In this section, the *Journal of Economic Education* publishes articles, notes, and communications describing innovations in pedagogy, hardware, materials, and methods for treating traditional subject matter. Issues involving the way economics is taught are emphasized.

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Abstract: The author presents a simple technique for teaching the Cournot model to first-year students. The approach involves demonstrating to the students that out of all rectangles with a common perimeter, the square has the greatest area. No use is made of derivatives. The same approach can be used to understand some other market forms.

Key words: Cournot model, derivatives, first-year students, rectangle method, teaching
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Many professors of first-year microeconomics avoid the Cournot model on the grounds that the students are uncomfortable using derivatives to find the firms' reaction functions. This is unfortunate in that the Cournot model has a very intuitive and illuminating outcome, which is intermediate to the polar cases of monopoly and perfect competition.

My purpose in this article is to explain an alternative technique for presenting the Cournot model that does not make use of derivatives. The key idea is to convince the students that out of all rectangles with a common perimeter, the square has the greatest area. Once this is recognized, the firms' reaction functions can easily be found. A similar technique can also be used to understand certain other

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market forms (Stackelberg, Bertrand with differentiated products, monopoly; see, e.g., Pindyck and Rubinfeld [1998] for a textbook presentation of these models).[1]

THE RECTANGLE METHOD

In this section, I explain the basic idea. I phrase the presentation in terms of a concrete example, because, in my experience, this is what first-year students like most.

Problem: Two firms compete in a market for a homogenous good, simultaneously choosing what quantities Q_1 and Q_2 to produce. There are no fixed costs of production, but there is a given variable cost of 3 per unit produced. The market price, P, automatically adjusts to clear the market, which happens at $P = 15 - Q_1 - Q_2$. A Cournot equilibrium, or equivalently a Nash equilibrium of this game, is a pair of quantities (Q_1^*, Q_2^*) such that each firm's choice of quantity maximizes its profit given the quantity choice of the other firm. Find all such equilibria.

Answer: Consider first the choice problem of firm 1. In equilibrium, this firm will choose a best response to its competitor's choice of quantity. Hence, given firm 2's equilibrium quantity, Q_2^*, firm 1 will choose Q_1 to maximize its profit $Q_1 (P - 3) = Q_1 (12 - Q_1 - Q_2^*)$. Note that this profit is the product of two numbers: Q_1 and $12 - Q_1 - Q_2^*$. Hence, the firm's profit corresponds to the area of the rectangle in Figure 1, where the lengths of the sides are as indicated.

The problem is to find the value of Q_1 (given Q_2^*) that maximizes the area of the rectangle. Note that if Q_1 is varied, the rectangle changes shape but its perimeter is kept constant at $2(12 - Q_2^*)$. Many students no doubt begin to see the answer now. To make it clear, to them, draw the set of rectangles illustrated in Figure 2, all with a given perimeter of 16.

The respective areas are 7, 12, 15, and 16. The square has the largest area. This is indicative of the following geometric truth: *Out of all rectangles with a given perimeter, the square has the greatest area.* In fact, a simple and illuminating

FIGURE 1
Firm 1's Profit

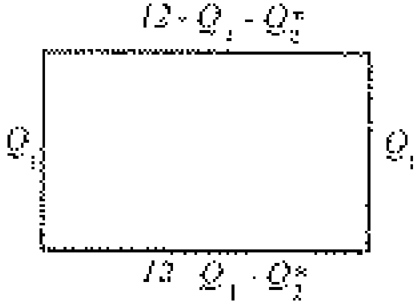


FIGURE 2
Equiperimetric Rectangles with Different Areas

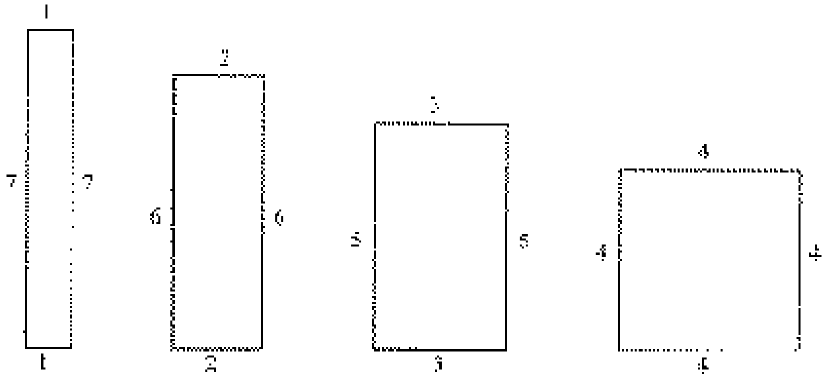
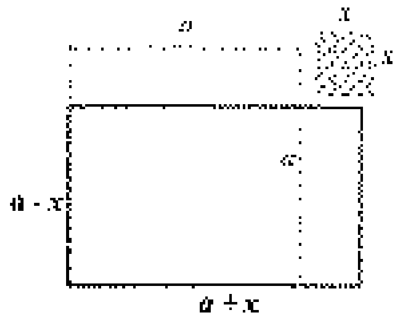


FIGURE 3
Geometric/Algebraic Proof



geometric/algebraic proof of this proposition may be supplemented[2]: For any given rectangle, let a be the average length of the sides, and let x be such that the height and width of the rectangle are $a - x$ and $a + x$, respectively (see Figure 3 for an illustration).

