder had seen others' bids when he made his bid(s)
- In others a bidder had to bid with no information about the other bidders

- In some the bidders had differing values, each knew his own value but not others' values
- In some the value was the same to each bidder, but no one knew the actual ("true") value

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The Auction Rules: (The first two pairs of bullets above)

The conductor of the auction can choose the format (the rules).

It's as if he's the "designer."

The Auction Environment: (The last pair of bullets above)

The conductor of the auction has to take this as given.

But the environment might influence what rules (i.e., "design") he wants to use.

Auction Environments

- Common Values

  The value is the same to each bidder, but no one knows the actual ("true") value

- Private Values

  The bidders have differing values, each knows his own value but not others' values

Examples:

- Oil, gas, timber leases: Mostly common value
- A painting: Both elements are generally present, one might predominate
- Procurement, construction: Both are important, and private value component often uncertain
- U of A b-ball ticket:

  If resale allowed: Common value

  If resale not allowed: Private value

Analyzing the Auctions

- As always, we want to begin with the simplest possible situation.
- The oral auctions have a temporal structure: bids are made sequentially.
  It therefore seems like we'll need to model them as extensive form games.
- In the sealed-bid auctions, bids are simultaneous, each bidder makes just one bid.
  We can therefore model them as strategic form games.
- We'll find that the private values case is easier than the common values case, and that the second-price auction is easier than the first-price auction. (The reasons for this are not obvious up front.)

So we begin by trying to model and analyze the second-price sealed-bid auction.

Second-Price Auction

We start with a very simple, specific example: Two bidders, private values, we specify each bidder's value, and we assume a much smaller strategy set for each bidder than they would really have.

We're choosing a sealed-bid auction as the first one to analyze because that's an auction in which each bidder makes only one bid, and the bids are simultaneous -- so we can model the auction as a strategic-form game.

Ann's value is $34

Bob's value is $32

Bob's Bid

Ann's Bid	**$31**	***$32***	**$33**	**$34**	**$35**	**$36**	**$37**
$31							
$32							
$33							
$34							
$35							
$36							
$37							

Second-Price Auction

Ann's value is $34 **Bob's value is $32**

Bob's Bid

Ann's Bid	$31	*$32*	$33	$34	$35	$36	$37
$31	0 or 3, 0 or 1	0 , 1	0 , 1	0 , 1	0 , 1	0 , 1	0 , 1
$32	3 , 0	0 or 2, 0 or 0	0 , 0	0 , 0	0 , 0	0 , 0	0 , 0
$33	3 , 0	2 , 0	0 or 1, 0 or -1	0 , -1	0 , -1	0 , -1	0 , -1
$34	3 , 0	2 , 0	1 , 0	0 or 0, 0 or -2	0 , -2	0 , -2	0 , -2
$35	3 , 0	2 , 0	1 , 0	0 , 0	0 or -1, 0 or -3	0 , -3	0 , -3
$36	3 , 0	2 , 0	1 , 0	0 , 0	-1 , 0	0 or -2, 0 or -4	0 , -4
$37	3 , 0	2 , 0	1 , 0	0 , 0	-1 , 0	-2 , 0	0 or -3, 0 or -5

We could replace the "0 or 3" type entries, when the two bids are the same, by (for example) the expected value of the player's profit. If the tie-breaking rule is to flip a coin, for example, then we would replace "0 or 3" by 1 1/2, and so on. It's probably more instructive for our purposes here, though, to leave these "tie" payoffs as they're shown above.

Let's determine whether any strategies are dominated.

<u>Important:</u> **Dominance doesn't consider others' payoffs, only my own.**
But IEDS and Nash equilibrium <u>do</u> consider others' payoffs.

Ann's Bids: Are any of Them Dominated?

We further simplify, by cutting down Bob's set of strategies.
And we write only Ann's payoffs, since Bob's payoffs are irrelevant for determining dominance among Ann's strategies.

Bob's Bid

Ann's Bid	$32	$33	$34	$35	$36
$31	0 ,	0 ,	0 ,	0 ,	0 ,
$32	0 or 2,	0 ,	0 ,	0 ,	0 ,
$33	2 ,	0 or 1,	0 ,	0 ,	0 ,
$34	2 ,	1 ,	0 or 0,	0 ,	0 ,
$35	2 ,	1 ,	0 ,	0 or -1,	0 ,
$36	2 ,	1 ,	0 ,	-1 ,	0 or -2,
$37	2 ,	1 ,	0 ,	-1 ,	-2 ,

A bid above $34 is never better than $34. When is it worse?

A bid below $34 is never better than $34. When is it worse?

Ann's Bids: Are any of Them Dominated?

Here we've put more strategies into the game for Bob.

Bob's Bid

Ann's Bid	$31	$32	$33	$34	$35	$36	$37
$31	0 or 3,	0 ,	0 ,	0 ,	0 ,	0 ,	0 ,
$32	3 ,	0 or 2,	0 ,	0 ,	0 ,	0 ,	0 ,
$33	3 ,	2 ,	0 or 1,	0 ,	0 ,	0 ,	0 ,
$34	3 ,	2 ,	1 ,	0 or 0,	0 ,	0 ,	0 ,
$35	3 ,	2 ,	1 ,	0 ,	0 or -1,	0 ,	0 ,
$36	3 ,	2 ,	1 ,	0 ,	-1 ,	0 or -2,	0 ,
$37	3 ,	2 ,	1 ,	0 ,	-1 ,	-2 ,	0 or -3,

You should be able to determine that
(a) bids above her value ($34) are weakly dominated by bidding her value, $34; why?
(b) bids below her value are also weakly dominated by bidding her value; why?

Consequently, it is a dominant strategy for Ann to bid her true value: she can never do better with any other bid. Of course, the same argument works for Bob … or for any bidder.

It should be clear that the arguments here will give the same result no matter how many strategies we include for either Ann or Bob: Bidding your value in the 2nd-price auction is a dominant strategy.