The perimeter of the rectangle in Figure 3 is $4a$ while the area is $(a - x)(a + x) = a^2 - x^2$. Now consider the effect of varying x, thereby creating new rectangles that still have the same perimeter $4a$. The closer to zero x is, the larger is the area. The area reaches a maximum of a^2 when $x = 0$, in which case the rectangle is, in fact, a square as indicated by the dotted lines. The difference in area between this square and the original rectangle is x^2, which corresponds to the area of the checkered elevated small square in the upper right corner of Figure 3.

Applying this process to the rectangle in Figure 1, one sees that firm 1's optimal choice of Q_1 given Q_2*, that is, Q_1*, satisfies $Q_1^* = 12 - Q_1^* - Q_2^*$. In an analogous way, one finds that firm 2's optimal choice of Q_2 given Q_1*, that is, Q_2*, satisfies $Q_2^* = 12 - Q_1^* - Q_2^*$. These two equations can be rearranged as

$$Q_1^* = 6 - 1/2Q_2^*. \qquad (1)$$

$$Q_2^* = 6 - 1/2Q_1^*. \qquad (2)$$

Solving equations (1) and (2) simultaneously yields the unique equilibrium in which $Q_1^* = Q_2^* = 4$ and where each firm makes a profit of $4(12 - 4 - 4) = 16$. Figure 2 was drawn so as to anticipate this solution. Note that equations (1) and (2) actually describe the firms' reaction functions that are usually derived using derivatives, but that no derivatives were used here.

VARIATIONS

With proper adjustments one can readily handle more general cases of Cournot competition, as well as some other market forms. A successful application of the rectangle method may, however, require some manipulations. To illustrate the typical reason, consider what happens if the preceding example is changed so that the market clearing price is determined as $P = 15 - k(Q_1 - Q_2)$, for some k π 1. *Mutatis mutandis,* firm 1's optimum choice of Q_1 given Q_2, maximizes firm 1's profit $Q_1(12 - kQ_1 - kQ_2)$. Describing the associated rectangle one detects a problem: as Q_1 changes, one does not stay within the class of rectangles with a given perimeter. Because of this, it is no longer true that the square has the greatest area.[3] The problem is that the coefficient in front of the Q_1 terms is of different magnitude for the two factors of the product. However, this is easily fixed by rewriting the firm's profit as $kQ_1(12/k - Q_1 - Q_2)$. It is now clear that the firm should choose Q_1 to maximize $Q_1(12/k - Q_1 - Q_2)$, or equivalently, the size of a rectangle with sides of lengths Q_1 and $12/k - Q_1 - Q_2$. Note that as Q_1 changes, the perimeter of the associated rectangles remains constant.

This manipulation technique comes in handy when one solves the Stackelberg model, which differs from Cournot's model only in that the firms move in sequence—first firm 1 (the leader) chooses Q_1, then firm 2 (the follower) chooses Q_2, after observing the leader's choice of Q_1. Starting with the parameterization described in the last section, the leader still estimates his profit as $Q_1(12 - Q_1 - Q_2)$ but substitutes the follower's reaction function $Q_2^* = 6 - 1/2Q_1$ (confer equation 2) for the choice of Q_2. This yields the leader's profit as $Q_1\{([12 - Q_1 - \{6 - 1/2Q_1\}] = Q_1(6 - 1/2Q_1)\}$. However, variations in Q_1 now generate rectangles of unequal perimeter, so we must rearrange the leader's profit as $1/2Q_1(12 - Q_1)$. This profit is proportional to the area $Q_1(12 - Q_1)$. The rectangle method is applicable and shows that the leader's equilibrium quantity will be $Q_1^* = 6$. Plugging this value into the follower's reaction function, we get $Q_2^* = 6 - 1/2Q_1^* = 6 - 6/2 = 3$, and one readily calculates the players' equilibrium profits as 18 and 9, respectively. An analogous technique can be used to analyze Bertrand competition with differentiated products, an exercise that traditionally makes use of derivatives.

Finally, it should be pointed out that the rectangle method may be useful also for analyzing monopoly markets. The usual approach is to let the students find the profit maximizing quantity at the point where the marginal revenue curve crosses the marginal cost curve. To many students, this approach is somewhat murky, because finding the marginal revenue curve typically requires taking a derivative. The rectangle method finesses all this, noting instead that the firm's profit can be written as a simple product.

CONCLUDING REMARK

The approach described here may be usefully combined with some other pedagogical tools for presenting the Cournot model in class. Fulton (1997) and Sarkar, Gupta, and Pal (1998) developed highly illuminating graphical techniques. Avoiding derivatives is not the objective of these authors, and the rectangle method can be meaningfully incorporated in the frameworks they consider.[4]

NOTES

1. The method has been successfully tried with 400 first-semester economics students at Stockholm University in the spring semester of 1999 (by the author and several teaching assistants).
2. I am grateful to a referee for suggesting that I include a proof along these lines.
3. The students may be skeptical of this claim. If this happens, give them an example. Consider the case at hand, where the rectangle's height is Q_1, and its width is $12 - kQ_1 - kQ_2$. Suppose that k = 1/2 and that $Q_2 = 0$. In this case, the rectangle's height is Q_1, and its width is $12 - 1/2Q_1$. If the rectangle is a square it holds that $Q_1 = 12 - 1/2Q_1$, or $Q_1 = 8$, in which case the area is 82 = 64. However, this is not the maximal rectangle. It makes sense to increase Q_1, intuitively speaking, because the height of the associated rectangles grows faster than the width decreases. For example, with $Q_1 = 12$, we get the area 12(12 – 12/2) = 72 > 64 (this is, in fact, the rectangle with the maximal area).
4. For example, using the rectangle method, one can calculate the first-order condition (1) of Sarkar, Gupta, and Pal (1998) without taking a derivative.

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